



srivenkateshwaraa
College of Engineering & Technology
(Approved by AICTE, New Delhi & Affiliated to Pondicherry University, Puducherry)
13-A, Pondy - Villupuram Main Road, Ariyur, Puducherry - 605 102.

ASPIRE TO EXCEL



DEPARTMENT OF MECHANICAL ENGINEERING

SUBJECT NOTES

SUB NAME: DESIGN OF TRANSMISSION SYSTEM

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**PREPARED BY,
Mr. M.NATARAJ., B.E., M.E., MISTE.(Ph.D)
ASSISTANT PROFESSOR
SVCET**

CHAPTER 1

1. V- BELT

Selection of V belts and pulleys

- ✓ Determine your drive requirements. How much power do you need to transmit and at what speed?

INTRODUCTION:

V- Belts are one type of flexible connectors for transmitting power from one pulley to another pulley. Whose center distance is approx. 3M. Their cross section is trapezoidal. The belts are operated on groove pulleys.

MATERIALS USED:

- Cord
- Fabric
- Cotton
- Rayon

POWER TRANSMISSION

Belts are the cheapest utility for power transmission between shafts that may not be axially aligned. Power transmission is achieved by specially designed belts and pulleys. The demands on a belt drive transmission system are large and this has led to many variations on the theme. They run smoothly and with little noise, and cushion motor and bearings against load changes, albeit with less strength than gears or chains. However, improvements in belt engineering allow use of belts in systems that only formerly allowed chains or gears.

Power transmitted between a belt and a pulley is expressed as the product of difference of tension and belt velocity

$$P = (T_1 - T_2) v$$

where, T_1 and T_2 are tensions in the tight side and slack side of the belt respectively. They are related as:

$$\frac{T_1}{T_2} = e^{\mu\theta/\sin(\alpha/2)}$$

TYPES OF V BELT

Generally V belts are classified into various grades based on their power transmitting capacity as A, B, C, D and E. the cross sectional areas are increased order from A –E

SELECTION OF V BELTS AND PULLEYS

V belts are designed based on

1. Fundamental formula
2. Manufactures catalogues

FUNDAMENTAL FORMULA:

1. Ratio of driving tensions

$$\frac{T_1}{T_2} = e^{\mu\theta/\sin(\alpha/2)}$$

Where T_1, T_2 are tensions.

θ = Angle of contact in radians

α = angle

2. Power Transmitted by belt

$$P = (T_1 - T_2)v \lambda^{-n/s}$$

MANUFACTURES CATALOGUES

1. At first based on amount of power to be transmitted , select the type of belt
2. Calculate design power

$$\text{Design power} = \frac{\text{rated power} \times \text{service factor}}{\text{arc factor} \times \text{belt pitch length factor}}$$

3. Pitch length

$$L = 2C + \frac{\pi}{2}(D+d) + \frac{(D-d)^2}{4C}$$

4. Note inside length

5. Determine Belt rating

6. Design no of belts = $\frac{\text{Design power}}{\text{Belt rating}}$

7. Correct the center Length

8. Also determine parameters of V groove pulleys using Manufactures data

Selection of Flat belts and pulleys

INTRTODUCTION

Flat Belts are one type of flexible connectors for transmitting power from one pulley to another pulley. Whose center distance is approx. 5-15m

CHARACTERISTICS OF BELT:

Belt Material	Ultimate Strength	Endurance limit	Factor of safety	Maximum velocity	density	width	thickn ess	$\frac{d}{t}$	Modulus of elasticity
leather	20-35	4	8-10	20-40	9.8	20-300	3-7.5	25-35	100-350
Rubber	35-45	8	6-10	20-30	12.5-15	20-500	2.5-14	30-40	80-120
Cotton	35-40	8	6-10	15-25	7.5-15	30-250	4.5-8.5	25-40	30-60
wollen	30	6	8-10	20-30	9-12.5	50-500	2-10	25-30	-----

TYPES OF FLAT BELT DRIVES:

1. Open belt drive
2. Cross belt drive

3. Quarter turn drive
4. Belt drive with idler pulley
5. Belt drive with many pulleys

DESIGN OF FLAT BELTS

1. Using fundamental formula
2. Using Manufactures catalogues

FUNDAMENTAL FORMULA:

3. Ratio of driving tensions

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

Where T_1, T_2 are tensions.

$\theta = \text{Angle of contact in radians}$

4. Power Transmitted by belt

$$P = (T_1 - T_2)v \text{ N-m/s}$$

MANUFACTURES CATALOGUES

1. How much power to be transmitted
2. What may be the power transmitting capacity

For determining the design power and belt rating, we must consider certain factors like service, arc of contact and so on.

i. Arc of contact

$$\text{Open belt drive } \theta = 180^\circ - \left(\frac{E-d}{c}\right) 60^\circ$$

$$\text{Cross belt drive } \theta = 180^\circ + \left(\frac{E+d}{c}\right) 60^\circ$$

ii. Load rating

The load ratings have been developed for 180° of arc of contact 10m/s belt speed per mm width.

iii. Length of belt

$$\text{Open belt drive } L = 2C + \frac{\pi}{2}(D+d) + \frac{(D-d)^2}{4C}$$

$$\text{Cross belt drive } L = 2C + \frac{\pi}{2}(D+d) + \frac{(D-d)^2}{4C}$$

iv. Belt tensions

1. Belt of 3 plies – 1.5% of L
2. Belt of 4,5,6 plies – 1% of L
3. Belt of 8 plies – 0.5% of L

v. Pulley width

Generally the pulleys should be slightly wider than belt width.

POWER TRANSMISSION

Belts are the cheapest utility for power transmission between shafts that may not be axially aligned. Power transmission is achieved by specially designed belts and pulleys. The demands on a belt drive transmission system are large and this has led to many variations on the theme. They run smoothly and with little noise, and cushion motor and bearings against load changes, albeit with less strength than gears or chains. However, improvements in belt engineering allow use of belts in systems that only formerly allowed chains or gears. Power transmitted between a belt and a pulley is expressed as the product of difference of tension and belt velocity

$$P = (T_1 - T_2) v$$

where, T_1 and T_2 are tensions in the tight side and slack side of the belt respectively. They are related as:

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

DESIGN PROCEDURE

1. From the given conditions like power, type of working conditions, diameters of pulleys, speed ratio etc, determine maximum power
Desin power = rated power x service factor x arc of contact factor
Select service factor based on nature of load and applications from PSG data book
2. Decide the type of belt
3. Then calculate the belt rating
4. Find the required width by design power by belt capacity and adopt the standard available
5. Determine the length of belt based on type of drive and reduce certain amount length
6. Find out the pulley dimension and draw the arrangement of belt drive.

Wire ropes and pulleys

SELECTION PROCEDURE

1. Based on the given data like nature of application, duty etc, select the type of rope
2. Estimate the design load by multiplying the dead weight by three times design factor.
3. Determine the net cross sectional area of the rope by choosing specific strength of wire.

$$A = \frac{P_d}{\sigma_u}$$

4. Find out the diameter of rope
5. Select the next standard dia of rope and note down the max breaking strength
6. Compute the load applied at normal working acceleration and starting etc. find out the actual factor of safety by dividing the breaking strength by above loads
7. For safe design the actual factor of safety should not be less than 5 at any circumstances
8. Then calculate the drum and pulley dimensions.

Selection of Transmission chains and Sprockets

SELECTION PROCEDURE

1. Depending upon the amount amount of power to be transmitted and another working conditions such as available space, chain speed, position of chain drive etc
2. Assuming the center distance between the chaun sprockets interms of pitches
3. Calculate the developed load for breaking the chain using expression as
4. For determining pitch, choose suitable chain from PSG Data Book
5. Find out the actual factor of safety
6. Determine the induced stress over the projected area of the chain using the relation as

$$\sigma = \frac{pk_s}{Av} N/mm^2$$

7. Find the length of chain and provide allowance for initial sagging.
8. Evaluate the pitch diameter of pinion sprocket (d_1) and wheel sprocket (d_2)

$$d_1 = \frac{p}{\sin \frac{180^\circ}{z_1}}$$

$$d_2 = \frac{p}{\sin \frac{180^\circ}{z_2}}$$

9. Draw a neat sketch of chain drive with calculated specifications.

SOLVED PROBLEMS

1. Design a V – Belt drive to the following specifications

Power transmitted = 75kw

Speed of driving wheel = 1440rpm

Speed of driven wheel = 400rpm

Diameter of driving wheel = 300mm

Center distance = 2500mm

Service = 16hrs/day

Solution

For the given power of 75 kw D type or E type belts are suited. Let us selected D type belt.

$$\text{Design power} = \frac{\text{rated power} \times \text{service factor}}{\text{arc factor} \times \text{belt pitch length factor}}$$

Service factor = 1.5 (for heavy duty and 16 hrs/ day with ac motor high torque)

Pitch length

$$L = 2C + \frac{\pi}{2}(D+d) + \frac{(L-d)^2}{4C}$$

Now = 300 mm

$$C = 2500\text{mm}$$

The next standard pitch length = 7648mm

Corresponding inside length = 7569mm

Length factor = 1.05

$$\begin{aligned} \text{Arc of contact } \theta &= 180^\circ - \frac{(L-d)}{c} \cdot 60^\circ \\ &= 161.3^\circ \end{aligned}$$

Arc of contact factor = 0.955

$$\text{Now design factor} = \frac{7.5 \times 1.5}{1.05 \times 0.955} = 112.0 \text{ kw}$$

Power rating for D type belt

$$= (3.22 S^{-0.009} - \frac{505.7}{d_e}) 4.78 \times 10^{-4} S^2$$

$$d_e = d_p \lambda F_b$$

Belt capacity = 15.96 kw

$$\begin{aligned} \text{Number of belts required} &= \frac{\text{design power}}{\text{Belt rating}} \\ &= 112 / 14.3 = 7.82 \end{aligned}$$

Total no of belts = 8

$$\text{Center distance} = A + \sqrt{A^2 - B}$$

$$\text{Where } A = \frac{L}{4} - \frac{\pi}{8} (D + d)$$

$$B = \frac{(D + d)^2}{8}$$

$$C = 2712 \text{ mm}$$

Initial Tension = 0.75%L

Final center distance = 2788 mm

Specification

Type of belt = D7569 50 IS294

Number of belts required = 8

Pitch diameter of small pulley = 1080mm

Center distance = 2788mm

2. Design a Flat belt drive to transmit 25 kw at 720 rpm to an aluminium rolling machine the speed reduction being 3.0. The distance between the shaft is 3m. Diameter of rolling machine pulley is 1.2m.

Solution:

Given

C = 3.0m

Diameter of larger pulley $D_1 = 1.2\text{m}$

Diameter of smaller pulley $D_2 = 0.4\text{m}$

Torque Transmitted = $P \times 1000 / \frac{2\pi N}{60}$

$$25 \times 1000 \times 60 / 2\pi \times 720 = 331.57 \text{ Nm}$$

Let T_1 and T_2 are the tensions on tight and slack sides respectively

$$T_1 - T_2 = 1657.86 \text{ N}$$

Coefficient of friction from PSG data Book

$$\mu = 0.2 \text{ for fabric belt on pulleys}$$

Angle of lap = $\theta = 2.86 \text{ rad}$

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

Using eqn :

$$T_1 = 3810.9 \text{ N}$$

From the table width of the pulley = width of belt + 40

$$= 305 + 40$$

$$= 345 \text{ mm}$$

$$\text{Length of the belt } L = 2C + \frac{\pi}{2}(D+d) + \frac{(L-d)^2}{4C}$$

$$L = 8566.6 \text{ mm}$$

Specification of the belt drive are

Dia of motor pulley are = 400mm

Dia of rolling machine pulley = 1200mm

Center distance = 30000 mm

Width of belt = 305mm

Width of the belt = 345 mm

Width of pulleys = 345mm

3. A roller chain is to be used on a paving machine to transmit 30 hp from the 4-cylinder Diesel engine to a counter-shaft; engine speed 1000 rpm, counter-shaft speed 500 rpm. The center distance is fixed at 24 in. The chain will be subjected to intermittent overloads of 100%. (a) Determine the pitch and the number of chains required to transmit this power. (b) What is the length of the chain required? How much slack must be allowed in order to have a whole number of pitches? A chain drive with significant slack and subjected to impulsive loading should have an idler sprocket against the slack strand. If it were possible to change the speed ratio slightly, it might be possible to have a chain with no appreciable slack. (c) How much is the bearing pressure between the roller and pin?

