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## DEPARTMENT OF MECHANICAL ENGINEERING

# SIXTH SEMESTER

# SUBJECT : MET-63 THERMAL ENGINEERING

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# UNIT-I

# INTERNAL COMBUSTION ENGINES

## INTRODUCTION

The Internal Combustion engine (I.C. engine) is a heat engine that converts chemical energy of fuel into mechanical energy. Chemical energy of a fuel is first converted into thermal energy by means of combustion or oxidation with air inside the engine. This thermal energy is again converted into useful work through mechanical mechanism of the engine. Most of the I.C. engines are reciprocating engines having pistons that reciprocate back and forth in cylinders internally within the engine. This chapter mainly concentrates on the study of this type of engine.

## CLASSIFICATION OF IC ENGINES

The I.C. engines are classified on the basis of

(i) Type of ignition;
   a. Spark Ignition engines (S.I. engines)
   b. Compression Ignition engines (C.I. engines)

(ii) Cycle of operation (Thermodynamics cycle):
   a. Otto cycle engine
   b. Diesel cycle engine
   c. Dual cycle engine

(iii) Engine cycle per stroke:
   a. Four stroke cycle
   b. Two stroke cycle

(iv) Types of fuel used:
   a. Petrol engine
   b. Diesel engine
   c. Gas engine

(v) Method of cooling:
   a. Air-cooled engines
   b. Water-cooled engines

(vi) Number of cylinders:
   a. Single cylinder engine
   b. Two cylinder engine
   c. Three cylinder engine
   d. Four cylinder engine
   e. Six cylinder engine
   f. Eight cylinder engine
   g. Twelve cylinder engine
   h. Sixteen cylinder engine

(vii) Valve location:
   a. L-head engine
   b. I-head engine
   c. F-head engine
   d. T-head engine

(viii) Arrangement of cylinders:
   a. Vertical engine
   b. Horizontal engine

c. Radial engine

d. V-engine engine

e. Opposed cylinder engine

(ix) Speed of the engine:

a. Low speed engine
b. Medium speed engine
c. High speed engine

(x) Types of lubrication system:

a. Wet sump lubrication system

b. Dry sump lubrication system

(xi) Method of governing:

a. Quantity governing
b. Quality governing
c. Hit and Miss governing

(xii) Field of application:

a. Automobile, truck, bus
b. Locomotive engine
c. Stationary engine
d. Marine engine
e. Aircraft engine

**ENGINE CONSTRUCTION**

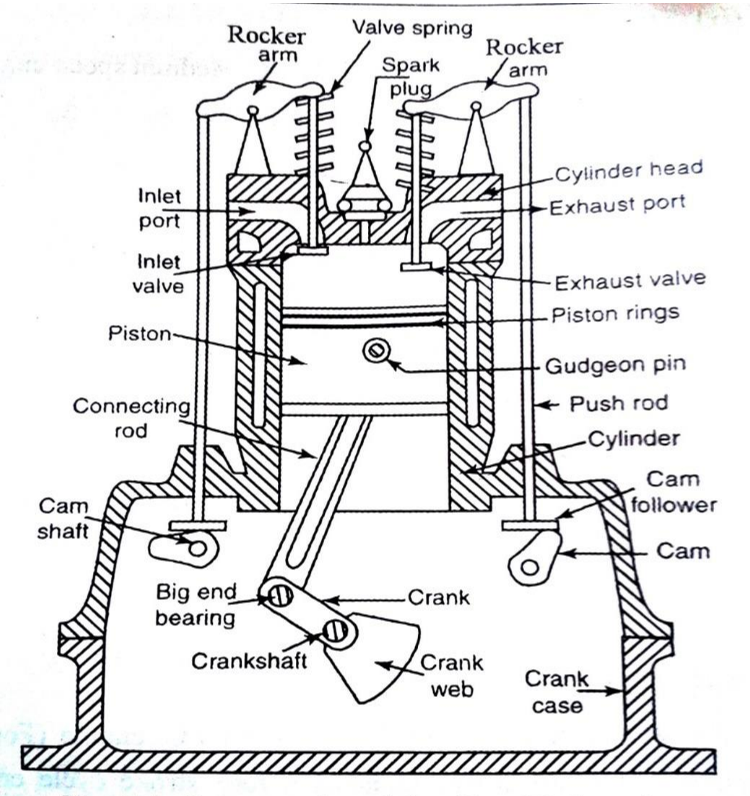


*Fig. 2.1. Constructional details of I.C engine*

Fig 2.1 shows the construction details of an I.C. engine (Four stroke petrol engines). The main components of a four stroke cycle engine are cylinder, piston, connecting rod, piston rings, cam shaft, crank shaft, crank case, inlet and outlet valves, spark plug, cylinder head, push rod, gudgeon or piston pin, rocker arm, cam follower, valve spring, big end bearing, inlet port etc. The piston reciprocates inside the cylinder; Piston rings are inserted in the circumferential grooves of the piston. The cylinder and cylinder head are bolted together.

The reciprocating motion of the piston is converted into rotary motion of the crankshaft by means of a connecting rod and crank. The small end of the connecting rod is connected to the piston by a gudgeon pin or piston pin. The big end of the connecting rod is connected to the crank pin. Crank pin is a bearing surface and it is rigidly fixed to the crankshaft.

The crankshaft is mounted on the main bearing. The main bearings are housed in the crankcase. Camshaft is driven by the crankshaft through timing gears. The camshaft actuates the inlet and outlet valves. The valve springs are provided to bring back the valves in the closed position.

The oil sump containing lubricating oil is provided at the bottom of the crankcase. Lubricating oil is circulated to the various parts of the engine from the oil sump. A spark plug is providing in petrol engines to ignite the air-fuel mixture in the engine cylinder. An injector is provided in diesel engines to inject the fuel into hot compressed air during power stroke.

## DIFFERENT PARTS OF I.C. ENGINE:

1. Parts common to both petrol and diesel engine

i)cylinder
ii)Cylinder head
iii)piston
iv)Piston rings
v)Gudgeon pin
vi)Connecting rod
vii)crankshaft
viii)Crank
ix)Engine bearing
x)Crank case
xi)flywheel
xii)Governor
xiii)Valve and valve operating mechanism

2. Parts of petrol engine only:

i)spark plug
ii)carburetor
iii)fuel pump

3.parts for diesel engine only

i)fuel pump
ii)injector

| Name of the parts | Material | function | Method of manufacturing |
|---|---|---|---|
| Cylinder | Hard grade Cast iron | Contains gas under pressure and guides the piston | Casting |
| Cylinder head | Cast iron or aluminum | Main function is to seal the working end of the cylinder and not to permit entry and exit of gases on overhead valve engine | Casting<br>Forging |
| Piston | Cast iron or aluminum alloy | It acts as a face to receive gas pressure and transmits the thrust to the connecting rod | Casting<br>Forging |
| Piston rings | Cast iron | Their main function is to provide a good sealing fit between the piston and cylinder | Casting |
| Gudgeon pin | Hardened steel | It supports and allows the connecting rod to swivel | Forging |
| Connecting rod | Alloy steel | It transmits the piston load to the crank causing the latter to turn thus converting the reciprocating motion of the piston into rotary motion of the crankshaft | Forging |
| Crankshaft | Special cast irons | It converts the reciprocating motion of the piston into the rotary motion | Forging |
| Main bearings | Steel or bronze | The function of bearing is to reduce. The friction and allow the parts to move easily | Casting |
| Flywheel | Steel or cast iron | In engine it takes care of fluctuations of speed during thermodynamic cycle | Casting |
| Inlet valve | Silicon chrome steel with about 3% carbon | Admits the air or mixture of air and fuel into engine cylinder | Forging |
| Exhaust valve | Austenitic steel | Discharges the product of combustion | |

**WORKING OF FOUR STROKE CYCLE ENGINES:**

In four-stroke cycle engines, one working cycle is completed in four stroke of the piston or two revolution of the crankshaft. Hence, it is called as four stroke engine. In 4-stroke engine, 2 valves are placed instead of port as that of two stroke engines. These are termed as inlet and exhaust values. The schematic diagram of four-stroke cycle engine is shown in fig. 2.12((a)

## Working of four stroke cycle (petrol) S.I. engine

It consists of the following four strokes

1. Suction stroke
2. Compression stroke
3. Power or expansion stroke
4. Exhaust stroke

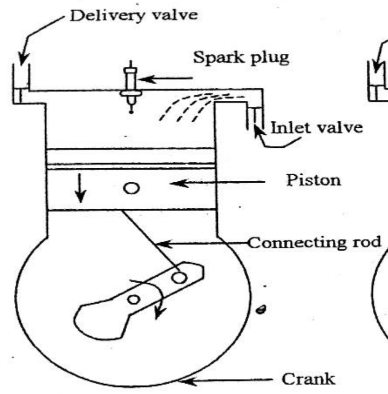


*Fig. 3.14. (a)*

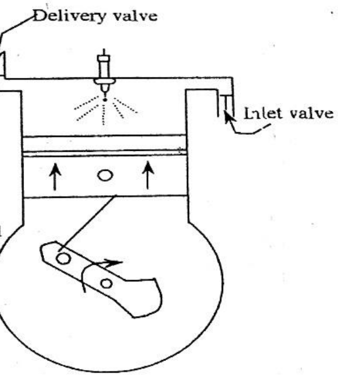


*Fig. 3.14. (b)*

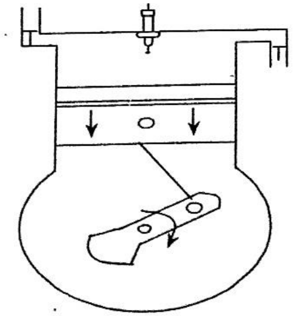


*Fig. 3.14.(c)*

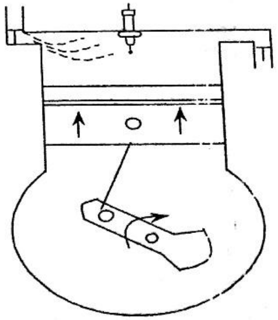


*Fig. 3.14. (d)*

### 1. Suction Stroke:

At the beginning of the stroke, the piston is at the top most position (TDC) and is ready to move downward. As the piston moves downwards, vacuum will create inside the cylinder. Due to this vacuum, air fuel mixture from the carburetor is sucked into the cylinder through inlet valves till the piston reaches bottom most position (BDC). During the suction stroke, exhaust valve remains in closed condition and inlet valve remains open. At the end of the suction stroke, the inlet valve will be closed Refer fig. 2.12.(a)

### 2. Compression stroke:

During the compression stroke, both the inlet and exhaust valves are in closed condition and the piston moves upward from BDC to compress the air fuel mixture. This process will continue till the piston reaches TDC as shown in fig. 2.12(b). The compression ratio of engine varies from 6 to 8. The pressure at the end of compression is about 600 to 1200kN/M$^2$.The temperature at the end of the compression is 250 to 300$^o$C. At the end of this stroke, the mixture is ignited by spark plug.It leads to increase in pressure and temperature of the mixture instantaneously.