Solution:

(a) *design hp* = 2(30) = 60 *hp* intermittent

$$\frac{D_2}{D_1} \approx \frac{n_1}{n_2} = \frac{1000}{500} = 2$$

$$D_2 = 2D_1$$

$$C = D_2 + \frac{D_1}{2} = 24 \text{ in}$$

$$2D_1 + \frac{D_1}{2} = 24$$

$$D_{1\max} = 9.6 \text{ in}$$

$$D_{2\max} = 2D_{1\max} = 2(9.6) = 19.2 \text{ in}$$

$$v_m = \frac{\pi D_1 n_1}{12} = \frac{\pi(9.6)(1000)}{12} = 2513 \text{ fpm}$$

Table 17.8, use Chain No. 35, Limiting Speed = 2800 fpm

Minimum number of teeth

$$\text{Assume } N_1 = 21$$

$$N_2 = 2N_1 = 42$$

[Roller-Bushing Impact]

$$hp = K_r \left(\frac{100N}{n} \right)^{1.5} P^{0.8}$$

Chain No. 35

$$P = \frac{3}{8} \text{ in}$$

$$N_{ts} = 21$$

$$n = 1000 \text{ rpm}$$

$$K_r = 29$$

$$hp = 29 \left[\frac{100(21)}{1000} \right]^{1.5} \left(\frac{3}{8} \right)^{0.8} = 40.3 \text{ hp}$$

[Link Plate Fatigue]

$$hp = 0.004 N_{ts}^{1.08} n^{0.9} P^{3-0.07 P}$$

$$hp = 0.004(21)^{1.08}(1000)^{0.9} \left(\frac{3}{8} \right)^{3-0.07} \left(\frac{3}{8} \right) = 2.91 \text{ hp}$$

$$\text{No. of strands} = \frac{\text{design hp}}{\text{rated hp}} = \frac{60}{2.91} = 21$$

Use Chain No. 35, $P = \frac{3}{8} \text{ in}$, 21 strands

Check for diameter and velocity

$$D_1 = \frac{P}{\sin\left(\frac{180}{N_t}\right)} = \frac{0.375}{\sin\left(\frac{180}{21}\right)} = 2.516 \text{ in}$$

$$v_m = \frac{\pi D_1 n_1}{12} = \frac{\pi(2.516)(1000)}{12} = 659 \text{ fpm}$$

Therefore, we can use higher size,

Max. pitch

$$P = D_1 \sin\left(\frac{180}{N_t}\right) = 9.6 \sin\left(\frac{180}{21}\right) = 1.43 \text{ in}$$

say Chain no. 80

$P = 1 \text{ in}$

$$hp = 0.004(21)^{1.08}(1000)^{0.9}(1)^{3-0.07(1)} = 53.7 \text{ hp}$$

A single-strand is underdesign, two strands will give almost twice over design,

Try Chain no. 60

$$P = \frac{3}{4} \text{ in}$$

$$hp = 0.004(21)^{1.08}(1000)^{0.9} \left(\frac{3}{4} \right)^{3-0.07} \left(\frac{3}{4} \right) = 23 \text{ hp}$$

$$\frac{\text{design hp}}{\text{rated hp}} = \frac{60}{23} = 2.61 \text{ or 3 strands}$$

$$D_1 = \frac{P}{\sin\left(\frac{180}{N_t}\right)} = \frac{0.75}{\sin\left(\frac{180}{21}\right)} = 5.0 \text{ in}$$

$$v_m = \frac{\pi D_1 n_1}{12} = \frac{\pi(5.0)(1000)}{12} = 1309 \text{ fpm}$$

The answer is Chain No. 60, with $P = \frac{3}{4} \text{ in}$ and 3 chains, limiting velocity is 1800 fpm

$$(b) L \approx 2C + \frac{1}{2} \frac{N_1 + N_2}{2} + \frac{2}{40C} \frac{1}{(N_1 - N_2)^2} \text{ pitches}$$

$$C = \frac{24}{2} = 32$$

$$N_1 = 21$$

$$N_2 = 42$$

$$L = 2(32) + \frac{21+42}{2} + \frac{(42-21)^2}{40(32)} = 95.845 \text{ pitches} \approx 96 \text{ pitches}$$

Amount of slack

$$h = 0.433(S^2 - L^2)^{\frac{1}{2}}$$

$$L = C = 24 \text{ in}$$

$$S = 24 \text{ in} + \frac{(96 - 95.845) \left(\frac{3}{4} \text{ in} \right)}{4} = 24.058 \text{ in}$$

$$h = 0.433 \left[(24.058)^2 - (24)^2 \right]^{\frac{1}{2}} = 0.7229 \text{ in}$$

(c) p_b = bearing pressure

Table 17.8, Chain No. 60

$$C = 0.234 \text{ in}$$

$$E = \frac{1}{2} \text{ in}$$

$$J = 0.094 \text{ in}$$

$$A = C(E + 2J) = 0.234 \left[\frac{1}{2} + 2(0.094) \right] = 0.160992 \text{ in}^2$$

$$\frac{FV}{33,000} = 60 \text{ hp}$$

$$\frac{F(1309)}{33,000} = 60 \text{ hp}$$

$$F = 1512.6 \text{ lb}$$

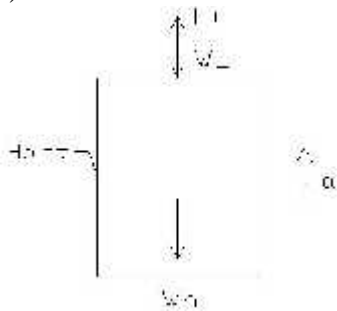
$$F = \frac{1512.6}{3} = 504.2 \text{ lb/strand}$$

$$p_b = \frac{504.2}{0.160992} = 3131 \text{ psi}$$

4. In a coal-mine hoist, the weight of the cage and load is 20 kips; the shaft is 400 ft. deep. The cage is accelerated from rest to 1600 fpm in 6 sec. A single 6 x 19, IPS, 1 3/4-in. rope is used, wound on an 8-ft. drum. (a) Include the inertia force but take the static view and compute the factor of safety with and without allowances for the bending load. (b) If $N = 1.35$, based on fatigue, what is the expected life? (c) Let the cage be at the bottom of the shaft and ignore the effect of the rope's weight. A load of 14 kips is gradually applied on the 6-kip cage. How much is the deflection of the cable due to the load and the additional energy absorbed? (d) For educational purposes and for a load of $0.2F_u$, compute the energy that this 400-ft rope can absorb and compare it with that for a 400-ft., 1 3/4-in., as-rolled-1045 steel rod. Omit the weights of the rope and rod. What is the energy per pound of material in each case?

Solution:

(a)



$$a = \frac{v_2 - v_1}{t} = \frac{(1600 \text{ fpm}) \left(\frac{1 \text{ min}}{60 \text{ sec}} \right)}{6 \text{ sec}} = 4.445 \text{ fps}^2$$

$$W_h = 20 \text{ kips}$$

For 6 x 19 IPS,

$$wL = 1.6 D_r^2 \left(\frac{400}{1000} \right) \text{ kips} = 0.64 D_r^2 \text{ kips}$$

$$F_t - wL - W_h = ma$$

$$20 + 0.64 D_r^2$$

$$m = \frac{32.2}{2} r$$

$$F_t - 0.64 D_r^2 - 20 = \left(\frac{20 + 0.64 D_r^2}{32.2} r \right) (4.445)$$

$$F_t = 22.76 + 0.73 D_r^2$$

$$D_r = 1 \frac{3}{4} \text{ in}$$

$$F_t = 22.76 + 0.73 \left(1 \frac{3}{4} \right)^2 = 25 \text{ kips}$$

$$N = \frac{F_u - F_b}{F_t}$$

Table AT 28, IPS

$$F_u \approx 42 D_r^2 \text{ tons}$$

$$F_u = 42 (1.75)^2 = 129 \text{ tons} = 258 \text{ kips}$$

with bending load

$$F_b = s_b A_m$$

$$s_b = \frac{E D_w}{D_s}$$

$$F_b = \frac{E A_m D_w}{D_s}$$

Table At 28, 6 x 19 Wire Rope

$$D_w = 0.067 D_r = 0.067(1.75) = 0.11725 \text{ in}$$

$$D_s = 8 \text{ ft} = 96 \text{ in}$$

$$E = 30,000 \text{ ksi}$$

$$A_m \approx 0.4 D_r^2$$

$$A_m = 0.4(1.75)^2 = 1.225 \text{ sq in}$$

$$F_b = \frac{(30,000)(1.225)(0.11725)}{(96)} = 45 \text{ kips}$$

$$N = \frac{F_u - F_b}{F_t} = \frac{258 - 45}{25} = 8.52$$

without bending load

$$N = \frac{F_u}{F_t} = \frac{258}{25} = 10.32$$

(b) $N = 1.35$ on fatigue

$$\text{IPS, } s_u \approx 260 \text{ ksi}$$

$$D_r D_s = \frac{2 N F_u s_u}{(p/s_u)} \\ (1.75)(96) = \frac{2(1.35)(25)}{(p/s_u)(260)}$$

$$p/s_u = 0.0015$$

Fig. 17.30, 6 x 19 IPS

Number of bends to failure = 7×10^5

$$(c) \delta = \frac{FL}{A_m E_r}$$

$$A_m = 1.225 \text{ sq in}$$

$$E_r \approx 12,000 \text{ ksi (6 x 19 IPS)}$$

$$F = 14 \text{ kips}$$

$$L = 400 \text{ ft} = 4800 \text{ in}$$

$$\delta = \frac{(14)(4800)}{(1.225)(12,000)} = 4.57 \text{ in}$$

$$U = \frac{1}{2} F \delta = \frac{1}{2} (14)(4.57) = 32 \text{ in-kips}$$

(d) $F = 0.2 F_u = 0.2(258) = 51.6 \text{ kips}$

$$\delta = \frac{FL}{A_m E_r}$$

$$\delta = \frac{(51.6)(4800)}{(1.225)(12,000)} = 16.85 \text{ in}$$

$$U = \frac{1}{2} F \delta = \frac{1}{2} (51.6)(16.85) = 434 \text{ in-kips}$$

For 1 3/4 in, as-rolled 1045 steel rod

$$s_u = 96 \text{ ksi}$$
$$F_u = s_u A = (96) \left(\frac{\pi}{4} (1.75)^2 \right) = 230.9 \text{ kips}$$

$$F = 0.2 F_u = 0.2 (230.9) = 46.2 \text{ kips}$$

$$\delta = \frac{FL}{AE}$$

$$\delta = \frac{(46.2)(4800)}{\left(\frac{\pi}{4} (1.75)^2 \right) (30,000)} = 3.073 \text{ in}$$

$$U = \frac{1}{2} F \delta = \frac{1}{2} (46.2)(3.073) = 71 \text{ in-kips} < U \text{ of wire rope.}$$

CHAPTER 2 SPUR GEARS AND HELICAL GEARS

Gear Terminology

The following terms, which will be mostly used in this chapter, should be clearly understood at this stage.

1. **Pitch circle.** It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear
2. **Pitch circle diameter.** It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also called as pitch diameter.
3. **Pitch point.** It is a common point of contact between two pitch circles.
4. **Pitch surface.** It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.
5. **Pressure angle or angle of obliquity.** It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by ϕ . The standard pressure angles are $14\frac{1}{2}^\circ$ and 20° .
6. **Addendum.** It is the radial distance of a tooth from the pitch circle to the top of the tooth.
7. **Dedendum.** It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.
8. **Addendum circle.** It is the circle drawn through the top of the teeth and is concentric with the pitch circle.
9. **Dedendum circle.** It is the circle drawn through the bottom of the teeth. It is also called root circle.
Note : Root circle diameter = Pitch circle diameter $\times \cos \phi$, where ϕ is the pressure angle.
10. **Circular pitch.** It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by p_c .
Mathematically,

$$\text{Circular pitch, } p_c = \pi D/T$$

Where, D = Diameter of the pitch circle, and
 T = Number of teeth on the wheel.