### 3. Power or Expansion stroke:

Both the pressure and temperature range of the ignited mixture are 1800 to 2000$^{o}$C and 3000 to 4000 KN/m2 respectively. During the expansion stroke, both the valves are remains closed. The rise in pressure of the mixture exerts an impulse on the piston and pushes it downward. Therefore, the piston moves from TDC to BDC. This stroke is known as power stroke Fig. 2.12.

### 4. Exhaust stroke:

During the exhaust stroke, the piston moves from DC to TDC, the exhaust valve is opened and inlet valve is closed. The burnt gases are released through the exhaust valve when the piston moves upward. As the piston reaches the TDC, again the inlet valves will open and the fresh air fuel mixture enters into the cylinder for the next cycle of operation.

It is obvious from the above operations; only one power stroke is produced in each and every four stroke of the piston or two revolution of the crankshaft. Hence, it is termed as four-stroke engine.

## VALVE TIMING DIAGRAM FOR A FOUR-STROKE CYCLE S.I. ENGINE:

The exact moment at which each of the valves open and close with reference to the position of piston and crank can be shown graphically in a diagram. This diagram is known as “Valve timing diagram”.

### Theoretical valve timing diagram:

In theoretical valve timing diagram, inlet and exhaust valves open and close at both the dead centers. Similarly, all the processes are sharply completed at the TDC of BDC. Fig. 2.13. Shows theoretical valve timing diagram for four stroke S.I. engines.

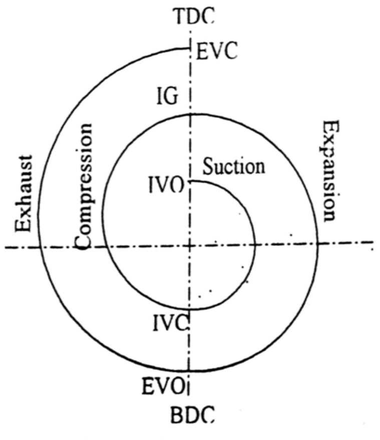


*Fig 3.16 Theoretical valve timing diagram*

| | |
|---|---|
| IVO- | Inlet Valve Open |
| IVC- | Inlet Valve Close |
| IS | - Ignition starts |
| EVO | - Exhaust valve Open |
| EVC | - Exhaust valve Close |
| TDC | - Top Dead Center |
| BDC | - Bottom Dead Center |

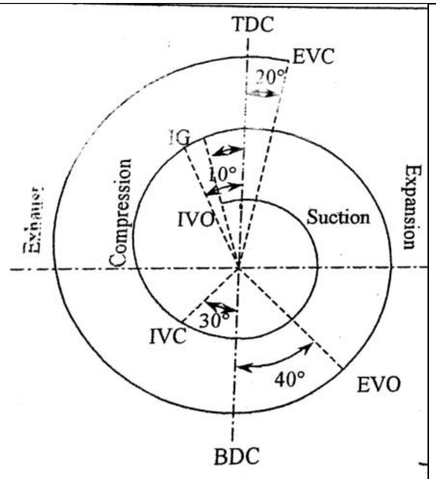


*Fig 3.17 Actual valve timing diagram*

| | |
|---|---|
| IVO- | Inlet Valve Open |
| IVC- | Inlet Valve Close |
| IS | -Ignition starts |
| EVO | - Exhaust valve Open |
| EVC | - Exhaust valve Close |
| TDC | - Top Dead Center |
| BDC | -Bottom Dead Center |

**Actual valve timing diagram:**

Fig.Shows actual valve timing diagram for four stroke S.I. engine. The inlet valve opens 10-30o before the TDC. The air-fuel mixture is sucked into the cylinder till the inlet valve closes. The inlet valve closes 30-40o or even 60o after the BDC. The charge is compressed till the spark occurs. The spark is produced 20-40o before the TDC. It gives sufficient time for the fuel to burn. The pressure and temperature increase. The burnt gases are expanded till the exhaust valve opens.

The exhaust valve opens 30-60o before the BDC. The exhaust gases are forced out from the cylinder till the exhaust valve closes. The exhaust valve closes 8-20o after the TDC. Before closing, again the inlet valve opens 10-30o before the TDC. The period between the IVO and EVC is known as valve overlap period. The angle between the inlet valve opening and exhaust valve closing is known as angle of valve overlap.

## FOUR STROKE CYCLE C.I. ENGINE (DIESEL ENGINE):

The working of four-stroke CI engine is similar to that of SI engine except. Here, fuel injector is placed instead of spark plug and only air is sucked into the cylinder during suction strokes. The operations are described as follows:

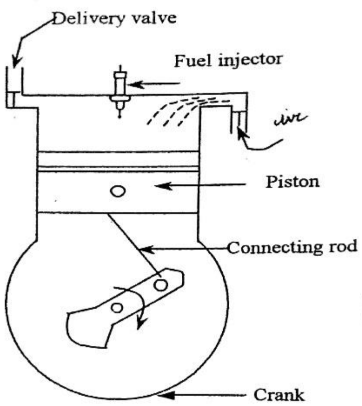


*Fig. 3.18. (a)*

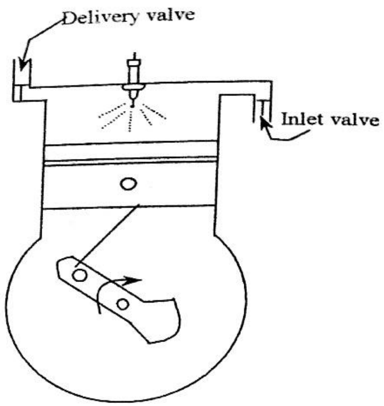


*Fig. 3.18. (b)*

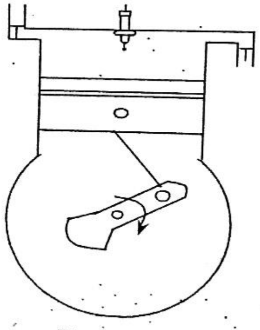


*Fig. 3.18. (c)*

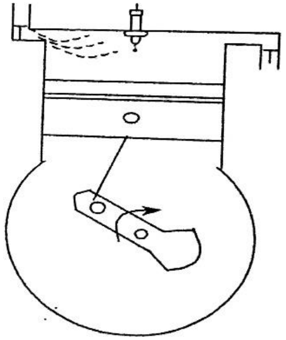


*Fig. 3.18. (d)*

1. Suction stroke:

   During the suction stroke, the piston moves from TDC to BDC. The inlet valve is in open condition whereas exhaust valve is closed. When the piston moves from top to bottom, the fresh air is admitted inside the cylinder through inlet valve as shown in fig. 2.15. (a).

2. Compression stroke

   During the compression stroke, both the inlet and exhaust valves are closed. The piston moves from BDC to TDC to compress the air. In case of CI engine, the compression ratio varies from 12 to 18. The pressure at the end of compression is about 3500 to 4000kN/M2. The temperature of the compressed air reaches 600 to 700$^{o}$C.

3. Power Stroke:

   In this stroke also, both the inlet and exhaust valves are in closed position. The fuel injector opens just before the beginning of the third stroke; it injects the fuel in atomized form. Ignition of the fuel takes place automatically by means of high pressure and temperature air. The pressure and temperature further will increase due to combustion, it pushes the piston towards down. Thus, it produces power stroke

4. Exhaust stroke:

   During this stroke, inlet valve is closed and the exhaust valve opens. The piston moves from BDC to TDC. Its blows out the burnt gases from the cylinder. Thus, one cycle of operation is completed and repeated again and again in the same manner.

## VALVE TIMING DIAGRAM FOR A FOUR STROKE C.I. ENGINE:

Fig.2.16 shows the actual valve-timing diagram for four-stroke diesel engine.

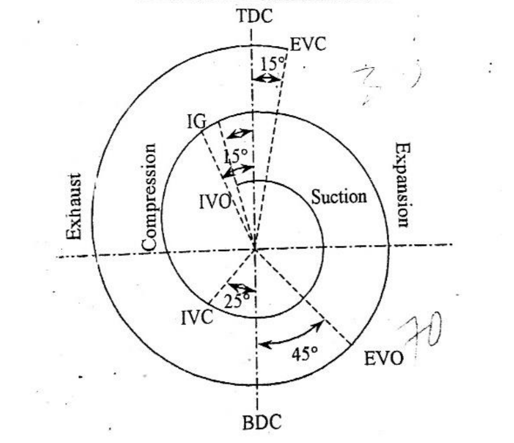


*Fig 3.20 Actual valve timing diagram*

| | |
|---|---|
| IVO - | Inlet Valve Open |
| IVC - | Inlet Valve Close |
| IS | - Ignition starts |
| EVO | - Exhaust valve Open |
| EVC | -Exhaust valve Close |
| TDC | -Top Dead Center |
| BDC | -Bottom Dead Center |

The inlet valve opens (IVO) $10^{o}$ to $25^{o}$ before the TDC. Fresh air is sucked into the engine cylinder till the inlet valve closes. The inlet valve closes (IVC) $25^{o}$ to $50^{o}$ after the BDC. The air is compressed till the fuel is injected.

The fuel injection starts (FIS) $5^{o}$ to $10^{o}$ before TDC in the compression stroke. The air-fuel mixture burns. Both the temperature and pressure increase. The burning gases are expanded till the exhaust valve opens.

The exhaust valve opens (EVO) $30^{o}$ to $50^{o}$ before the BDC. The exhaust gases are forced out of the cylinder till the exhaust valve closes.

The exhaust valves close (EVC) $10^{o}$ to $15^{o}$ after the TDC. Before closing the exhaust valve, again the inlet valve opens $10^{o}$ to $25^{o}$ before the TDC. The period when both the inlet and exhaust valves are opened is known as valve overlap period. The angle between these two events is known as angle of valve overlap

## TWO STROKE CYCLE ENGINE:

In two stroke engines, one working cycle is completed in two strokes of the piston (i.e. one up and down movement) or one revolution of the crankshaft. Hence, it is called as two stroke engines. One working cycle consists of the following four processes.

1. Suction
2. Compression
3. Power or expansion
4. Exhaust

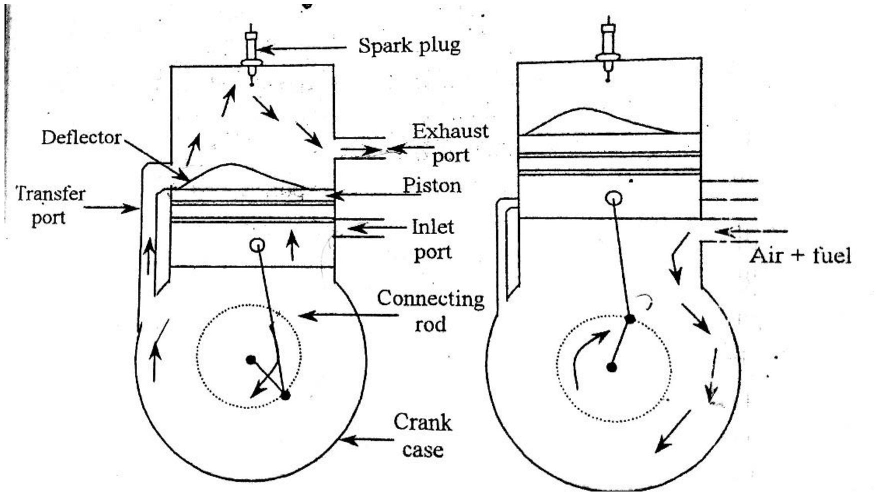
The schematic diagram of two-stroke engine is given in fig. 2.17. It consists of a cylinder in which all the operations are performed. The piston reciprocates inside the cylinder.

The piston is connected to the crankshaft through connecting rod and the crank. Inside of the cylinder, valve ports are placed. Opening and closing of ports are performed by the position of the piston.

The above diagram describes the spark ignition (SI) engine. in the case of compression ignition (CI) engines, the spark plug is replaced by fuel injector. A two-stroke engine consists of ports instead of valves. In two stroke engines, both the suction and compression process are carried out in the first stroke, expansion and exhaust processes in the second stroke. Therefore, it develops power in each revolution of the clerk shaft.

## WORKING OF TWO STROKE CYCLE S.I. ENGINE (PETROL ENGINE)

The working principles of two stroke S.I. engine is described as follows.



**First stroke (Suction and Compression):**

The first stroke consists of the suction and compression processes. During the first stroke, the piston moves upward from BDC to TDC. When the piston is at BDC, the partially compressed air fuel mixture from crank case enters into the cylinder through transfer port as shown fig. 2.18 (a). Then the piston moves upward and compresses the air contained in it till the piston reaches TDC. At the end of the compression stroke, the spark plug produces spark, it will ignite the compressed high pressure fuel air mixture.

When the piston is at TDC, the inlet port opens and the air fuel mixture from the carburetor enters into the crank case as shown in fig. 2.18. (b). Thus, one stroke of the piston is completed.

**Second stroke (expansion or power and exhaust stroke):**

When air fuel mixture is ignited, both the pressure and temperature of the products of combustion will suddenly increase. Therefore, the piston receives power impulse from the expanded gas and it pushes the piston downward and also produces the power stroke. This process is described in fig. 2.18. (c).During the expansion stroke, some of the heat energy produced is converted into mechanical work.

During downward stroke of piston, already entered air fuel mixture in the crank case is partially compressed by the underside of the piston. This pre compression process is called crank case compression. At the end of power stroke, the exhaust port opens and burnt gases are sent out of the engine through this port as shown in fig 2.18. (d).

At the same time, all the burnt gases are not exhausted. Some portion of it will remain in the cylinder. When the piston moves to BDC, the fresh air fuel mixture from crank case enters into the cylinder to sweep out the burnt gases. The process of sweeping out the exhaust gases with help of fresh air fuel mixture is known as scavenging. The scavenging helps to remove the burnt gases from the cylinder.

## PORT TIMING DIAGRAM FOR TWO-STROKE CYCLE S.I. ENGINE:

Two stroke cycle engines are provided with ports. The timing sequence of events like opening and closing of various ports are shown graphically in terms of crank angles from dead center positions. This diagram is known as port timing diagram. Fig. 2.19 shows the port timing diagram for two stroke cycle SI engines. The inlet port is opened by the piston 45° to 55° before TDC. The inlet port is closed by the piston 45° to 55° after TDC position.

The exhaust port is opened by the piston 40° to 50° before BDC. The exhaust port is closed by the piston 40° to 55° after BDC position.

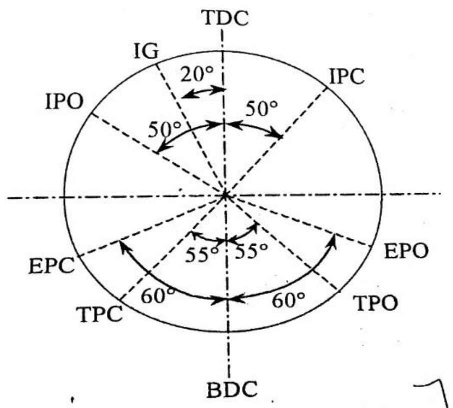


*Fig 3.24 Port timing diagram for S.I.*

IOP=inlet port open

IPC=inlet port closed

IG= ignition starts

EPO= exhaust port open

TPO= transfer port open

TPC= transfer port closed

EPC= exhaust port closed

TDC= top dead center

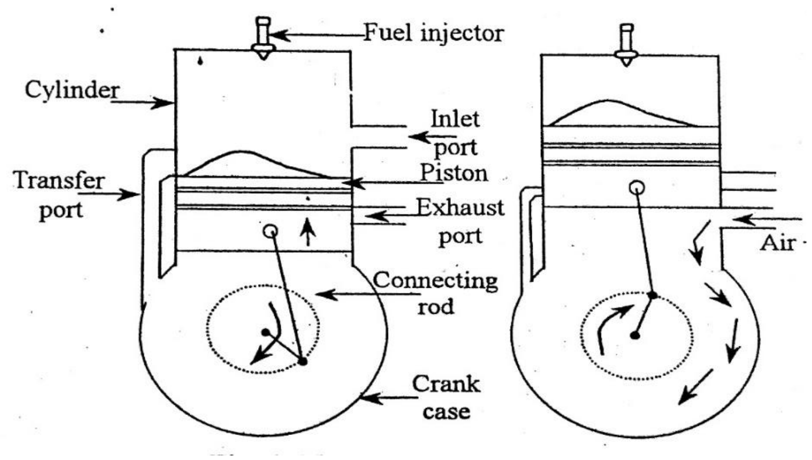
BDC= bottom dead center

The transfer port is opened by the piston 30°to 40° before BDC. The transfer port is closed by the piston 30°to 40° after BDC. Ignition occurs 15° to 20° before TDC.

| Si no | Port valve timing | Events during port timing |
|---|---|---|
| 1 | IPO-IPC | Air fuel mixture is sucked into the crank case |
| 2 | IPC-TPO | Air fuel mixture is partially compressed in the crank case which is known as crank case compression. |
| 3 | TPO-TPC | Partially compressed air-fuel mixture is transferred into the engine cylinder. |
| 4 | TPC-IG | Air-fuel mixture is compressed in the cylinder. |
| 5 | IG | Air-fuel mixture is ignited by the spark plug and burns. |
| 6 | IG-EPO | The burning gases expand for doing work on the piston. |
| 7 | EPO-EPC | Burnt gases are pushed out of the engine. |

WORKING OF TWO STROKE CYCLE C.I. ENGINE (DIESEL ENGINE)

The working of two stroke C.I. engine slightly differs from the S.I. engine instead of sparkplug the fuel injector is placed on the top of the cylinder.



First stroke:

In the first stroke, the piston move from BDC to TDC. When the piston is at BDC, partially compressed air from the crank case enters into the cylinder through transfer port as shown in fig. 2.20.(a). then the piston move upward and further compresses the air into high pressure and temperature till the piston reaches TDC. At the end of the compression stroke the fuel injector injects the fuel in atomized from and ignited automatically by the compressed air. During the upward movement of the piston, a slight vacuum will be produced at the crank cast to suck the air from the atmosphere.

Second stroke:

When the fuel and air are ignited, it suddenly increases the pressure and temperature of the gas. Therefore, the gases will expand and it pushes the piston downward and producing the power stroke as shown in fig. 2.20.(c). During the expansion, some of the heat energy produced is converted in to mechanical work.

During the downward stroke of the piston, first it uncovers the exhaust port and the burnt gases are sent out of the engine, as shown in fig. 2.20. (d). At the same time, all the burnt gases are not exhaust. Therefore, scavenging takes place in the cylinder. (Scavenging process already defined in previous section).