A little consideration will show that the two gears will mesh together correctly, if the two wheels have the same circular pitch.

Note : If D_1 and D_2 are the diameters of the two meshing gears having the teeth T_1 and T_2 respectively; then for them to mesh correctly,

$$p_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} = \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

11. **Diametral pitch.** It is the ratio of number of teeth to the pitch circle diameter in millimetres. It denoted by pd. Mathematically,
Diametral pitch,

$$pd = \frac{T}{D} = \frac{\pi}{p_c}$$

where T = Number of teeth, and
 D = Pitch circle diameter.

12. **Module.** It is the ratio of the pitch circle diameter in millimetres to the number of teeth. It is usually denoted by m. Mathematically,

$$\text{Module, } m = D / T$$

Note : The recommended series of modules in Indian Standard are 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40 and 50.

The modules 1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36 and 45 are of second choice.

13. **Clearance.** It is the radial distance from the top of the tooth to the bottom of the tooth, in a meshing gear. A circle passing through the top of the meshing gear is known as clearance circle.

14. **Total depth.** It is the radial distance between the addendum and the dedendum circle of a gear. It is equal to the sum of the addendum and dedendum.

15. **Working depth.** It is radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.

16. **Tooth thickness.** It is the width of the tooth measured along the pitch circle.

17. **Tooth space.** It is the width of space between the two adjacent teeth measured along the pitch circle.

18. **Backlash.** It is the difference between the tooth space and the tooth thickness, as measured on the pitch circle.

19. **Face of the tooth.** It is surface of the tooth above the pitch surface.

20. **Top land.** It is the surface of the top of the tooth.

21. **Flank of the tooth.** It is the surface of the tooth below the pitch surface.

22. **Face width.** It is the width of the gear tooth measured parallel to its axis.

23. **Profile.** It is the curve formed by the face and flank of the tooth.

24. **Fillet radius.** It is the radius that connects the root circle to the profile of the tooth.

25. **Path of contact.** It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.

26. **Length of the path of contact.** It is the length of the common normal cut-off by the addendum circles of the wheel and pinion.

27. **Arc of contact.** It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. The arc of contact consists of two parts, i.e.

(a) **Arc of approach.** It is the portion of the path of contact from the beginning of the engagement to the pitch point.

(b) **Arc of recess.** It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.

Note: The ratio of the length of arc of contact to the circular pitch is known as contact ratio i.e. number of pairs of teeth in contact.

Speed ratio

Speed ratios and

Number of teeth

It is the ratio of speed of driving gear to the speed of the driven gear.

$$i = \frac{N_A}{N_B} = \frac{Z_B}{Z_A}$$

where, N_A and N_B = speed of the driver and driven respectively, and
 Z_A and Z_B = Number of teeth on driver and driven respectively.

Force analysis

1. Tangential component:

The tangential component F_t is a useful component. Because it transmit power. Using the value of F_t , the magnitude of torque and transmitted power can be determined.

$$\text{Transmitted power } W_t = F_t$$

2. Radial component:

The radial component F_r is a separating force which is always directed towards the centre of the gear. F_r does not really a useful component. It is also called as transverse force or bending force.

Let P = Power transmitted (W)

M_t = Torque transmitted (N-m)

N_1 and N_2 = Speed of pinion and gear respectively

d_1 and d_2 = Pitch circle diameters of pinion and gear respectively

ϕ = Pressure angle.

The torque transmitted by gear

$$M_t = \frac{60 \times P}{2\pi N}$$

DESIGN PROCEDURE FOR SPUR GEAR:

1. Calculation of gear ratio (i):

$$i = \frac{N_A}{N_B} = \frac{Z_B}{Z_A}$$

where, N_A and N_B = speed of the driver and driven respectively, and
 Z_A and Z_B = Number of teeth on driver and driven respectively.

2. Selection of material

Consulting Table 5.3, knowing the gear ratio i , choose the suitable material.

3. If not given, assume gear life (say 20000 hrs)

4. Calculation of initial design torque:

$$[M_t] = M_t \cdot K \cdot K_d$$

where, $[M_t]$ = transmission torque
 K = Load factor, Table 5.11
 K_d = Dynamic load factor, Table 5.12

Assume $K \cdot K_d = 1.3$ (if not given)

5. Calculation of E_{eq} , $[\sigma_b]$ and $[\sigma_c]$:

- ✓ From table 5.20 Calculate E_{eq}
- ✓ From table 5.16 Calculate Design bending stress $[\sigma_b]$
- ✓ Calculate Design contact stress $[\sigma_c]$ by

$$[\sigma_c] = C_B \cdot HB \cdot K_{cl} \text{ (or)}$$

$$[\sigma_c] = C_R \cdot HRC \cdot K_{cl}$$

where, C_B C_R = Coefficient of surface hardness from table 5.18

HB HRC = Hardness number

6. Calculation of centre distance (a):

$$a \geq (i + 1) \sqrt{\left(\frac{.74}{[c]}\right)^2 \times \frac{Eeq [Mt]}{i\varphi}}$$

$$\varphi = b/a \text{ from table 5.21}$$

7. Select number of teeth on gear and pinion:

- On pinion, $Z_1 = \text{Assume } 18$
- On gear, $Z_2 = i \times Z_1$

8. Calculation of module:

$$m = \frac{2a}{(z_1 + z_2)}$$

Choose standard module from table 5.8

9. Revision of centre distance (m):

$$a = \frac{n(z_1 + z_2)}{2}$$

10. Calculate b, d₁, v and ψ_p :

- ✓ Calculate face width, $b = \psi \cdot a$
- ✓ Calculate pitch dia, $d = m \cdot z_1$
- ✓ Calculate pitch line velocity, $v = (\pi d_1 N_1) / 60$
- ✓ Calculate value of $\psi_p = b / d_1$

11. Selection of quality of gear:

Knowing the pitch line velocity and consulting table 5.22, select a suitable quality of gear.

12. Revision of design torque [M_t]:

Revise K:

Using the calculated value of ψ_p revise the K value by using table 5.11

Revise K_d:

Using the selected quality of gear and pitch line velocity, revise the K_d value

$$[M_t] = M_t \cdot K \cdot K_d$$

13. Check for bending:

Calculate induced bending stress,

$$\sigma_b = \frac{(i \pm 1)}{am b Y} [Mt]$$

- ✓ Compare σ_b and $[\sigma_b]$.
- ✓ If $\sigma_b \leq [\sigma_b]$, then design is safe.

14. Check for wear strength:

Calculate induced contact stress,

$$\sigma_c = 0.74 \frac{(i \pm 1)}{a} \sqrt{\frac{(i \pm 1)}{ib} Eeq} [Mt]$$

- ✓ Compare σ_c and $[\sigma_c]$.
- ✓ If $\sigma_c \leq [\sigma_c]$, then design is safe.

15. If the design is not satisfactory ($\sigma_b > [\sigma_b]$ and / or $\sigma_c > [\sigma_c]$), then increase the module of face width value of the gear material.

16. Check for gear:**a. Check for bending:**

Using $\sigma_{b1} y_1$ and $\sigma_{t1} y_1$.

$$\sigma_{b2} = \frac{t_1 y_1}{y_2}$$

- ✓ Compare σ_{b2} and $[\sigma_{b2}]$.
- ✓ If $\sigma_{b2} \leq [\sigma_{b2}]$, then design is safe.

b. Check for wear strength:

Calculate induced contact stress will be same for pinion and gear,

So,

$$\sigma_{c2} = \sigma_c$$

- ✓ Compare σ_c and $[\sigma_c]$
- ✓ If $\sigma_c \leq [\sigma_c]$, then design is safe

DESIGN PROCEDURE FOR HELICAL GEAR:

1. Calculation of gear ratio (i):

$$i = \frac{N_A}{N_B} = \frac{Z_B}{Z_A}$$

where, N_A and N_B = speed of the driver and driven respectively, and
 Z_A and Z_B = Number of teeth on driver and driven respectively.

2. Selection of material

Consulting Table 5.3, knowing the gear ratio i , choose the suitable material.

3. If not given, assume gear life (say 20000 hrs)

4. Calculation of initial design torque:

$$[M_t] = M_t \cdot K \cdot K_d$$

where, $[M_t]$ = transmission torque
 K = Load factor, Table 5.11
 K_d = Dynamic load factor, Table 5.12

Assume $K \cdot K_d = 1.3$ (if not given)

5. Calculation of E_{eq} , $[\sigma_b]$ and $[\sigma_c]$:

- ✓ From table 5.20 Calculate E_{eq}
- ✓ From table 5.16 Calculate Design bending stress $[\sigma_b]$
- ✓ Calculate Design contact stress $[\sigma_c]$ by

$$[\sigma_c] = C_B \cdot HB \cdot K_{cl} \text{ (or)}$$

$$[\sigma_c] = C_R \cdot HRC \cdot K_{cl}$$

C_B C_R = Coefficient of surface hardness from table 5.18

HB HRC = Hardness number

6. Calculation of centre distance (a):

$$a \geq (i + 1) \sqrt{\left(\frac{.74}{[c]}\right)^2 \times \frac{Eeq [Mt]}{i\varphi}}$$

$$\varphi = b/a \text{ from table 5.21}$$

7. Select number of teeth on gear and pinion:

- On pinion, $Z_1 = \text{Assume} \geq 17$
- On gear, $Z_2 = i \times Z_1$

8. Calculation of module:

$$m = \frac{2a}{(z_1 + z_2)} \times \cos \beta$$

Choose standard module from table 5.8

9. Revision of centre distance (m):

$$a = \frac{m}{\cos \beta} \frac{(z_1 + z_2)}{2}$$

10. Calculate b, d₁, v and ψ_p :

- ✓ Calculate face width, $b = \psi \cdot a$
- ✓ Calculate pitch dia, $d = (m_n \cdot z_1) / \cos \beta$
- ✓ Calculate pitch line velocity, $v = (\pi d_1 N_1) / 60$
- ✓ Calculate value of $\psi_p = b / d_1$

11. Selection of quality of gear:

Knowing the pitch line velocity and consulting table 5.22, select a suitable quality of gear.

12. Revision of design torque [M_t]:

Revise K:

Using the calculated value of ψ_p revise the K value by using table 5.11

Revise K_d:

Using the selected quality of gear and pitch line velocity, revise the K_d value

$$[M_t] = M_t \cdot K \cdot K_d$$

13. Check for bending:

Calculate induced bending stress,

$$\sigma_b = \frac{0.7 (i \pm 1)}{am \cdot bY} [Mt]$$

- ✓ Compare σ_b and $[\sigma_b]$.
- ✓ If $\sigma_b \leq [\sigma_b]$, then design is safe.

14. Check for wear strength:

Calculate induced contact stress,

$$\sigma_c = 0.7 \frac{(i \pm 1)}{a} \sqrt{\frac{(i \pm 1)}{ib} Eeq} [Mt]$$

- ✓ Compare σ_c and $[\sigma_c]$.
- ✓ If $\sigma_c \leq [\sigma_c]$, then design is safe.

15. If the design is not satisfactory ($\sigma_b > [\sigma_b]$ and / or $\sigma_c > [\sigma_c]$), then increase the module of face width value of the gear material.