At the time off downward movement of the piston, already entered air in the crankcase is partially compressed by the underside of the piston. This process is called crank case compression.

## PORT TIMING DIAGRAM FOR TWO STROKE C.I. ENGINE

Fig 2.21. Shows the port timing diagram for two stroke C.I. Engine. The port timing diagram for two strokes C.I. Engine is exactly similar to the two-stroke cycle S.I. Engine except the position of opening and closing of fuel valve. Fuel is injected into the cylinder 10o to 15o before TDC.Fuel injection continues till 15$^{o}$ to 20$^{o}$ after TDC. The other difference between these two is, the charging and scavenging period of C.I. engine (90$^{o}$) is greater than the SI. Engine (70$^{o}$).Since, there is no danger of loss of fuel during scavenging of diesel engine as only air is charged.

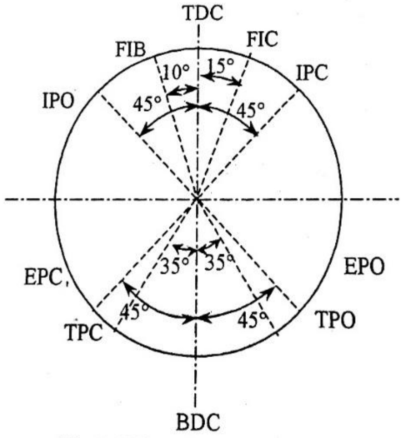


*Fig 3.27 Port timing diagram*

| | |
|---|---|
| IPO - | Inlet port open |
| IPC - | Inlet port close |
| IG | - Ignition starts |
| EPO - | Exhaust Port Open |
| TPO - | Transfer port Open |
| TPC - | Transfer port Close |
| EPC - | Exhaust port close |
| TDC | - Top Dead Center |
| BDC- | Bottom Dead Center |

**The various events are given below:**

| | | |
|---|---|---|
| 1. | IPO-IPC | Air is sucked into the crankcase through plate valve |
| 2. | IPC-TPO | Air is compressed partially in the crankcase. (Crankcase compression |
| 3. | TPO-TPC | Partially compressed air is transferred to the engine cylinder through transfer port. |
| 4. | TPC-FIB | Air is compressed in the cylinder. Both the pressure and temperature of the air increase. |
| 5. | FIB-FIC | Fuel is injected into the hot compressed air. Fuel mixes with hot air and burns. |
| 6. | FIC-EPO | The burning gases expand and move the piston t do work |
| 7. | EPO-EPC | Burnt gases escape to the atmosphere through exhaust port. |

## COMPARISON OF TWO STROKES AND FOUR STROKE ENGINES:

| **S. No** | **Two stroke cycle engine** | **Four stroke cycle engine** |
|---|---|---|
| 1. | Cycle is completed in 2 stroke or one revolution of the crankshaft | One cycle is completed in 4 strokes or two revolution of the crank shaft. |
| 2. | It develops twice the number of power strokes than the four stroke engines. | It develops half the number of power stroke than two stroke engine. |
| 3. | For the same power developed, the two stroke engine is much lighter, less bulky and occupies less floor area. | For the same power developed, the four stroke engine is bulky, heavier and occupies more floor area. |
| 4. | Turning moment is more uniform and hence lighter flywheel is required. | Turning moment is not uniform and hence heavier flywheel is required. |
| 5. | It contains ports which are operated by the piston itself therefore no separate mechanisms are required. | It contains valves which are operated by separate mechanisms. |
| 6. | Initial cost is low due to less complexity in mechanism. | Initial cost is high because of heavier and complicated mechanisms. |
| 7. | Mechanical efficiency is more. | Mechanical efficiency is low. |
| 8. | Easy to start. | It requires separate starting motor. |
| 9. | It can be run in either direction, which is useful in marine engines. | It can be run in only one direction. |
| 10. | Thermal efficiency is low. | Thermal efficiency is high. |
| 11. | Volumetric efficiency is low. | Volumetric efficiency is more. |
| 12. | Greater cooling and lubrication are required. | Lesser cooling and lubrication are required. |
| 13. | Overall efficiency is less. | Overall efficiency is more. |
| 14. | Greater rate of wear and tear. | Lesser rate of wear and tear. |
| 15. | It is used in light vehicles only (e.g.) scooters, motor cycles, mopeds etc. | Used in heavy vehicles such as cars, buses, trucks etc. |

| | | |
|---|---|---|
| 16. | Sudden release of exhaust gases makes the exhaust more noisy. | Release of exhaust gas is more uniform and hence noiseless operation. |
| 17. | Specific fuel consumption is more because of escaping of the fresh charge with exhaust gases. | Specific fuel consumption is less because of separate exhaust stroke. |
| 18. | Less compression ratio. | More compression ratio. |

## COMPARISON OF S.I. AND C.I. ENGINES

| **SL. No.** | **S.I. or Petrol engine** | **C.I. or Diesel engine** |
|---|---|---|
| 1. | During the suction stroke, air fuel mixture is drawn from carburetor. | During the suction stroke, air is only drawn from the atmosphere. |
| 2. | Carburetor is used to mix the air and fuel in required proportion. | Fuel injector or atomizer is required to inject the fuel into cylinder in atomized form. |
| 3. | Spark plug is required to ignite the fuel air mixture. | Fuel is ignited automatically by high pressure & temperature air. |
| 4. | Compression ratio varies from 6 to 8. | Compression ratio varies from 12 to18. |
| 5. | It is operated by Otto cycle or constant volume cycle. | It is operated by Diesel cycle or constant pressure cycle. |
| 6. | The starting is easy due to lower compression ratio. | The starting is little difficult due to higher compression |
| 7. | Running cost is high because of high cost of fuel. | Running cost is less because of lower cost of fuel. |
| 8. | For the same power, less space is required. | For the same power, more space is required. |
| 9. | Initial cost is low. | Initial cost is high. |
| 10. | Maintenance cost is less because of fewer parts. | Maintenance cost is more because of more number of parts. |
| 11. | Thermal efficiency is low. | Thermal efficiency is considerably more. |
| 12. | These are used for high speed applications. | These are used for low speed operations. |

## SCAVENGING OF I.C. ENGINE:

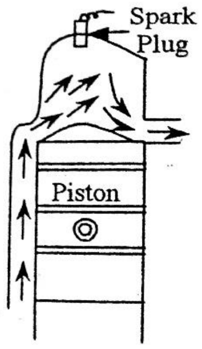
The sequence of operations in a cycle of an IC engine. The last stroke of an IC engine is the exhaust, which means the removal of burnt gases from the engine cylinder. It has been experienced that the burnt gases in the engine cylinder are not completely exhausted before the suction stroke. But a part of the gases still remain inside the cylinder and mix with the fresh charge. As a result of this mixing, the fresh charge gets diluted and its strength is reduced. The scientists and engineers, all over the world, have concentrated on the design of their IC engine so that the burnt gases are completely exhausted from the engine cylinder before the suction starts. The process of removing burnt gases, from the combustion chamber of the engine cylinder, is known as scavenging.

Now we shall discuss the scavenging in four-stroke and two- stroke cycle engine.

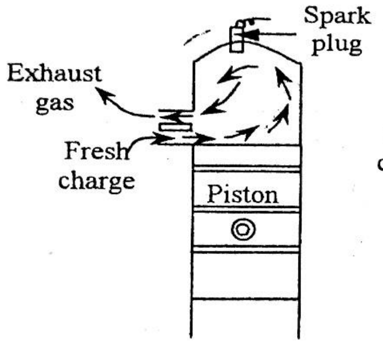
1. **Four-stroke cycle engine:**
   In a four stroke cycle engine, the scavenging is very effective as the piston during the exhaust stroke, pushes out the burnt gases from the engine cylinder. It may be noted that a small quantity of burnt gases remain in the engine cylinder clearance space.
2. **Two -stroke cycle engine:**
   In a four stroke cycle engine, the scavenging is less effective as the exhaust port is open from a small fraction of the crank revolution. Moreover, as the transfer and exhaust port are open simultaneously during a part of the crank revolution, therefore fresh change also escapes out along with the burnt gases. This difficulty is overcome by designing the piston crown of a particular shape

### Types of scavenging:

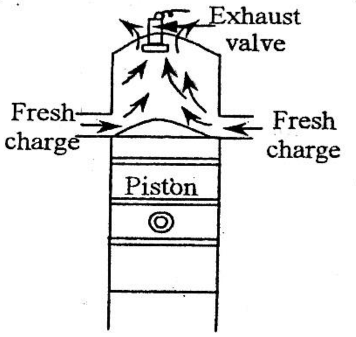
There are three types of scavenging.



(a) *Cross flow scavenging*



(b) *Loop or Back flow scavenging*



(c) *Uniflow scavenging*

1. **Cross flow scavenging**

   In this method, the transfer port (or inlet port for the engine cylinder) and exhaust port are situated on the opposite sides of the engine cylinder (as is done in case of two –stroke cycle engine). The piston crown is designed in to a particular shape, so that the fresh charge moves up words and pushes out the burnt gases in the form of cross flow as shown in fig.(a)

2. Black flow or loop scavenging

   In this method, the inlet and outlet port are situated on the same side of the engine cylinder. The fresh charge, while entering into the cylinder, forms a loop and pushes out the burnt gases as shown in fig.(b)
3. Uniflow scavenging

   In this method, the fresh charge, while entering from one side (or some time two sides) of the engine cylinder pushes out the gases through the exit value situated on the top of the cylinder in unfloor scavenging, both the fresh charge and burnt gases move in the same upward direction as shown in fig.(c)

## COOLING SYSTEM IN I.C. ENGINE:

### Purpose of cooling system

When the air- fuel mixture is ignited and combustion takes place at about $2500^{o}$c for producing power inside an engine the temperature of the cylinder, cylinder head, piston and valve, continuous to rise when the engine runs.

It these parts are not cooled by some means then they are liked to get damaged and even melted. The piston may cease inside the cylinder. To prevent this, temperature of the parts around the combustion chamber is maintained 200oc to 250oc. Too much of cooling will lower the thermal efficiency of the engine. Hence, the purpose of cooling is to keep the engine at its most efficient operating temperature at all engine speeds and all driving condition. The cooling system is so designed that it prevents cooling until the engine reaches in to its normal operating temperature. When the engine warms up the cooling system will begin to function. It cools rapidly when the engine is too hot and it cools slowly or not at all when the engine is cold or warming up. Thus, the duty of the cooling system is to keep the engine from getting too hot-not to keep it cool.

**Types of cooling system:**

There are two types of cooling system

1. Air cooling system
2. Water cooling system

**Air cooling system:**

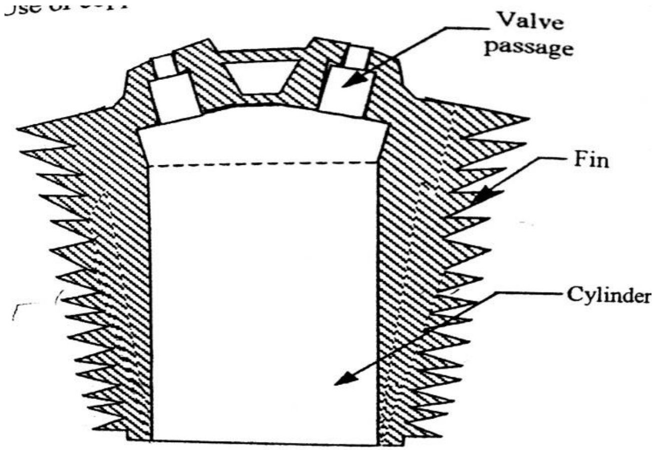


*Fig. 3.57. Cylinder with Fins*

The air cooling system is shown in fig, is used in the engine of motor cycle, scooters, aero planes and other stationary installations. In countries with cold climate, this system is also used in car engines. In this system, the heat is dissipated directly to the atmosphere air by conduction thorough the cylinder walls. In order to increase the rate of cooling, the Surface area of the cylinder and cylinder head is increased by providing radiating fins and flanges in bigger units, fans are provided to circulate the air around the cylinder walls and cylinder head.

Advantages

1. Light in weight since there are no radiators, cooling water and pipelines.
2. No coolant is used and so no leak and no anti-freeze required.
3. Warming up is faster.
4. Maintenance is easy and hence cheaper.

Disadvantages

1. Less efficient since air is poor conductor of heat compared with water.
2. Since it is not possible to maintain even cooling some time distortion may take place.
3. Noisy operation.
4. It can be used only in small engines.
5.

## Water cooling system:

Thermosyphon system:

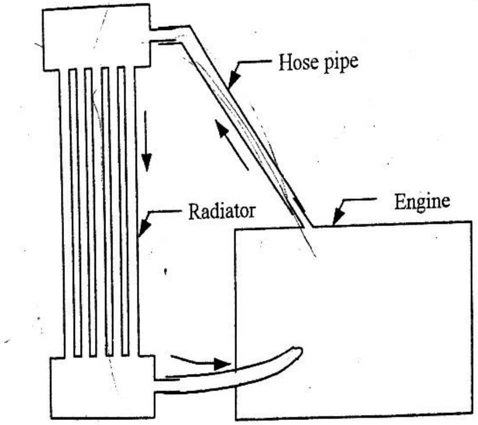


Fig. 3.58. Thermosyphon System of Cooling

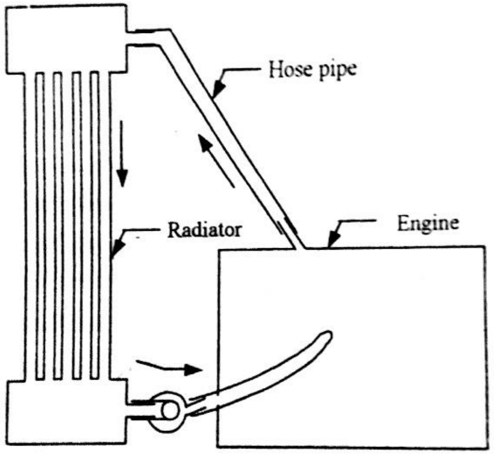


Fig. 3.59.Pump Circulation System

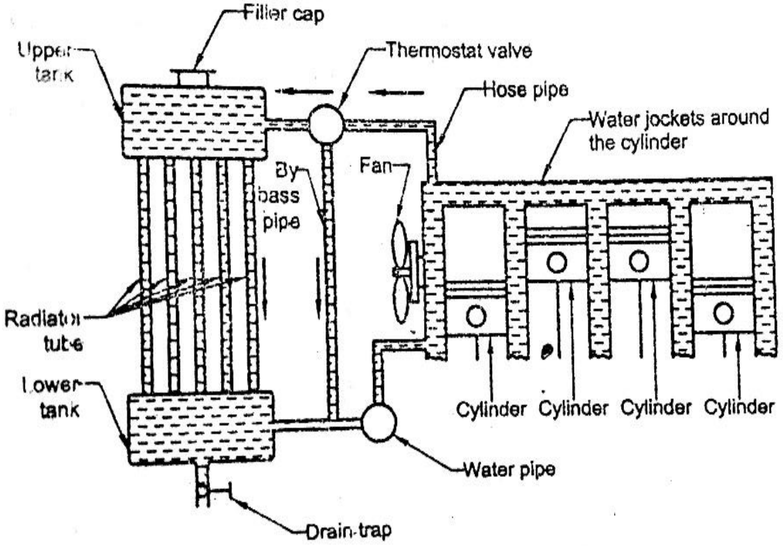


Fig 3.60 Water-cooling system for 4 cylinder engine

The water cooling system as shown in fig. is used in the engine of car, bus, trucks etc. in this system; the water is circulated through water jackets around each of combustion chamber, cylinder, valve seats and valve stems. The water is kept continuously in motion by a centrifugal pump which is driven by a V-belt from the pulley on the engine crank shaft. After passing through the engine jackets in the cylinder block and heads, the water is passed through the radiator, the water is cooled by air drawn through the radiator by a fan. Usually, fan and water pump are mounted and driven on a common shaft. After passing through the radiator, the water pumps through a cylinder inlet passage. The water is again circulated through the engine jackets.

**LUBRICATION SYSTEM IN IC ENGINE:**

In an I.C. engine, moving parts rub against each other causing frictional force. Due to the frictional force, heat is generated and the engine parts wear easily. Power is also lost due to friction. To reduce the power loss and also wear and tear of the moving parts, a foreign substance called lubricant is introduced in between the rubbing surface .the lubricant keeps the mating surface apart. Lubricant may be solid (graphite), or semi-solid (grease) or liquid lubricant generally used is mineral oil. This is obtained by refining petroleum. Grease is also used to lubricant certain parts of the engine.

Purposes of lubrication (or) function of lubrication:

1. It reduced friction between moving parts.
2. It reduces wear and tear of the moving parts.
3. It minimizes power loss due to friction.
4. It reduces noise

**Types of lubrication**

The various methods adopted for lubrication of I.C. engines are

(i) Petroil lubrication
(ii) Wet sump system, and
  a. Gravity lubrication system
  b. Splash lubrication system
  c. Pressure lubrication system
  d. Semi-pressure lubrication system
(iii) Dry sump system.

**Gravity lubrication system:**

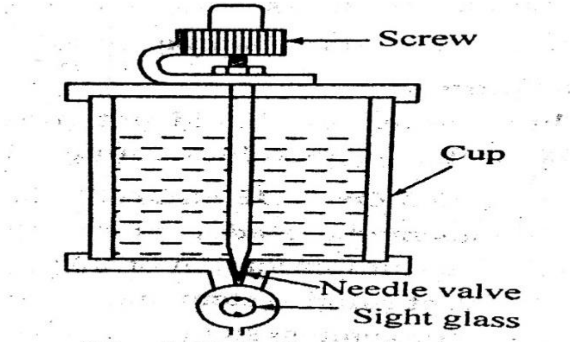


*Fig. 3.61.Drop feed oiler*

In this method, oil is supplied to the parts to be lubricated by means of gravity. This system uses a drop feed oiler shown fig. it consist of a cup and needle valve arrangement. The needle valve is operated by means of screw. The valve is raised to increase the flow of oil and lowered to decrease the oil flow. This system is used for lubricating external moving parts such as bearing, cross head, crank pins of simple steam engine.

**Splash lubrication system:**

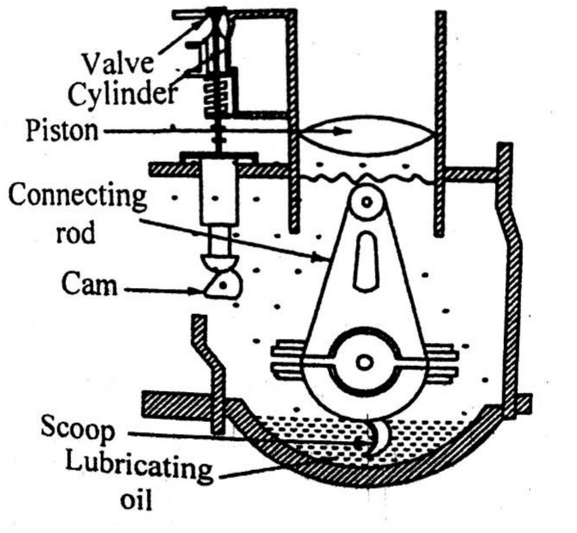


Fig.3.62. Splash lubrication

In this system, oil is stored in the crank case. A small scoop is attached with the big end of connecting rod as shown in fig.

When the crank is rotated, the scoop dips in the oil and splashes the oil. The oil is splashes on cylinder wall, connecting rod ends and valve mechanisms. This method is used in some motorcycle and single cylinder stationary engines. Greater care should be taken that the oil in the crank case is filled up to the desired mark. There will be insufficient lubrication when the oil level is low.