16. Check for gear:**c. Check for bending:**

Using $\sigma_{b1} \cdot y_1$ and $\sigma_{t1} \cdot y_1$.

$$\sigma_{b2} = \frac{t_1 \cdot y_1}{y_2}$$

- ✓ Compare σ_{b2} and $[\sigma_{b2}]$.
- ✓ If $\sigma_{b2} \leq [\sigma_{b2}]$, then design is safe.

d. Check for wear strength:

Calculate induced contact stress will be same for pinion and gear,
So,

$$\sigma_{c2} = \sigma_c$$

- ✓ Compare σ_c and $[\sigma_c]$
- ✓ If $\sigma_c \leq [\sigma_c]$, then design is safe

SOLVED PROBLEMS

1. The following particulars of a single reduction spur gear are given :
Gear ratio = 10 : 1; Distance between centres = 660 mm approximately; Pinion transmits 500 kW at 1800 r.p.m.; Involute teeth of standard proportions (addendum = m) with pressure angle of 22.5°; Permissible normal pressure between teeth = 175 N per mm of width. Find :

1. The nearest standard module if no interference is to occur;
2. The number of teeth on each wheel;
3. The necessary width of the pinion; and
4. The load on the bearings of the wheels due to power transmitted.

Solution : Given :

$$G = TG / TP = DG / DP = 10 ;$$

$$L = 660 \text{ mm} ;$$

$$P = 500 \text{ kW} = 500 \times 10^3 \text{ W};$$

$$NP = 1800 \text{ r.p.m.} ; \phi = 22.5^\circ ;$$

$$WN = 175 \text{ N/mm width}$$

1. Nearest standard module if no interference is to occur

Let m = Required module,

TP = Number of teeth on the pinion,

TG = Number of teeth on the gear,

DP = Pitch circle diameter of the pinion, and

DG = Pitch circle diameter of the gear.

We know that minimum number of teeth on the pinion in order to avoid interference,

$$T_P = \frac{2Aw}{G \left[\sqrt{1 + \frac{1}{G} \left(\frac{1}{G} + 1 \right) (\sin \phi)^2} - 1 \right]}$$

$$= \frac{2 \times 1}{10 \cdot \left[\sqrt{1 + \frac{1}{10} \left(\frac{1}{10} + 1 \right) \left((\sin 22.5)^2 - 1 \right)} \right]}$$

$$\mathbf{T_G = 140}$$

we already know that,

$$\begin{aligned} L &= (D_G / 2) + (D_p / 2) \\ &= 5.5 D_p \end{aligned}$$

We also know that

$$DP = m \cdot TP$$

$$\begin{aligned} \therefore m &= DP / TP = 120 / 14 \\ &= 8.6 \text{ mm} \end{aligned}$$

Since the nearest standard value of the module is 8 mm, therefore we shall take

$$\mathbf{m = 8 \text{ mm Ans.}}$$

2. Number of teeth on each wheel

We know that number of teeth on the pinion,

$$TP = DP / m = 120 / 8 = \mathbf{15 \text{ Ans.}}$$

and number of teeth on the gear,

$$TG = G \times TP = 10 \times 15 = \mathbf{150 \text{ Ans.}}$$

3. Necessary width of the pinion

We know that the torque acting on the pinion,

$$\begin{aligned} T &= 60P / (2\pi N) \\ &= \mathbf{2652 \text{ N-m}} \end{aligned}$$

Tangential load, $WT = T / (D_p / 2)$

$$= 2652 / (20.12 / 2)$$

$$T = 44200 \text{ N}$$

$$\begin{aligned} W_N &= W_t / \cos \phi \\ &= 44200 / \cos 22.5 \end{aligned}$$

$$W = 47\,840 \text{ N}$$

Since the normal pressure between teeth is 175 N per mm of width, therefore necessary width of

the pinion,

$$\begin{aligned} b &= 47840 / 175 \\ &= 273.4 \text{ mm Ans.} \end{aligned}$$

4. Load on the bearings of the wheels

We know that the radial load on the bearings due to the power transmitted,

$$\begin{aligned} W_R &= W_N \cdot \sin \phi \\ &= 47\,840 \times \sin 22.5^\circ \\ &= 18\,308 \text{ N} \\ &= 18.308 \text{ kN An} \end{aligned}$$

-
2. A bronze spur pinion rotating at 600 r.p.m. drives a cast iron spur gear at a transmission ratio of 4 : 1. The allowable static stresses for the bronze pinion and cast iron gear are 84 MPa and 105 MPa respectively. The pinion has 16 standard 20° full depth involute teeth of module 8 mm. The face width of both the gears is 90 mm. Find the power that can be transmitted from the standpoint of strength.

Solution.

Given :

$$N_P = 600 \text{ r.p.m. ;}$$

$$V.R. = T_G / T_P = 4 ;$$

$$\sigma_{OP} = 84 \text{ MPa} = 84 \text{ N / mm}^2 ;$$

$$\sigma_{OG} = 105 \text{ MPa} = 105 \text{ N/mm}^2 ;$$

$$T_P = 16 ;$$

$$m = 8 \text{ mm} ;$$

$$b = 90 \text{ mm}$$

We know that pitch circle diameter of the pinion,

$$D_P = m.T_P = 8 \times 16 = 128 \text{ mm} = \mathbf{0.128 \text{ m}}$$

\therefore Pitch line velocity,

$$\begin{aligned} v &= \pi D_N / 60 \\ &= \mathbf{4.02 \text{ m/s}} \end{aligned}$$

We know that for 20° full depth involute teeth, tooth form factor for the pinion,

$$\begin{aligned} y_P &= 0.154 - (.912 / T_P) \\ &= \mathbf{0.097} \end{aligned}$$

and tooth form factor for the gear

$$\begin{aligned} y_G &= 0.154 - (.912 / T_G) \\ &= \mathbf{0.14} \end{aligned}$$

$$\sigma_{OP} \times y_P = 84 \times 0.097 = 8.148$$

and $\sigma_{OG} \times y_G = 105 \times 0.14 = 14.7$

Since $(\sigma_{OP} \times y_P)$ is less than $(\sigma_{OG} \times y_G)$, therefore the pinion is weaker. Now using the Lewis equation for the pinion, we have tangential load on the tooth (or beam strength of the tooth),

$$\begin{aligned} W_T &= \sigma_{WP} . b . \pi m . y_P \\ &= (\sigma_{OP} \times C_v) b . \pi m . y_P \quad (Q \sigma_{WP} = \sigma_{OP} . C_v) \\ &= 84 \times 0.427 \times 90 \times \pi \times 8 \times 0.097 = 7870 \text{ N} \end{aligned}$$

\therefore Power that can be transmitted = $W_T \times v$

$$\begin{aligned}
&= 7870 \times 4.02 \\
&= 31\,640 \text{ W} \\
&= \mathbf{31.64 \text{ kW Ans.}}
\end{aligned}$$

- 3. A pair of helical gears are to transmit 15 kW. The teeth are 20° stub in diametral plane and have a helix angle of 45°. The pinion runs at 10 000 r.p.m. and has 80 mm pitch diameter. The gear has 320 mm pitch diameter. If the gears are made of cast steel having allowable static strength of 100 MPa; determine a suitable module and face width from static strength considerations and check the gears for wear, given $\sigma_{es} = 618 \text{ MPa}$.**

Solution.

Given :

$$P = 15 \text{ kW} = 15 \times 10^3 \text{ W};$$

$$\phi = 20^\circ ;$$

$$\alpha = 45^\circ ;$$

$$NP = 10\,000 \text{ r.p.m.} ;$$

$$DP = 80 \text{ mm} = 0.08 \text{ m} ;$$

$$DG = 320 \text{ mm} = 0.32 \text{ m} ;$$

$$\sigma_{OP} = \sigma_{OG} = 100 \text{ MPa} = 100 \text{ N/mm}^2 ;$$

$$\sigma_{es} = 618 \text{ MPa} = 618 \text{ N/mm}^2$$

Module and face width

Let m = Module in mm, and

b = Face width in mm.

Since both the pinion and gear are made of the same material (i.e. cast steel), therefore the pinion is weaker. Thus the design will be based upon the pinion.

We know that the torque transmitted by the pinion,

$$T = (60P) / (2\pi N)$$

$$= 14.32 \text{ N-m.}$$

Tangential tooth load on the pinion,

$$W_T = T / (D_p/2)$$

We know that number of teeth on the pinion,

$$T_P = D_P / m = 80 / m$$

and formative or equivalent number of teeth for the pinion,

$$\begin{aligned} T_E &= T_P / \cos^3 \alpha \\ &= 226.4 / m \end{aligned}$$

∴ peripheral velocity,

$$\begin{aligned} v &= (\pi D_p N_p) / 60 \\ &= 42 \text{ m/s} \end{aligned}$$

Velocity factor, $C_v = 0.75 / (0.75 + \sqrt{v})$

$$= 0.104$$

Since the maximum face width (b) for helical gears may be taken as 12.5 m to 20 m, where m is the module, therefore let us take

$$b = 12.5 \text{ m}$$

We know that **the tangential tooth load (WT),**

$$\begin{aligned} 358 &= (\sigma_{OP} \cdot C_v) b \cdot \pi m \cdot y_P \\ &= (100 \times 0.104) 12.5 \text{ m} \times \pi m (0.175 - 0.0037 m) \\ &= 409 m^2 (0.175 - 0.0037 m) \\ &= 72 m^2 - 1.5 m^3 \end{aligned}$$

Solving this expression by hit and trial method, we find that

$$m = 2.3 \text{ say } \mathbf{2.5 \text{ mm Ans.}}$$

and **face width,** $b = 12.5 \text{ m} = 12.5 \times 2.5 = 31.25 \text{ say } \mathbf{32 \text{ mm Ans.}}$

Checking the gears for wear

We know that velocity ratio,

$$\begin{aligned} \text{V.R.} &= D_G / D_P \\ &= 320 / 80 \\ &= 4 \end{aligned}$$

We know that the maximum limiting load for wear,

$$\begin{aligned} W_w &= (D b Q K) / \cos^2 \alpha \\ &= 5554 \text{ N} \end{aligned}$$

A helical cast steel gear with 30° helix angle has to transmit 35 kW at 1500 r.p.m. If the gear has 24 teeth, determine the necessary module, pitch diameter and face width for 20° full depth teeth. The static stress for cast steel may be taken as 56 MPa. The width of face may be taken as 3 times the normal pitch. What would be the end thrust on the gear? The tooth factor for 20° full depth involute gear may be taken as $0.154 - 0.912/T$ where TE represents the equivalent number of teeth.

Solution.

Given :

$$\begin{aligned} \alpha &= 30^\circ ; \\ P &= 35 \text{ kW} = 35 \times 10^3 \text{ W} ; \\ N &= 1500 \text{ r.p.m.} ; \\ T_G &= 24 ; \\ \phi &= 20^\circ ; \\ \sigma_o &= 56 \text{ MPa} = 56 \text{ N/mm}^2 ; \\ b &= 3 \times \text{Normal pitch} = 3 p_N \end{aligned}$$

Module

Let m = Module in mm, and

D_G = Pitch circle diameter of the gear in mm.