Disadvantages:

- It is not efficient, if the bearing loads are heavy.
- It is very difficult to introduce oil in the minute gaps between the sliding surface.

**Pressure lubrication system:**

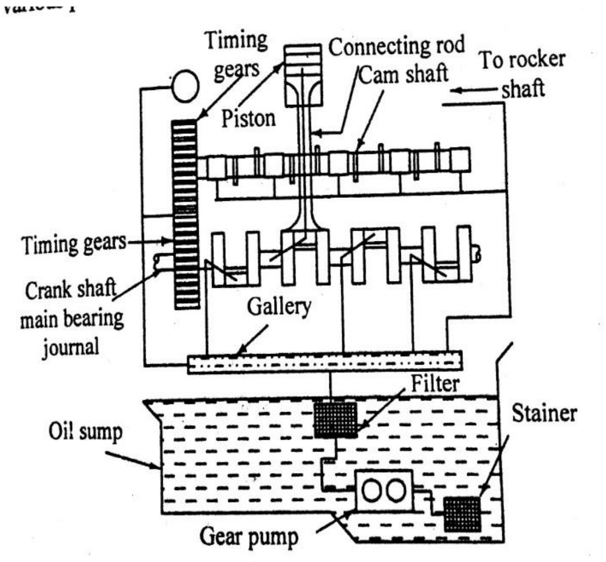


***Fig.3.63. Pressure lubrication***

In this system, lubrication oil is forced under pressure b hole is a pump at a pressure of 2 to 4Mpa. Fig. shows a line diagram of this system. It consist of oil sump, oil pump, oil gallery, pressure release valve, oil filter, oil pressure gauge and oil dipstick. The lubrication oil from the sump or oil pan is sucked by oil pump and lifted to oil main gallery though oil filter and strainer. The oil pump is driven by the camshaft. Oil pump and filter are always immersed in the oil. From the gallery, the oil is described under pressure to various parts of the engine to be lubricated by the oil tubes. Oil from gallery enters the crank pin bearing through a taper hole in the crank shaft. A though provided at the center of the connecting rod. The oil from the big end bearing enters the gudgeon pin bearing (small and bearing) through the hole in the connecting rod. Separate oil tubes carry oil for lubrication timing gears, rocker arm assembly, and cam shaft etc. another oil line is connected to the pressure gauge to show the pressure of the oil.

The excess supplied oil drips back into the oil pump. A pressure relief valve is provided to avoid any damages in case of excess oil pressure an oil dipstick is provided to measure the oil level in the sump.

Advantages

1. All the parts of the engine are efficiently lubricated.
2. The minute gap between the sliding surfaces can be lubricated since the oil is supplied under pressure.
3. Engine parts lubricated under pressure: crank shaft main beginning cam shaft main bearings, big end bearings of the connecting rod, timing gears, chain and sprocket, rocker arm, springs, valve guides etc.

### Semi pressure lubrication:

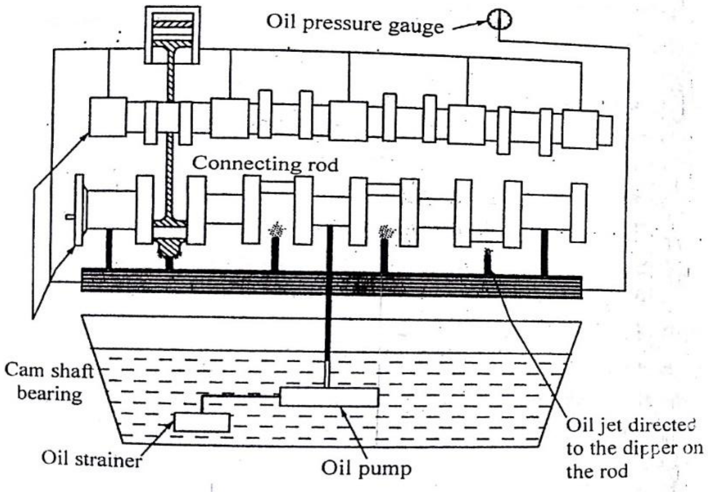


*Fig 3.64.Partial pressure lubrication*

It is also called as partial pressure lubrication system. This is a modification of flash lubrication system. This system is used if the Bering loads are heavy and splash lubrication is not sufficient. It is a combination of splash and pressure lubrication.

This system consists of oil pump, oil gallery, oil filter, oil pressure gauge scoops attached to connecting rod big end. The pump pumps the oil to the main gallery. From the gallery, oil is forced under pressure to the engine parts to be lubricated. Scoops or attached to the big end of the connecting rod.

The lubricating oil is directed to the scoops through oil jets from gallery. The scoops splash this oil in all direction to lubricate the engine parts such as piston, cylinder wall etc.

Engine parts lubricated by splash piston, cylinder walls, corns, piston pin and rings, springs and guides of valve stems, oil pump drive gear etc.

### Dry sump lubrication system:

The lubricating oil stored in the oil sump is called wet sump system. But the system in which the lubricating oil is not kept in the oil sump is known as dry sump system. In this system.in this system, oils cares in a separate tank and fed to the engine.

The oil which falls into the oil sump after lubricating is sent back to the oil tank by a separate delivery pump. Thus the system consists of two pumps. One pump is used to feed the oil. The other pump is used to deliver the oil the oil tank. This system is used in aircrafts.

The main advantages of this system are that there is no chance of break down in the oil supply during up and down movement of the vehicle.

## IGNITION SYSTEM OF I.C. ENGINE:

In I.C. engines, the ignition of fuel-air mixture should take place at the end of compression stroke. It ensures efficient and smooth running of an engine. There are mainly two different types of ignition used in I.C. engines.

1. Compression ignition
2. Spark ignition.

1. Compression ignition

This system is used in heavy oil engines working on diesel cycle.in these engines, fresh air are drawn into the engine cylinder during suction stroke. This air is compressed to a high compression ratio (12 to 18) during compression stroke. The air pressure at the end of compression is about 3500Kn/m2. The temperature at the end of compression is about 600 c. at the end of compression stork; the fuel is injected into the engine cylinder in the form of fine spray. The temperature of the air is higher than the self-ignition temperature of the fuel injected. Hence, the fuel-air mixture is ignited simultaneously. In diesel, the engines, the ignition of fuel-air mixture takes place due to high pressure and temperature of the air. Hence, they are known as compression ignition (C.I.) engines.

2. Spark ignition:

Spark ignition is mostly used in gas engines, petrol engines and light oil engines working on Otto cycle. In this system the fuel –air mixture is ignition by a high-tension electric spare. Hence, they are known as spark ignition (S.I.) engines.

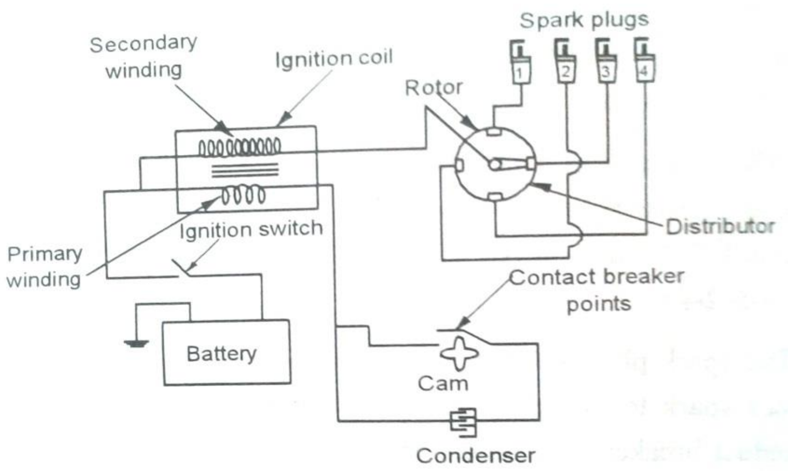
There are different types of spark ignition systems used in Otto cycle engines.

1. Coil ignition system,
2. Magneto ignition system,
3. Electronic ignition and,
4. Transistorized ignition system.

**Battery coil ignition system:**

It is employed in engines fig. shows the wiring diagram of a simple coil ignition system of a cylinder engine. This system is used in automobile.

**Construction:**



It consists of a battery, ignition coil, condenser, contact breaker, distributor and spark plugs. Generally, 6 or 12 volt battery is used. The ignition coil consists of two winding primary and secondary. The primary winding consists of thinks wire with less number of turns. The primary winding is formed of 200-300 turns of thick wire of #20-gage to produce a resistance of about 1.5ohms.

The secondary winding located inside the primary winding consists of about 21,000 turns of thin enameled wire of #38-40 gage with sufficiently insulted to withstand high voltage. It is wound close to the core with one end connected to the secondary terminal and the other end grounded either to the metal case or the primary coil. The condenser is connected across the contact breaker. It prevents excess arcing and pitting of contact breaker points. The contact breaker is housed in the distributor itself. It makes and breaks the primary ignition circuit. The distributor distributes the high voltage to the respective spark plugs having regular interval in the sequence of firing order of the engine.

(The sequence in which the firing or power occurs in a multi cylinder engine is known as firing order of a 4-cylinder in-line engine is 1-3-4-2 or 1-4-3-2. The firing order of a 6-cylinder in-line engine is 1-5-3-6-2-4).

The spark is fitted on the combustion chamber of the engine. it produces spark to ignite the fuel-air mixture. The rotor of the distributor and contact breaker cam are driven by the engine. there are two circuits in this system. One is the primary circuit. It consists of battery, primary coil of the ignition coil, condenser and contact breaker. The other circuit is the secondary circuit. It consists of secondary coil, distributor and spark plugs.

Working:

The ignition switch is switched on and the engine is cranked. The cranking of the engine opens and closes the contact breaker points through a cam.

**When the contact breaker points are closed:**

1. The current flows from the battery to the contact breaker points through the switch and primary winding and then returns to battery through the earth
2. This current builds up a magnetic field in the primary winding of the ignition coil.
3. When the primary current is at the highest peak, the contact breaker points will be opened by the cam.

**When the contact breaker points are closed:**

1. The magnetic field set up in the primary winding is suddenly collapsed.
2. A high voltage (15000 volts) is generated in the secondary winding of the ignition coil.
3. This high voltage is directed to the rotor of the distributor.
4. The rotor directs this high voltage to the individual spark plugs in the sequence of the firing order of the engine.
5. This high voltage tries to cross the spark plug gap (0.45 to 0.6mm) and the spark is produced. This spark ignites the fuel-air mixture.

**Advantages:**

1. It provides better sparks at low speeds of the engine during starting and idling due to availability of maximum current throughout the engine speed range.
2. The initial cost is low as compared with magneto ignition system.
3. The maintenance cost is negligible except battery.
4. Spark efficiency remains unaffected by various position of the timing control mechanism.

**Disadvantages:**

1. Frequent battery down occurs when the engine is not is use continuously. This causes starting trouble.
2. The weight is greater than magneto ignition system.
3. Wiring mechanism is more complicated.

**Magneto ignition system:**

In this system, the battery is replaced with a magneto fig shown the wiring diagram of a magneto ignition system. It consists of a switch, magneto, contact breaker, condenser, distributor and spark plugs. This system is used in two wheelers like motor cycles etc.

Construction:

The magneto ignition system consists of a rotating magnet assembly driven by a engine and a fixed armature. The armature consists of primary and secondary windings. The primary circuit consists of a primary winding condenser and contact breaker. The secondary circuit consists of a secondary windings, distributor and spark plugs.

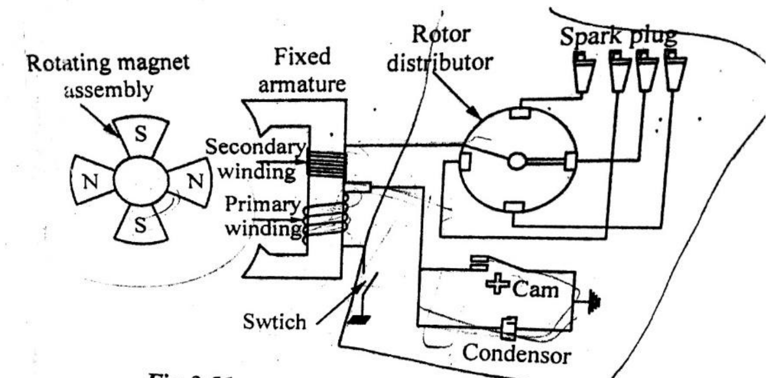


*Fig 3.51 magneto ignition system*

**When the contact breaker points are closed:**

1. There current flows in the primary circuit.
2. This produces a magnetic field in the primary winding.
3. When the primary current is at the highest peak, the contact breaker points will be opened by the cam.

**When the contact breaker points are opened:**

1. There is a break in the primary circuit.
2. The magnetic field in the primary winding is suddenly collapsed.
3. A high voltage (15000 volts) is generated in the secondary winding.
4. This high voltage is distributed to the respective spark plugs through the rotor of the distributor.
5. The high voltages tries to cross the spark plug gap and a spark is produced in the gap. This spark ignites the fuel-air mixture in the engine cylinder.

**Advantages:**

1. It has no maintenance problem like coil ignition (i.e. for battery).so, it is more reliable.

2. When the speed increases, it provides better intensity of spark and thus provides better combustion as compared to batter coil ignition system.
3. Less space is required as compared to battery ignition system.
4. It is very light in weight and compact in size.

**Disadvantages:**

1. Initial cost is very high as compared to coil ignition system.
2. Minimum 75 rpm is necessary to start the engine.
3. For higher power engine, some other devices are necessary to start ignition.

## ELECTRONIC IGNITION SYSTEM:

Construction

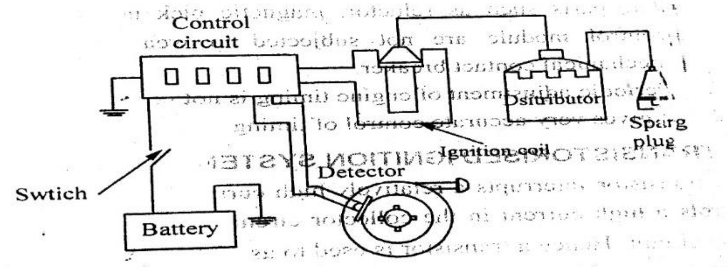


*Fig 3.53 Electronic ignition system*

A schematic diagram of an electronic ignition system is shown in fig .it consists of a battery, ignition switch, electronic control unit, magnetic pick-up, reluctor or armature, ignition coil, distributor and spark plug. The construction of battery, ignition switch, ignition coil, distributor and spark plug is same as previous methods. In this system, a magnetic pick-up is used of contact breaker points in conventional system. Also cam is replaced by a reluctor or armature.

The magnetic pick-up is shown in fig .it consists of a sensor coil through which passes the magnetic flus is generated by a permanent magnet. A star shaped rotor called reluctor or armature is mounted on the distributor shaft which modulates the flux density in the coil and due to the consequent changes in the flux voltage induced in the coil. This voltage serves as a trigger signal for the high voltage generator circuit. Since there is one spark plug per cylinder,the number of teeth of armature is equal to the number of engine cylinders.

Working:

When the ignition switch is closed (i.e. Switch is ‘ON’.), the reluctor rotates which makes the teeth of the reluctor cone closed to the permanent magnet. It reduces the air gap between the reluctor tooth and the sensor coil.

Thus, the reluctor provides a path for the magnetic lines from the magnet. The magnetic field is passed onto the pick up every time when the reluctor teeth pass the pick up coil in which an electric pulse is generated. This small current then triggers the electronic control unit which stops the flow of battery current to the ignition coil.

The magnetic field in the primary winding collapses and the high voltage is generated in the secondary. it leads to the spark in spark plug via distributor. Meanwhile, the reluctor teeth pass the pickup coil. Therefore, the pulse unit is ended. It causes the electronic control unit to close the primary circuit.

Advantages:

1. The parts such as reluctor, magnetic pickup and electronic control unit are not subjected to wear as incase of mechanical contact breaker.
2. Periodic adjusted of engine timing is not necessary.
3. It gives very accurate control of timing.

## GOVERING OF I.C.ENGINES:

As a matter of fact, all the I.C. engines like other engines are always designed to run at a particular speed. But in actual practice, load on the engine keeps on fluctuating from time to time. A little considertation will show ,that change of load ,on an I.C.engine is sure to change it has been observed that if load on an I.C. engine is decreased without changing the quantity of fuel,the engine will run at a higher speed . similarly, if load on the engine is increased without changing the quantity of fuel, the engine will run at a lower speed.

Now, in order to have a high efficiency of an I.C. engine, at different load conditions, its speed must be kept constant as far as possible. The process of providing any arrangement, which will keep the speed constant (according to the changing load conditions) is known as governing of I.C. engines.

### Methods of Governing I.C. Engines

Through there are many methods for the governing of I.C. engines, yet the following are important from the subject point of view:

1. **Hit and miss governing**: This method of governing is widely used for I.C. engines of smaller capacity or gas engines. This method is most suitable for engines, which are frequently subjected to reduced loads and as a result of this, the engines tend to run at higher speeds. In this system of governing, whenever the engine starts running at higher speed (due to decreased load), some explosion are omitted or missed. This is done with help of centrifugal governor (Art.24.11) in which the inlet valve of fuel is closed and the explosions are omitted till the engine speed reaches its normal value. The only disadvantage of this method is that there is uneven turning moment due to missing of explosions. As a result of this, it requires a heavy fly wheel.

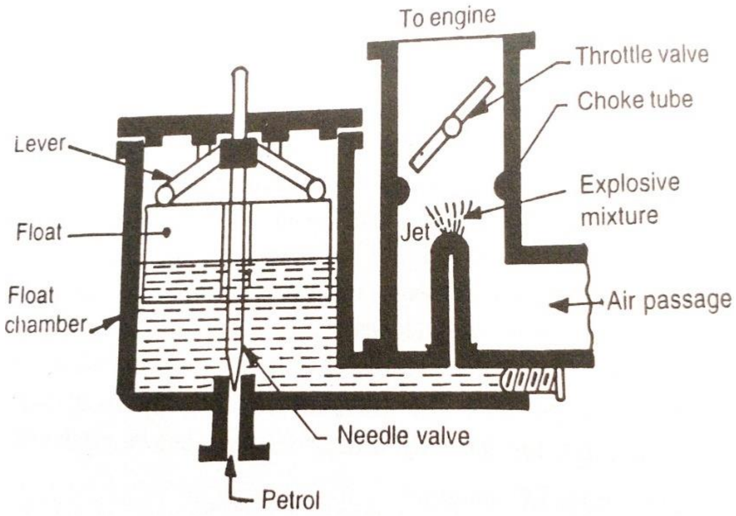
**2. Qualitative governing:** In this system of governing, a control valve is fitted in the fuel delivery pipe, which controls the quantity of fuel to be mixed in the charge. The movement of control valve is regulated by the centrifugal governor through rack and pinion arrangement. It may be noted that in this system, the amount of air used in each cycle remains the same. But with the changed in the quantity of fuel (with quantity of air remaining constant), the quality of charge (i.e. air-fuel ratio of mixture) changes. Whenever the engine starts running at higher speed (due to decreased load), the quantity of fuel is reduced till the engine speed reaches its normal value. Similarly, whenever the engine starts running at lower speed (due to increased load), the quantity of fuel is increased. In automobile engines, the rack and pinion arrangement is connected with the accelerator.

**3. Quantitative governing**. In this system of governing, the quality of charge (i.e. air-fuel ratio of the mixture) is kept constant. But quantity of mixture supplied to the engine cylinder is varied by means of a throttle valve which is regulated by the centrifugal governor through rack and pinion arrangement. Whenever the engine starts running at higher speed (due to decreased load), the quantity of charge is increased. This method is used for governing large engines.