We know that torque transmitted by the gear,

$$T = (60P) / (2\pi N)$$

$$= 223 \times 10^3 \text{ N-mm.}$$

Formative or equivalent number of teeth,

$$T_E = T_P / \cos^3 \alpha$$

$$= 37.$$

\therefore **Tooth factor, $y' = 0.154 - 0.912/T$**

$$= 0.129$$

Tangential tooth load on the pinion,

$$W_T = T / (D_p/2)$$

$$= 18600 / m$$

\therefore **peripheral velocity,**

$$v = (\pi D_p N_p) / 60$$

$$= 1.885m \text{ m/s}$$

Velocity factor, $C_v = 0.75 / (0.75 + v)$

$$= 0.75 / (0.75 + 1.885m)$$

We know that tangential tooth load,

$$W_T = (\sigma_o \times C_v) b \cdot \pi m \cdot y' = (\sigma_o \times C_v) 3pN \times \pi m \times y' \quad \dots (Q \text{ b} = 3 pN)$$

$$= (\sigma_o \times C_v) 3 \times p_c \cos \alpha \times \pi m \times y' \quad \dots (Q \text{ pN} = p_c \cos \alpha)$$

$$= (\sigma_o \times C_v) 3 \pi m \cos \alpha \times \pi m \times y' \quad \dots (Q \text{ p}_c = \pi m)$$

Solving this equation by hit and trial method, we find that

$$m = 5.5 \text{ say } 6 \text{ mm Ans}$$

Pitch diameter of the gear

We know that the pitch diameter of the gear,

$$DG = m \times TG = 6 \times 24 = 144 \text{ mm Ans.}$$

Face width

It is given that the face width,

$$b = 3 pN = 3 pc$$

$$\cos \alpha = 3 \times \pi m \cos \alpha$$

$$= 3 \times \pi \times 6 \cos 30^\circ$$

$$= 48.98 \text{ say } 50 \text{ mm Ans.}$$

End thrust on the gear

We know that end thrust or axial load on the gear,

$$WA = Wr \tan \alpha$$

$$= 1790 \text{ N}$$

CHAPTER 3

BEVEL AND WORM GEARS

Common terms used:

1. **Pitch cone.** It is a cone containing the pitch elements of the teeth.
2. **Cone centre.** It is the apex of the pitch cone. It may be defined as that point where the axes of two mating gears intersect each other.
3. **Pitch angle.** It is the angle made by the pitch line with the axis of the shaft. It is denoted by 'θP'.
4. **Cone distance.** It is the length of the pitch cone element. It is also called as a pitch cone radius. It is denoted by 'OP'. Mathematically, cone distance or pitch cone radius,

$$OP = \frac{\text{pitch radius}}{\sin \theta_p} = \frac{D_p/2}{\sin \theta_{p1}} = \frac{D_g/2}{\sin \theta_{p2}}$$

5. **Addendum angle.** It is the angle subtended by the addendum of the tooth at the cone centre. It is denoted by 'α' Mathematically, addendum angle,

$$\alpha = \tan^{-1} (a/OP)$$

where a = Addendum, and OP = Cone distance.

6. **Dedendum angle.** It is the angle subtended by the dedendum of the tooth at the cone centre. It is denoted by 'β'. Mathematically, dedendum angle,

$$\beta = \tan^{-1} (d/OP)$$

where d = Dedendum, and OP = Cone distance.

7. **Face angle.** It is the angle subtended by the face of the tooth at the cone centre. It is denoted by 'φ'. The face angle is equal to the pitch angle plus addendum angle.

8. **Root angle.** It is the angle subtended by the root of the tooth at the cone centre. It is denoted by 'θR'. It is equal to the pitch angle minus dedendum angle.

9. **Back (or normal) cone.** It is an imaginary cone, perpendicular to the pitch cone at the end of the tooth.

10. **Back cone distance.** It is the length of the back cone. It is denoted by 'RB'. It is also called back cone radius.

11. **Backing.** It is the distance of the pitch point (P) from the back of the boss, parallel to the pitch point of the gear. It is denoted by 'B'.

12. **Crown height.** It is the distance of the crown point (C) from the cone centre (O), parallel to the axis of the gear. It is denoted by 'HC'.

13. **Mounting height.** It is the distance of the back of the boss from the cone centre. It is denoted by 'HM'.

14. **Pitch diameter.** It is the diameter of the largest pitch circle.

15. **Outside or addendum cone diameter.** It is the maximum diameter of the teeth of the gear. It is equal to the diameter of the blank from which the gear can be cut. Mathematically, outside diameter,

$$DO = DP + 2 a \cos \theta P$$

where DP = Pitch circle diameter,

a = Addendum, and

θP = Pitch angle.

16. **Inside or dedendum cone diameter.** The inside or the dedendum cone diameter is given by

$$Dd = DP - 2d \cos \theta P$$

where Dd = Inside diameter, and

d = Dedendum.

Design procedure for Bevel Gear

17. **Calculation of gear ratio (i) and pitch angle:**

$$i = \frac{N_A}{N_B} = \frac{Z_B}{Z_A}$$

where, N_A and N_B = speed of the driver and driven respectively, and
 Z_A and Z_B = Number of teeth on driver and driven respectively.

18. **Selection of material**

Consulting Table 5.3, knowing the gear ratio i, choose the suitable material.

19. If not given, assume gear life (say 20000 hrs)

20. **Calculation of initial design torque:**

$$[M_t] = M_t \cdot K \cdot K_d$$

where, $[M_t]$ = transmission torque
K = Load factor, Table 5.11
 K_d = Dynamic load factor, Table 5.12

Assume $K \cdot K_d = 1.3$ (if not given)

21. Calculation of E_{eq} , $[\sigma_b]$ and $[\sigma_c]$:

- ✓ From table 5.20 Calculate E_{eq}
- ✓ Calculate Design bending stress $[\sigma_b]$

$$[\sigma_b] = (1.4 K_{bl}/n.K_\sigma) \sigma_{-1}, \text{ for one rotation}$$

$$[\sigma_b] = (K_{bl}/n.K_\sigma) \sigma_{-1}, \text{ for both rotation}$$

- ✓ Calculate Design contact stress $[\sigma_c]$ by

$$[\sigma_c] = C_B \cdot HB \cdot K_{cl} \text{ (or)}$$

$$[\sigma_c] = C_R \cdot HRC \cdot K_{cl}$$

where, $C_B C_R =$ Coefficient of surface hardness from table 5.18

HB HRC = Hardness number

22. Calculation if cone distance (a):

$$R \geq \psi_y \sqrt{(i^2 + 1)} \sqrt[3]{\left(\frac{.72}{[y-.5] [c]}\right)^2 \times \frac{E_{eq} [Mt]}{i}}$$

$\psi_y = R/b$ from table 5.21

23. Select number of teeth on gear and pinion:

- On pinion, $Z_1 =$ Assume 18
- On gear, $Z_2 = i \times Z_1$

24. Calculation of module:

$$m = \frac{2a}{(z_1 + z_2)}$$

Choose standard module from table 5.8

25. Revision of cone distance (m):

$$R = 0.5m_t \sqrt{z_1^2 + z_2^2}$$

26. Calculate b, d_{1av}, v and ψ_y :

- ✓ Calculate face width, $b = R/\psi_y$
- ✓ Calculate pitch dia, $d_{1av} = m_{av} \cdot Z_1$
- ✓ Calculate pitch line velocity, $v = (\pi d_{1av} N_1)/60$
- ✓ Calculate value of $\psi_p = b/d_{1av}$

27. Selection of quality of gear:

Knowing the pitch line velocity and consulting table 5.22, select a suitable quality of gear.

28. Revision of design torque [M_t]:

Revise K:

Using the calculated value of ψ_y revise the K value by using table 5.11

Revise K_d:

Using the selected quality of gear and pitch line velocity, revise the K_d value.

$$[M_t] = M_t \cdot K \cdot K_d$$

29. Check for bending:

Calculate induced bending stress,

$$\sigma_b = \frac{R \sqrt{(i^2 + 1)}}{(R - 0.5b)^2 \cdot m_t \cdot h Y} [M_t]$$

- ✓ Compare σ_b and [σ_b].
- ✓ If σ_b ≤ [σ_b], then design is safe.

30. Check for wear strength:

Calculate induced contact stress,

$$\sigma_c = \frac{.77}{(R - 0.5b)} \sqrt{\frac{\sqrt{(i^2 + 1)}^3}{ib} Eeq} [M_t]$$

- ✓ Compare σ_c and [σ_c].
- ✓ If σ_c ≤ [σ_c], then design is safe.

31. If the design is not satisfactory (σ_b > [σ_b] and / or σ_c > [σ_c]), then increase the module of face width value of the gear material.

32. Check for gear:

e. Check for bending:

Using $\sigma_{b1,y1}$ and $\sigma_{t1,y1}$.

$$\sigma_{b2} = \frac{t1.y1}{y2}$$

- ✓ Compare σ_{b2} and $[\sigma_{b2}]$.
- ✓ If $\sigma_{b2} \leq [\sigma_{b2}]$, then design is safe.

f. Check for wear strength:

Calculate induced contact stress will be same for pinion and gear,
So,

$$\sigma_{c2} = \sigma_c$$

- ✓ Compare σ_c and $[\sigma_c]$
- ✓ If $\sigma_c \leq [\sigma_c]$, then design is safe

WORM GEAR

Common terms used :

The following terms, in connection with the worm gearing, are important from the subject point of view :

1. **Axial pitch.** It is also known as linear pitch of a worm. It is the distance measured axially (i.e. parallel to the axis of worm) from a point on one thread to the corresponding point on the adjacent thread on the worm, as shown in Fig. 31.3. It may be noted that the axial pitch (p_a) of a worm is equal to the circular pitch (p_c) of the mating worm gear, when the shafts are at right angles.

2. **Lead.** It is the linear distance through which a point on a thread moves ahead in one revolution of the worm. For single start threads, lead is equal to the axial pitch, but for multiple start threads, lead is equal to the product of axial pitch and number of starts. Mathematically,

$$\text{Lead, } l = p_a \cdot n$$

where p_a = Axial pitch ; and

n = Number of starts.

3. **Lead angle.** It is the angle between the tangent to the thread helix on the pitch cylinder and the plane normal to the axis of the worm. It is denoted by λ .

A little consideration will show that if one complete turn of a worm thread be imagined to be unwound from the body of the worm, it will form an inclined plane whose base is equal to the pitch circumference of the worm and altitude equal to lead of the worm.

$$\begin{aligned}\tan \lambda &= \text{Lead of the worm} / \text{Pitch circumference of the worm} \\ &= \frac{m \cdot n}{D_W}\end{aligned}$$

where m = Module, and

D_W = Pitch circle diameter of worm.

The lead angle (λ) may vary from 9° to 45° . It has been shown by F.A. Halsey that a lead angle less than 9° results in rapid wear and the safe value of λ is $12\frac{1}{2}^\circ$.

4. Tooth pressure angle. It is measured in a plane containing the axis of the worm and is equal to one-half the thread profile angle as shown in Fig. Normal pitch. It is the distance measured along the normal to the threads between two corresponding points on two adjacent threads of the worm.

Mathematically,

$$\text{Normal pitch, } p_N = p_a \cdot \cos \lambda$$

Note. The term normal pitch is used for a worm having single start threads. In case of a worm having multiple start threads, the term normal lead (l_N) is used, such that

$$l_N = l \cdot \cos \lambda$$

6. Helix angle. It is the angle between the tangent to the thread helix on the pitch cylinder and the axis of the worm. It is denoted by α_W . The worm helix angle is the complement of worm lead angle, i.e.

$$\alpha_W + \lambda = 90^\circ$$

It may be noted that the helix angle on the worm is generally quite large and that on the worm gear is very small. Thus, it is usual to specify the lead angle (λ) on the worm and helix angle (α_G) on the worm gear. These two angles are equal for a 90° shaft angle.

7. Velocity ratio. It is the ratio of the speed of worm (N_W) in r.p.m. to the speed of the worm gear (N_G) in r.p.m. Mathematically, velocity ratio,

$$\text{V.R.} = N_W / N_G$$

Let l = Lead of the worm, and

D_G = Pitch circle diameter of the worm gear.

We know that linear velocity of the worm,

$$vW = 1 \cdot NW/60$$

where n = Number of starts of the worm.

DESIGN PROCEDURE FOR WORM GEAR:

1. Selection of the material:

2. Calculation of teeth:

- ✓ Assume Z_1 depending upon the number of stat.
- ✓ $Z_2 = i \times Z_1$

3. Calculation of diameter factor and lead angle

$$q = d1/m_x$$

If not given assume $q = 11$

$$\gamma = \tan^{-1} (z1/q)$$

4. Calculation of Tangential load:

$$F_t = (p/v) \times K_0$$

5. Calculation of Dynamic load:

$$F_d = F_t / c_v$$

6. Calculation of Beam strength:

$$F_s = \pi m_x b [\sigma_b] y$$

7. Calculation of Axial load:

Calculate axial load by equating F_d and F_s

8. Calculate b, d2, v.

9. Recalculation of beam strength

$$F_s = \pi m_x b [\sigma_b] y^1$$

10. Recalculation of dynamic load

$$F_d = F_t / c_v$$

11. Check for beam strength:

If $F_d \leq F_d$, design is satisfactory.

12. Calculation for maximum wear load:

$$F_w = d^2 \cdot b \cdot K_w$$

13. Check for wear strength:

If $F_d \leq F_w$, design is satisfactory.

14. Calculate power loss and area:

15. Calculate basic dimensions.

SOLVED PROBLEMS

1. For continuous duty in a speed reducer, two helical gears are to be rated at 7.4 hp at a pinion speed of 1750 rpm; $m_w \approx 2.75$; the helix angle 15° ; 20° F.D. teeth in the normal plane; let $N_p = 21$ teeth, and keep $b < 2D_p$. Determine the pitch, face, N_g , and the material and heat treatment. Use through-hardened teeth with a maximum of 250 BHM (teeth may be cut after heat treatment).

Solution:

$$\psi = 15^\circ$$

$$\phi_n = 20^\circ$$

$$v_m = \frac{\pi D_p n_p}{12}$$

$$D_p = \frac{N_p}{P_d} = \frac{21}{P_d}$$

$$n_p = 1750 \text{ rpm}$$

$$v_m = \frac{\pi \left(\frac{21}{P_d} \right) (1750)}{12} = \frac{9621}{P_d}$$

$$F_t = \frac{33,000 \text{ hp}}{v_m} = \frac{(33,000)(7.4)}{\left(\frac{9621}{P_d} \right)} = 25.38 P_d$$

$$b \leq 2D_p$$

$$b = 2 \left(\frac{21}{P_d} \right) = \frac{42}{P_d}$$

$$F_d = F_t + \frac{(F_t + C_b \cos^2 \psi) \phi_s \psi}{+(F_m + C_b \cos^2 \psi)^{1/2}} lb$$

0

.

0

5

v

Table AT 25

Assume $C = 1660$

$$\psi = 15^\circ$$

$$F_d = 25.38P_d + \frac{0.05 \left(\frac{9621}{P_d} \right) \left[25.38P_d + 1660 \right] \left(\frac{42}{P_d} \right)^2 \cos 15}{\cos 15} \text{ lb}$$

$$F_d = 25.38P_d + \frac{465 \left(25.38P_d + \frac{65050}{P_d} \right)}{481 + \left(25.38P_d + \frac{65050}{P_d} \right)} \text{ lb}$$

For continuous service: $F_w \geq F_d$

$$F_w = \frac{bD_p Q K_g}{\cos^2 \psi}$$

$$Q = \frac{2m_g}{m_g + 1} = \frac{2(2.75)}{2.75 + 1} = 1.467$$

Table At 26, Bhn = 250

Sum of BHN = 500, $\phi_n = 20^\circ$

$$K_g = 131$$

$$F_w = \frac{\left(\frac{42}{P_d} \right) \left(\frac{21}{P_d} \right) (1.467)(131)}{\cos^2 15} = \frac{181,670}{P_d^2}$$

$$F_w \geq F_d$$

By trial and error method

P_d	F_d	F_w
7	3967	3708
6	4758	5046

use $P_d = 6$

$$D_p = \frac{21}{P_d} = \frac{21}{6} = 3.5 \text{ in}$$

$$b = \frac{42}{P_d} = \frac{42}{6} = 7 \text{ in}$$

$$v_m = \frac{9621}{P_d} = \frac{9621}{6} = 1604 \text{ fpm}$$

Fig. AF 19, permissible error = 0.0018 in

Fig. AF 20

Use carefully cut gears, $P_d = 6$

Error = 0.001 in is o.k.

For material

Strength

$$F_s = \frac{sbY \cos \psi}{K_f P_d}$$

$$N_{ep} = \frac{N_p}{\cos^3 \psi} = \frac{21}{\cos^3 15} = 23$$

Table AT 24, Load near middle

$$N_{ep} = 23, \phi_n = 20^\circ \text{ FD}$$

$$Y = 0.565$$

assume $K_f = 2.0$

$$F_s = N_{sf} F_d$$

assume $N_{sf} = 2.0$

$$\frac{s(7)(0.565)\cos 15}{(2)(6)} = (4758)(2)$$

$$s = 29,892 \text{ psi}$$

$$\text{use } s_n = \frac{s_u}{3}$$

$$s_u = 3(29,892) = 89,676 \text{ psi}$$

Use C1050, OQT 1100 F,

$$s_u = 122 \text{ ksi}, BHN = 248 < 250$$

Ans.

$$P_d = 6$$

$$b = 7 \text{ in}$$

$$N_g = m_w N_p = (2.75)(21) = 58$$

Material. C1050, OQT 1100 F

2. A pair of helical gears, subjected to heavy shock loading, is to transmit 50 hp at 1750 rpm of the pinion.; $m_g = 4.25$; $\psi = 15^\circ$; minimum $D_p = 4\frac{3}{4} \text{ in.}$; continuous service, 24 hr/day; 20° F.D. teeth in the normal plane, carefully cut; through-hardened to a maximum BHN = 350. Decide upon the pitch, face width, material and its treatment.

Solution:

$$v_m = \frac{\pi(4.75)(1750)}{12} = 2176 \text{ fpm}$$

$$F_t = \frac{33,000 \text{ hp}}{v_m} = \frac{(33,000)(50)}{(2176)} = 758 \text{ lb}$$

Dynamic load:

$$F_d = F_t + \frac{0.05 v_m (F_t + C b \cos^2 \psi) \cos \psi}{0.05 v_m + (F_t + C b \cos^2 \psi)^{1/2}} \text{ lb}$$

Fig. AF 19, $v_m = 2176 \text{ fpm}$

Permissible error = 0.0014 in

Use carefully cut gears, $e = 0.001 \text{ in}$, $P_d = 5$ as standard

Table AT 25,

Steel and steel, 20° FD

$$C = 1660$$

$$F_d = 758 + \frac{0.05(2176)(758 + 1660b \cos^2 15) \cos 15}{0.05(2176) + (758 + 1660b \cos^2 15)^{\frac{1}{2}}} lb$$

$$F_d = 758 + \frac{105.1(758 + 1548.8b)}{108.8 + (758 + 1548.8b)^{\frac{1}{2}}} lb$$

Wear load:

$$F_w = \frac{bD_p Q K_g}{\cos^2 \psi}$$

$$Q = \frac{2m_g}{m_g + 1} = \frac{2(4.25)}{4.25 + 1} = 1.619$$

Table At 26, 20° FD,

Sum of BHN = 2(350) = 700

$$K_g = 270$$

$$F_w = \frac{b(4.75)(1.619)(270)}{\cos^2 15} = 2225b$$

$$F_w \geq F_d, b_{\min} = 2P = \frac{2\pi}{P_d \tan \psi} = 4.69 \text{ in.}$$

By trial and error method

b	F_d	F_w
5	5203	11125
6	5811	13350

use $b = 5 \text{ in}$

Material:

Strength:

$$F_s = \frac{sbY}{K_f P_{dn}} = \frac{sbY \cos \psi}{K_f P_d}$$

$$N_{ep} = \frac{N_p}{\cos^3 \psi}$$

$$N_p = P_d D_p = (5)(4.375) = 22$$

$$N_{ep} = \frac{22}{\cos^3 15} = 25$$

Table AT 24, Load near middle

$$N_{ep} = 25, \phi_n = 20^\circ \text{ FD}$$

$$Y = 0.580$$

assume $K_f = 1.7$

$$F_s = \frac{s(5)(0.580)\cos 15}{(1.7)(5)} = 0.32955s$$

$$F_s = N_{sf} F_d$$

for 24 hr/day service, heavy shock loading

$$N_{sf} = 1.75$$

$$0.32955s = (1.75)(5203)$$

$$s = 27,629 \text{ psi}$$

$$\text{use } s_n = \frac{s_u}{3}$$

$$s_u = 3(27,629) = 82,887 \text{ psi}$$

Table AT 9

Use 4150, OQT 1200 F,

$$s_u = 159 \text{ ksi}, BHN = 331 < 350$$

Ans.

$$P_d = 5$$

$$b = 5 \text{ in}$$

Material. 4150, OQT 1200 F

Two helical gears are used in a single reduction speed reducer rated at 27.4 hp at a motor speed of 1750 rpm; continuous duty. The rating allows an occasional 100 % momentary overload. The pinion has 33 teeth. $P_{dn} = 10$, $b = 2 \text{ in.}$, $\phi_n = 20^\circ$, $\psi = 20^\circ$, $m_w = 2.82$. For both gears, the teeth are carefully cut from SAE 1045 with BHN = 180. Compute (a) the dynamic load, (b) the endurance strength; estimate $K_f = 1.7$. Also decide whether or not the 100 % overload is damaging. (c) Are these teeth suitable for continuous service? If they are not suitable suggest a cure. (The gears are already cut.)

Solution:

$$D_p = \frac{N_p}{P_d}$$

$$P_d = P_{dn} \cos \psi = (10) \cos 15 = 9.66$$

$$D_p = \frac{33}{9.66} = 3.42 \text{ in}$$

$$v_m = \frac{\pi D_p n_p}{12} = \frac{\pi (3.42)(1750)}{12} = 1567 \text{ fpm}$$

$$F_t = \frac{33,000 \text{ hp}}{v_m} = \frac{33,000(27.4)}{1567} = 577 \text{ lb}$$

(a) Dynamic load

v

$$F_d = F_t +$$

v

m

0

.

0

5

$$\frac{F + \frac{Cb \cos \psi}{\cos^2 \psi} \left(\frac{F + Cb \cos^2 \psi}{\cos^2 \psi} \right)^{1/2}}{\cos^2 \psi} lb$$

Fig. AF 20, carefully cut gears, $P_{dn} = 10$, $e = 0.001 in$

Table AT 25, steel and steel, $20^\circ FD$

$C = 1660$

$b = 2 \text{ in}$

$$F_d = 577 + \frac{0.05(1567) \left[77 + 1660(2) \cos^2 15 \right] \cos 15}{0.05(1567) + \left[77 + 1660(2) \cos^2 15 \right]} = 2578 \text{ lb}$$

(b) Endurance strength

$$F_s = \frac{sbY}{K_f P_{dn}} \text{ lb}$$

For SAE 1045, BHN = 180

$$s_u = 0.5BHN = 0.5(180) = 90 \text{ ksi}$$

$$s_n = 0.5s_u = 0.5(90) = 45 \text{ ksi}$$

$$N_{ep} = \frac{N_p}{\cos^3 \psi} = \frac{33}{\cos^3 15} = 37$$

Table AT 24, Load near middle, $\phi_n = 20^\circ$

$$Y = 0.645$$

$$K_f = 1.7$$

$$F_s = \frac{sbY}{K_f P_{dn}} = \frac{(45,000)(2)(0.645)}{(1.7)(10)} = 3415 \text{ lb}$$

For 100 % overload

$$F_t = 2(577) = 1154 \text{ lb}$$

$$F_d = F_t + \frac{(F_t + C_b \cos^2 \psi) \cos \psi}{(F_m + C_b \cos^2 \psi)^{2.1}} \text{ lb}$$

0

.

0

5

$m \quad y$
 t

$$F_d = 1154 + \frac{0.05(1567) \left[154 + 1660(2) \cos^2 15 \right] \cos 15}{0.05(1567) + \left[154 + 1660(2) \cos^2 15 \right]} = 3475 \text{ lb}$$

Since $F_s \approx F_d$, 100 % overload is not damaging

$$(c) \left(\frac{F}{w} \right) = \frac{b D_p Q K_g}{\cos^2 \psi}$$

$$b = 2 \text{ in.}$$

$$Q = \frac{2m_w}{m_w + 1} = \frac{2(2.82)}{2.82 + 1} = 1.476$$

Table AT 26, $\phi_n = 20^\circ$

$$\text{Sum of BHN} = 2(180) = 360$$

$$K_g = 62.5$$

$$F_w = \frac{(2)(3.42)(1.476)(62.5)}{\cos^2 15} = 676 \text{ lb} < F_d (=2578 \text{ lb})$$

Therefore not suitable for continuous service.

Cure: Through hardened teeth

For Bhn

$$K_g = \frac{2578}{676} (62.5) = 238$$

$$\text{min Bhn} = 0.5(650) = 325.$$

CHAPTER 5 DESIGN OF GEAR BOXES

Standard progression

When the spindle speed are arranged in geometric progression then the ratio between the two adjacent speeds is known as step ratio or progression ratio.

$$\frac{N_2}{N_1} = \frac{N_3}{N_2} = \frac{N_4}{N_3} = \dots = \frac{N_n}{N_{(n-1)}} = \text{constant} = \phi$$

Step ratio

Basic series	Step ratio (ϕ)
R5	1.58
R10	1.26
R20	1.12
R40	1.06
R80	1.03

Preferred basic series

Basic series	Preferred number
R5 ($\phi=1.6$)	1.00, 1.60, 2.50, 4.00, 6.30, 10.00
R10 ($\phi=1.26$)	1.00, 1.25, 1.60, 2.00, 2.50, 3.15, 4.00, 5.00, 6.30, 8.00, 10.00
R20 ($\phi=1.12$)	1.00, 1.06, 1.25, 1.18, 1.60, 1.25, 2.00, 2.24, 2.50, 2.80, 3.15, 3.55, 4.00, 4.50, 5.00, 5.60, 6.30, 7.10, 8.00, 9.00, 10.00
R40 ($\phi=1.06$)	1.00, 1.06, 1.18, 1.25, 1.32, 1.18, 1.40, 1.60, 1.70, 1.25, 1.80, 1.90, 2.00, 2.10, 2.24, 2.36, 2.50, 2.65, 2.80, 3.00, 3.15, 3.35, 3.55, 4.00, 4.25, 4.50, 4.75, 5.00, 5.30, 5.60, 6.00, 6.30, 6.70, 7.10, 7.50, 8.00, 8.50, 9.00, 9.50, 10.00

Design of gear box

1. Selection of Spindle speed:

Find the standard step ratio by using the relations,

$$\frac{N_{max}}{N_{min}} = \phi_{n-1}$$

2. Structural formula

It can be selected based on the number of speed:

Number of speed	Structural formula
6	3(1) 2(3) 2(1) 3(2)
8	2(1) 2(2) 2(4) 4(1) 2(4)
9	3(1) 3(3)
12	3(1) 2(3) 2(6) 2(1) 3(2) 2(6) 2(1) 2(2) 3(4)
14	3(1) 3(3) 2(5) 4(1) 2(4) 2(6)
15	3(1) 3(3) 2(6)
16	4(1) 2(4) 2(8) 2(1) 4(2) 2(8) 2(1) 2(2) 4(4)
18	3(1) 3(3) 2(9) 3(1) 2(3) 3(6) 2(1) 3(2) 3(6)

3. Ray Diagram

The ray diagram is the graphical representation of the drive arrangement in general form. In other words, The ray diagram is the graphical representation of the structural formula.

The basic rules to be followed while designing the gear box as

- ✓ Transmission ratio (i):

$$\frac{1}{4} \leq i \leq 2$$

(or)

$$N_{\min} / N_{\text{input}} \geq \frac{1}{4}$$

$$i_{\max} = N_{\max} / N_{\text{input}} \leq 2$$

- ✓ For stable operation the speed ratio at any stage should not be greater than 8.

$$N_{\max} / N_{\min} \leq 8$$

4. Kinematic Layout:

The kinematic arrangement shows the arrangement of gears in a gear box.

Formula for kinematic arrangement,

$$n = p_1(X_1) \cdot p_2(X_2)$$

5. Calculation of number of teeth.

In each stage first pair,

Assume, driver $Z_{\min} \geq 17$,

Assume $Z = 20$ (driver)

SOLVED PROBLEMS

1. Design a 4- speed gear box for a machine. The speed vary approximately from 200 to 400 rpm. The input shaft speed is 600 rpm.

Sol:

$$\phi = (Rn)^{1/z-1}$$

$$Rn = 450/200 = 2.25; Z=4$$

$$\phi = 2.25^{0.334} = 1.31$$

the nearest standard value of ϕ is 1.25

the speed of shafts are 180,224,280 and 355 rpm or 224,280,355,450 rpm

$$T_a + T_b = T_c + T_d$$

$$N_a / N_b = T_a / T_b = 355/600$$

$$N_d / N_c = T_c / T_d = 450/600$$

Assuming minimum number of teeth on the smaller gear as 18($T_a = 18$)

$$18 / T_b = 355/600$$

$$T_b = 30$$

$$T_a + T_b = 48 = T_c + T_d$$

$$T_c / T_d = 450/600$$

$$T_c = 20 \text{ and } T_d = 28$$

Considering the transmission between the intermediate and output shafts

$$T_e + T_f = T_g + T_h$$

$$N_f / N_e = T_e / T_f = 224/355$$

$$N_a / N_b = T_a / T_b = 355/355$$

Assuming minimum number of teeth on the smaller gear as 20($T_e = 20$)

$$20 / T_f = 224/355$$

$$T_b = 32$$

$$T_e + T_f = 52 = T_g + T_h$$

$$T_g / T_h = 450/600$$

$$T_g = T_h = 26$$

2. Design a gear box for a drilling machine to give speed variation between 100 and 560 rpm in six steps. The input shaft speed is 560 rpm. The intermediate shaft is to have three speeds.

Sol:

The progression ratio,

$$\phi = (Rn)^{1/Z-1}$$

$$Rn = 560/100 = 5.6; Z = 6$$

$$\phi = 5.6^{0.2} = 1.411$$

the nearest standard value of ϕ is 1.4

the speed of shafts are 100,140,200,280,400 and 560 rpm.

$$T_a + T_b = T_c + T_d = T_e + T_f$$

$$N_b / N_a = T_a / T_b = 280/560$$

$$N_d / N_c = T_c / T_d = 400/560$$

$$N_f / N_e = T_e / T_f = 560/560$$

Assuming minimum number of teeth on the smaller gear as 20($T_a = 20$)

$$20 / T_b = 280/560$$

$$T_b = 40$$

$$T_c / T_d = 400/560$$

$$T_a + T_b = 60 = T_c + T_d$$

$$T_c = 25 \text{ and } T_d = 35$$

$$T_e / T_f = 560/560 \text{ and } T_e + T_f = 60$$

$$T_e = T_f = 30$$

Considering the transmission between the intermediate and output shafts

$$T_g + T_h = T_i + T_j$$

$$N_h / N_g = T_g / T_h = 100/280$$

$$N_j / N_i = T_i / T_j = 280/280$$

Assuming minimum number of teeth on the smaller gear as 20 ($T_g = 20$)

$$20 / T_h = 100/280$$

$$T_h = 56$$

$$T_i / T_j = 280/280$$

$$T_i + T_j = 76 = T_g + T_h$$

$$T_i = T_j = 38$$

- 2. Design a nine speed gear box for a grinding machine with a minimum speed of 100 rpm and a maximum speed of 700 rpm. The motor speed is 1400 rpm. Determine the speed at which the input shaft is to be driven.**

Sol:

The progression ratio,

$$\phi = (Rn)^{1/z-1}$$

$$Rn = 700/100 = 7; Z = 9$$

$$\phi = 7^{0.125} = 1.275$$

The nearest standard value of ϕ is 1.25

The speeds of shafts are 112, 140, 180, 224, 280, 355, 450, 560 and 710 rpm

$$T_a + T_b = T_c + T_d = T_e + T_f$$

$$N_b / N_a = T_a / T_b = 224/560$$

$$N_d / N_c = T_c / T_d = 280/560$$

$$N_f / N_e = T_e / T_f = 355/560$$

Assuming minimum number of teeth on the smaller gear as 24($T_a = 24$)

$$24 / T_b = 224/560$$

$$T_b = 60$$

$$T_c / T_d = 280/560$$

$$T_a + T_b = 84 = T_c + T_d$$

$$T_c = 28 \text{ and } T_d = 56$$

$$T_e / T_f = 355/560 \text{ and } T_e + T_f = 84$$

$$T_e = 33 \text{ and } T_f = 51$$

Considering the transmission between the intermediate and output shafts

$$T_g + T_h = T_i + T_j = T_k + T_l$$

$$N_h / N_g = T_g / T_h = 112/224$$

$$N_j / N_i = T_i / T_j = 224/224$$

$$N_l / N_k = T_k / T_l = 450/224$$

Assuming minimum number of teeth on the smaller gear as 20($T_g = 20$)

$$20 / T_h = 112/224$$

$$T_h = 40$$

$$T_i / T_j = 224/224$$

$$T_i + T_j = 60 = T_g + T_h$$

$$T_i = T_j = 30$$

$$T_k / T_l = 450/224$$

$$T_k + T_j = 60$$

$$T_k = 40 \text{ and } T_l = 20$$

4. Design an all geared speed gear box for a radial machine, with the following specifications:

Maximum size of the drill to be used = 50 mm

Minimum size of the drill to be used = 10mm

Maximum cutting speed = 40m/mt

Minimum cutting speed = 6 m /mt

Number of speeds = 12

Sol:

$$\text{Max. Speed } N_{\max} = 100 * V_{\max} / \pi D_{\min} = 1000 * 40 / \pi * 10 = 1280 \text{ rpm} = n_z$$

$$\text{Min. Speed } N_{\min} = 100 * V_{\min} / \pi D_{\max} = 1000 * 6 / \pi * 50 = 38 \text{ rpm} = n_1$$

The progression ratio,

$$\phi = (Rn)^{1/z-1}$$

$$Rn = 1280/38 = 33.68; Z=12$$

$$\phi = 33.68^{1/12-1} = 1.38$$

The nearest standard value of ϕ is 1.4

The speeds of shafts are 31.5, 45, 63, 90, 125, 180, 250, 355, 500, 710, 1000 and 1400 rpm

$$T_a + T_b = T_c + T_d = T_e + T_f$$

$$N_b / N_a = T_a / T_b = 500/1400$$

$$N_d / N_c = T_c / T_d = 710/1400$$

$$N_f / N_e = T_e / T_f = 1000/1400$$

Assuming minimum number of teeth on the smaller gear as 18 ($T_a = 18$)

$$18 / T_b = 500/1400$$

$$T_b = 50$$

$$T_c / T_d = 710/1400$$

$$T_a + T_b = 68 = T_c + T_d$$

$$T_c = 23 \text{ and } T_d = 45$$

$$T_e / T_f = 1000/1400 \text{ and } T_e + T_f = 68$$

$$T_e = 28 \text{ and } T_f = 40$$

Considering the transmission between the second and third shafts

$$T_g + T_h = T_i + T_j$$

$$N_h / N_g = T_g / T_h = 125/500$$

$$N_j / N_i = T_i / T_j = 355/500$$

Assuming minimum number of teeth on the smaller gear as 18 ($T_g = 18$)

$$18 / T_h = 125/500$$

$$T_h = 72$$

$$T_i / T_j = 355/500$$

$$T_i + T_j = 90 = T_g + T_h$$

$$T_i = 37 \text{ and } T_j = 53$$

Considering the transmission between the third and output shafts

$$T_k + T_l = T_m + T_n$$

$$N_l / N_k = T_k / T_l = 31.5/125$$

$$N_n / N_m = T_m / T_n = 250/125$$

Assuming minimum number of teeth on the smaller gear as 18 ($T_k = 18$)

$$18 / T_l = 31.5/125$$

$$T_l = 72$$

$$T_m / T_n = 250/125$$

$$T_m + T_n = 90 = T_k + T_l$$

$$T_m = 60 \text{ and } T_n = 30$$

QUESTION BANK

UNIT- I

A (2 Marks)

1. Give the relationship of ratio of tensions in a V-belt drive.
2. Define maximum tension in a belt.
3. What is the effect of centre distance and diameter of pulley on the life of belts?
4. What are the various losses in the power transmission by belts?
5. In what way the timing belt is superior to ordinary belt?
6. Why V belts are preferred than flat belts?
7. Define creep in belts.
8. What is the advantage of V belt over flat belt?
9. Define slip.
10. Distinguish regular lay and long lay ropes.
11. Give some application of wire ropes.
12. In what way silent chain is better than ordinary driving chain?
13. What do you understand by simplex, duplex and triplex chain?
14. Explain the chordal action of chain drive.
15. Explain the term "Crowning of Pulley".

UNIT- I

PART-B (11 Marks)

1. A workshop crane is lifting a load of 25kN through a wire rope and a hook. The weight of the hook etc is 15kN. The rope drum diameter may be taken as 30 times the diameter of the rope. The load is to be lifted with an acceleration of 1 m/s². Calculate the diameter of the wire rope. Take a factor of safety of 6 and Young's modulus for the wire rope 80 kN/mm². The ultimate stress may be taken as 1800 MPa. The cross-sectional area of the wire rope may be taken as 0.38 times the square of the wire rope diameter.
2. Two shafts whose centers are 1 meter apart are connected by a V-belt drive. The driving pulley is supplied with 95 kW power and has an effective diameter of 300mm. It runs at 1000 rpm, while the driven pulley runs at 375 rpm. The angle of groove on the pulleys is 40°. Permissible tension in 400 mm² cross-sectional area belt is 2.1 MPa. The material of the belt has density of 1100 kg/mm³. The driven pulley is overhung, the distance of the centre from the nearest bearing being 200 mm. The coefficient of friction between belt and pulley rim is 0.28. Estimate the number of belts required.
3. A V-belt drive is to transmit 15kW to a compressor. The motor runs at 1150 rpm and the compressor is to run at 400 rpm. Determine.
 - (i) Belt specifications
 - (ii) Number of belts
 - (iii) Correct centre distance and
 - (iv) Drive pulley diameter.
4. Design a chain drive to actuate a compressor from a 12 kW electric motor at 900 rpm, the compressor begins 250 rpm. Minimum centre distance should be 500 mm; the chain tension may be adjusted by shifting the motor on rails. The compressor is to work 8 hour/day.

5. Design a chain drive to actuate a compressor from 15kW electric motor running at 1,000 rpm, the compressor speed being 350 rpm. The minimum centre distance is 500 mm. the compressor operates 15 hours per day. The chain tension may be adjusted by shifting the motor.

6. Design a V-belt drive and calculate the actual belt tension and average stress for the following data. Driven pulley diameter, $D= 500$ mm, driver pulley diameter, $d=150$ mm, center distance $C=925$ mm, speed $N_1 = 1000$ rpm, $N_2 = 300$ rpm and power, $P = 7.5$ kW.

7) A crane is lifting a load of 18 kN through a wire rope and a hook. The weight of the hook etc. is 10kN. The load is to be lifted with an acceleration of 1m/sec^2 . Calculate the diameter of the wire rope. The rope diameter may be taken as 30 times the diameter of the rope. Take a factor of safety of 6 and Young's modulus for the wire rope $0.8 \times 10^5\text{N/mm}$. The ultimate stress may be taken as 1800 N/mm. The cross-sectional area of the wire rope may be taken as 0.38 times the square of the wire rope diameter.

8. A 15 kW squirrel cage motor, 1250 rpm is driving a centrifugal pump at 550 rpm. The centrifugal pump is located at 700 mm from the motor. Design a chain drive.

UNIT- II

PART- A (2 Marks)

- 1 Why is a gear tooth subjected to dynamic loading?
2. Differentiate the following terms with respect to helical gears:
 - (a) Transverse circular pitch
 - (b) Normal circular pitch and
 - (c) Axial pitch.
3. Why the crossed helical gear drive not used for power transmission
4. State the advantage of herringbone gear
5. What is interference in involutes profile?
6. How many number of teeth affects the design of gears?
- 7 What are the advantages of the helical gear over spur gear?
8. What is Herringbone gear? State its application.
9. What is working depth of a gear-tooth?
10. What is virtual number of teeth?
11. Define the term back lash.
12. What are the forms of gear tooth profile?
13. State some materials used for gear materials.
14. What are the conditions required for interchangeability?

UNIT- II PART-B (11 Marks)

1. Design a pair of helical gears to transmit 30kW power at a speed reduction ratio of 4:1. The input shaft rotates at 2000 rpm. Take helix and pressure angles equal to 25° and 20° respectively. The number of teeth on the pinion may be taken as 30.

2. Design a straight spur gear drive to transmit 8 kW. The pinion speed is 720 rpm and the speed ratio is 2. Both the gears are made of the same surface hardened carbon steel with 55RC and core hardness less than 350 BHN. Ultimate strength is 720 N/mm² and yield strength is 360 N/mm².
3. A motor shaft rotating at 1500 rpm has to transmit 15kW to a low speed shaft with a speed reduction of 3:1. Assume starting torque to be 25% higher than the running torque. The teeth are 20° involutes with 25 teeth on the pinion. Both the pinion and gear are made of C45 steel. Design a spur gear drive to suit the above conditions and check for compressive and bending stresses and plastic deformations. Also sketch the spur gear drive.
4. A helical gear with 30° helix angle has to transmit 35kW at 1500 rpm. With a speed reduction ratio 2.5. If the pinion has 24 teeth, determine the necessary module, pitch diameter and face width for 20° full depths the teeth. Assume 15Ni2Cr1Mo15 material for both pinion and wheel.
5. An electric motor is to be connected to a reciprocating pump through a gear pair. The gears are overhanging in their shafts. Motor speed = 1440 rpm. Speed reduction ratio = 5. Motor power = 36.8 kW. The gears are to have 20° pressure angles. Design a spur gear drive.
6. A pair of helical gears subjected to moderate shock loading is to transmit 37.5kW at 1750 rpm of the pinion. The speed reduction ratio is 4.25 and the helix angle is 15°. The service is continuous and the teeth are 20° FD in the normal plane. Design the gears, assuming a life of 10,000 hours.
7. A compressor running at 300 rpm is driven by a 15 Kw, 1200 rpm motor through a 14½° full depth spur gears. The centre distance is 375 mm. The motor pinion is to be of C30 forged steel hardened and tempered, and the driven gear is to be of cast iron. Assuming medium shock condition, design the gear drive.

UNIT- III

PART- A (2 Marks)

1. Why is multistart worm more efficient than the single start one?
2. What factors influence backlash in gear drives?
3. State true or false and justify.
“Miter gears are used for connecting non-intersecting shafts”
4. What is zero bevel gear?
- 5 Define the following terms:
 - (a) Cone distance
 - (b) Face angle
6. In which gear drive, self-locking is available?
7. State the use of bevel gears.
8. What is irreversibility in worm gears?
9. How can you specify a pair of worm gear?
10. What are the materials commonly used for worm gears?
11. List out the main types of failure in worm gears.
12. What are the various losses in worm gear?
13. What are forces acting on bevel gears?
14. What is a crown gear?
15. Where do we use worm gears?

UNIT- III

PART-B (11 Marks)

1. A kW motor running at 1200 rpm drives a compressor at 780 rpm through a 90° bevel gearing arrangement. The pinion has 30 teeth. The pressure angle of the teeth is 20° . Both the pinion and gear are made of heat treated cast iron grade 35. Determine the cone distance, average module and face width of the gears.
2. Design a pair of bevel gears for two shafts whose axes are at right angles. The power transmitted is 25kW. The speed of the pinion is 300 rpm and the gear is 120 rpm.
3. A 2 kW power is applied to a worm shaft at 720 rpm. The worm is of quadruple start with 50mm as pitch circle diameter. The worm is of quadruple start type with 50mm as pitch circle diameter. The worm gear has 40 teeth with 5mm module. The pressure angle in the diametral plane is 20° . Determine (i) the lead angle of the worm, (ii) velocity ratio, and (ii) centre distance. Also, calculate efficiency of the worm gear drive, and power lost in friction
4. A pair of straight tooth bevel gears has a velocity ratio of $4/3$. The pitch diameter of the pinion is 150 mm. The face width is 50mm. The pinion rotates at 240 rev/min. The teeth are 5mm module, $14\ 1^\circ$ involutes. If 6 kW is transmitted, determine (i) the tangential force at the Mean radius (ii) the pinion thrust force (iii) the gear thrust force. Draw the free body diagrams indicating the forces.
5. Design a worm gear drive with a standard centre distance to transmit 7.5 kW from a worm rotating at 1440 rpm to a worm wheel at 20 rpm.
6. Design the teeth of a pair of bevel gears to transmit 18.75 kW at 600 rpm of the pinion. The velocity ratio should be about 3 and the pinion should have about 20 teeth which are full depth 20° involutes. Find the module, face width, diameter of the gears and pitch core angle for both gears.
7. A 90° degree straight bevel gear set is needed to give a 3:1 reduction. Determine the pitch cone angle, pitch diameter, and gear forces if the, 25 degree pressure angle pinion has 15 teeth of pitch circle diameter, 4, and the transmitted power is 8 HP at 550 pinion rpm.
8. Design a worm gear drive to transmit 22.5 kW at a worm speed of 1440 rpm. Velocity ratio is 24:1. An efficiency of at least 85% is desired. The temperature rise should be restricted to 40°C . Determine the required cooling area.
9. A speed reducer unit is to be designed for an input of 11.25kW with a transmission ratio of 20. The speed of the hardened steel worm is 1500 rpm. The worm wheel is to be made of bronze. The tooth form is to be 20in involute.
- 10 Design a bevel gear drive to transmit 3.5 kW with the following specifications: speed ratio = 4; driving shaft speed = 200 r.p.m.; drive is non-reversible; material for pinion is steel; material for wheel is cast iron; and life 25000 hours.
- 11 Design a worm gear drive to transmit a power of 22.5 kW. The worm speed is 1440 r.p.m. and the speed of the wheel is 60 rpm. The drive should have a minimum efficiency of 80%

and above. Select suitable materials for worm and wheel and decide upon the dimensions of the drive.

UNIT- IV

PART- A (2 Marks)

1. What is the function of spacers in a gear – box?
2. What is backlash in gears?
3. State the advantage of gear box.
4. List six standard speeds starting from 18 rpm with a step ratio 1.4.
5. Sketch the kinematics layout of gears for 3 speeds between two shafts.
6. What does the ray diagram of a gear box indicates?
7. What are preferred numbers?
8. List any two methods used for changing speeds in gear box.
9. What situation demands the use of gear box?
10. State any three basic rules followed in designing a gear box.

PART-B (11 Marks)

1. Sketch the arrangements of a six speed gear box. The minimum and maximum speeds required are around 460 and 1400 rpm. Drove speed is 1440 rpm. Construct speed diagram of the gear box and obtain various reduction ratios. Use standard output speeds and standard step ratio. Calculate number of teeth in each gear and verify whether the actual output speeds are within + 2% of standard speeds.
2. Design the layout of a 12 speed gear box for a milling machine having an output of speeds ranging from 180 to 2000 rpm. Power is applied to the gear box from a 6 kW induction motor at 1440 rpm. Choose standard step ratio and construct the speed diagram. Decide upon the various reduction ratios and number of teeth on each gear wheel sketch the arrangement of the gear box.
3. In a milling machine, 18 different speeds in the range of 35 rpm and 650 rpm are required. Design a three stage gear box with a standard step ratio. Sketch the layout of the gear box, indicating the number of teeth n each gear. The gear box receives 3.6 kW from an electric motor running at 1,440 rpm. Sketch also the speed diagram.
4. Design a nine-speed gear box for a machine to provide speed ranging from 100 to 1500rpm. The input is from a motor of 5KW at 1440rpm. Assume any alloy steel for the gear.
5. A machine tool gear box is to have 9 speeds. The gear box is driven by an electric motor whose shaft rotational speed is 1400 r.p.m. The gear box is connected to the motor by a belt drive. The maximum and minimum speeds required at the gear box output are 1000 r.p.m. and 200 r.p.m. respectively. Suitable speed reduction can also be provided in the belt drive. What is the step ratio and what are the values of 9 speeds? Sketch the arrangement. Obtain the number of teeth on each gear and also the actual output speeds.
6. A six speed gear box is required to provide output speeds in the range of 125 to 400 r.p.m. with a step ratio of 1.25 and transmit a power of 5 kW at 710 r.p.m. Draw the speed diagram and kinematics diagram. Determine the number of teeth module and face width of all gears, assuming suitable materials for the gears. Determine the length of the gear box along the axis of the gear shaft.