**4. Combination system of governing**. In this system of governing, the above mentioned two methods of governing (i.e. qualitative and quantitative) are combined together, so that quality as well as quantity of the charge is varied according to the changing conditions. This system is complicated, and has not proved to be successful.

## CARBURETOR:

The carburetor is a device for *atomizing and ** vaporizing the fuel and mixing it with the air in the varying proportions to suit the changing operating conditions of the engine. The process of breaking up and mixing the fuel with the air is called carburation.

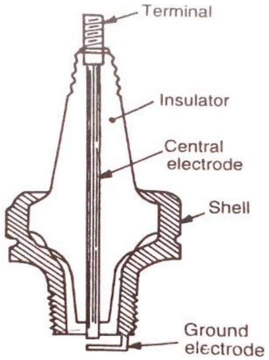


There are many types off the carburetors in use, but the simplest form off the carburetor is shown in Fig. It consists of a fuel jet located in the center of the choke tube. A float chamber is provided for maintaining the level of the fuel in the jet and is controlled by a float and lever which operates its needle valve. The fuel is pumped into the float chamber and when the correct level of the fuel is reached, the float closes the needle valve, and shuts off the petrol supply.

The suction produced by the engine draws air through the choke tube. The reduced diameter of the choke tube increases the velocity of air and reduces the pressure. The high velocity and low pressure in the tube facilities the breaking up of fuel and its admixture with the air. A throttle valve controls the flow of the mixture delivered to the engine cylinder.

## SPARK PLUG:

It is always screwed into the cylinder head for igniting the charge of petrol engines. It is, usually, designed to withstand a pressure up to 35 bar and operate under a current of 10000 to 30000 volts.



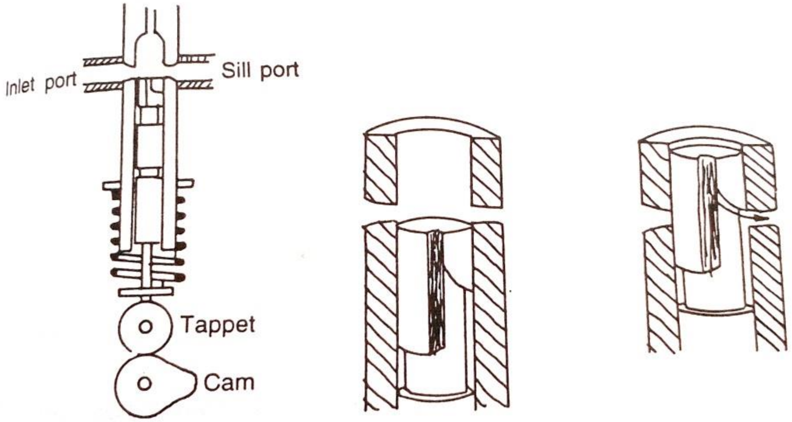
A park plug consists of central porcelain insulator, containing an axial electrode length wise and ground electrode welded to it. The central electrode has an external contact at the top, which is connected to the terminal and communicates with the distributor. A metal tongue is welded to the ground electrode, which bends over to lie across the end of the central electrode. There is a small gap known as spark gap between the end of the central electrode and the metal tongue, as shown in Fig. The high tension electric spark jumps over the gap to ignite the charge in the engine cylinder.

The electrode material should be such which can withstand corrosiveness, high temperature having good thermal conductivity. The electrodes are generally made from the alloys of platinum, nickel, chromium, barium etc.

Note: The spark plug gap is kept from 0.3 mm to 0.7 mm. The experiments have shown that efficiency of the ignition system is greatly reduced if the gap is too large or too small. Sometimes, foreign materials (such as carbon) get deposited in the spark gap. It is a source of nuisance, as it permits some of the high voltage current to bypass the gap and reduce the intensity of spark as well as engine efficiency

## FUEL PUMP

The main object of a fuel pump in a diesel engine is to deliver a fuel to the injector which sprays the finely divided particles of the fuel suitable for rapid combustion.



(*a*) Fuel pump. (*b*) Beginning of stroke. (*c*) End of stroke.

The simplified sketch of a fuel pump is shown in Fig. (a). It consists of a plunger which moves up and down in the barrel by the cam and spring arrangement provided for pushing and lowering the plunger respectively. The fuel oil is highly filtered by means of felt-pack filter before entering the barrel of the pump.

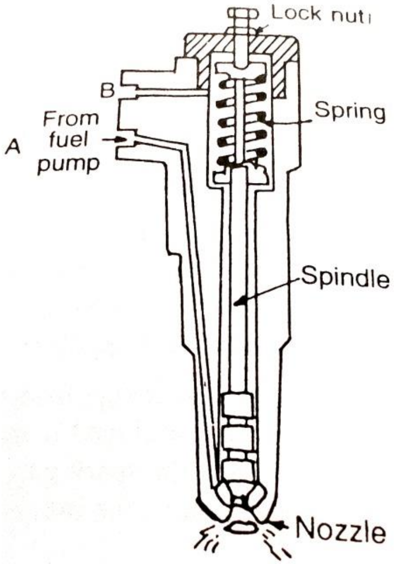
The upper end part of the plunger is cut away in a helix shaped piece forming a groove between the plunger and barrel, which is the most important one. Therefore, the amount of fuel delivered and injected into the engine cylinder depends upon the rotary position of the plunger in the barrel, Fig. (b) And (c) shows how the top part of the plunger is designed so that the correct amount of fuel is delivered to the injector.

When the plunger is at the bottom of its stroke as shown in Fig. (b), the fuel enters the barrel through the inlet port. As the plunger rises, it forces this fuel up into the injector, until the upper part cut away comes opposite the sill port. Then the fuel escapes down the groove and out through the sill port so that injection ceases, as shown in Fig.(c). The plunger can be made to rotate in the barrel and therefore more fuel is injected. When the plunger is rotated so that the groove is opposite to the sill port, no fuel at all is injected and thus the engine stops.

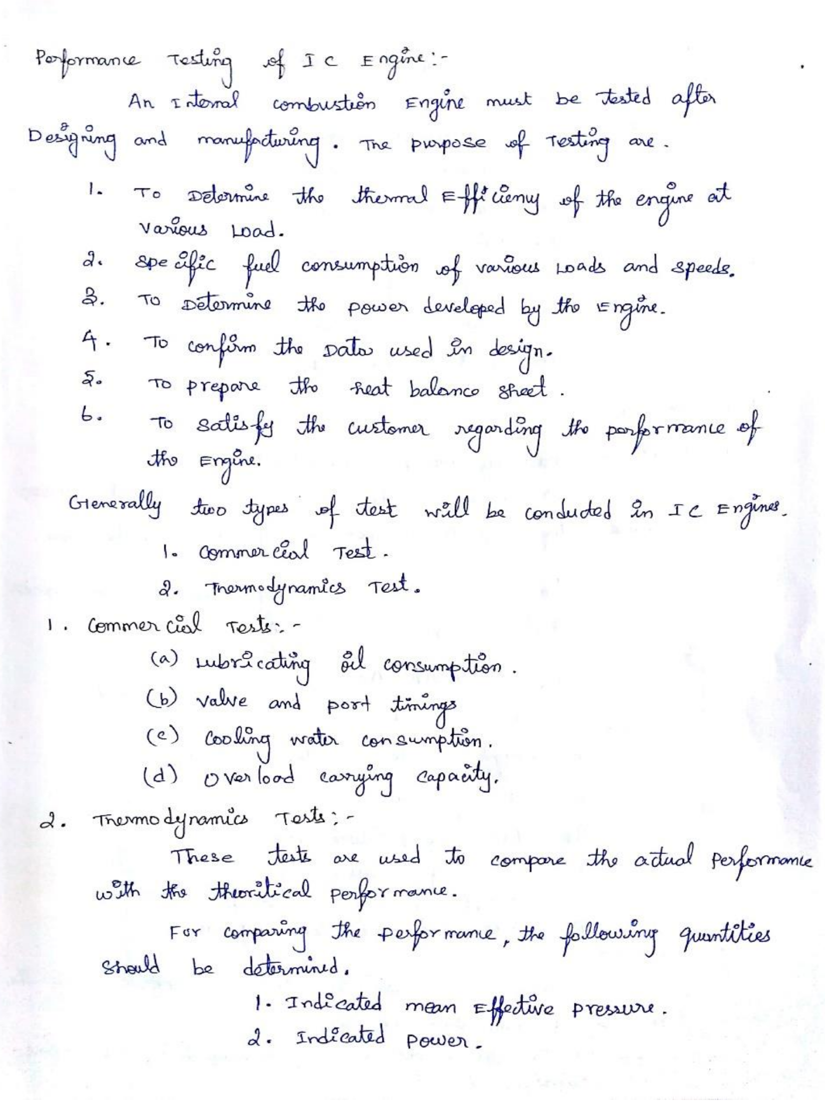
## INJECTOR OR ATOMISER

The injector or atomizer is also an important part of the diesel engine which breaks up the fuel and sprays into the cylinder into a very fine divided particle

Fig. shows the type of an injector in which fuel is delivered from the pump along the horizontal pipe connected at A. The vertical spindle of the injector is spring loaded at the top which holds the spindle down with a pressure of 140 bars so that the fuel pressure must reach this value before the nozzle will lift to allow fuel to be injected into the engine cylinder. The fuel which leaks past the vertical spindle is taken off by means of an outlet pipe fitted at B above the fuel inlet pipe.



## Performance Calculation:



Performance Testing of I C Engine :-

An Internal combustion Engine must be tested after Designing and manufacturing. The purpose of Testing are.

1. To Determine the thermal Efficiency of the engine at various Load.
2. Specific fuel consumption of various Loads and speeds.
3. To Determine the power developed by the Engine.
4. To confirm the Data used in design.
5. To prepare the heat balance sheet.
6. To satisfy the customer regarding the performance of the Engine.

Generally two types of test will be conducted in I C Engines.

1. Commercial Test.
2. Thermodynamics Test.

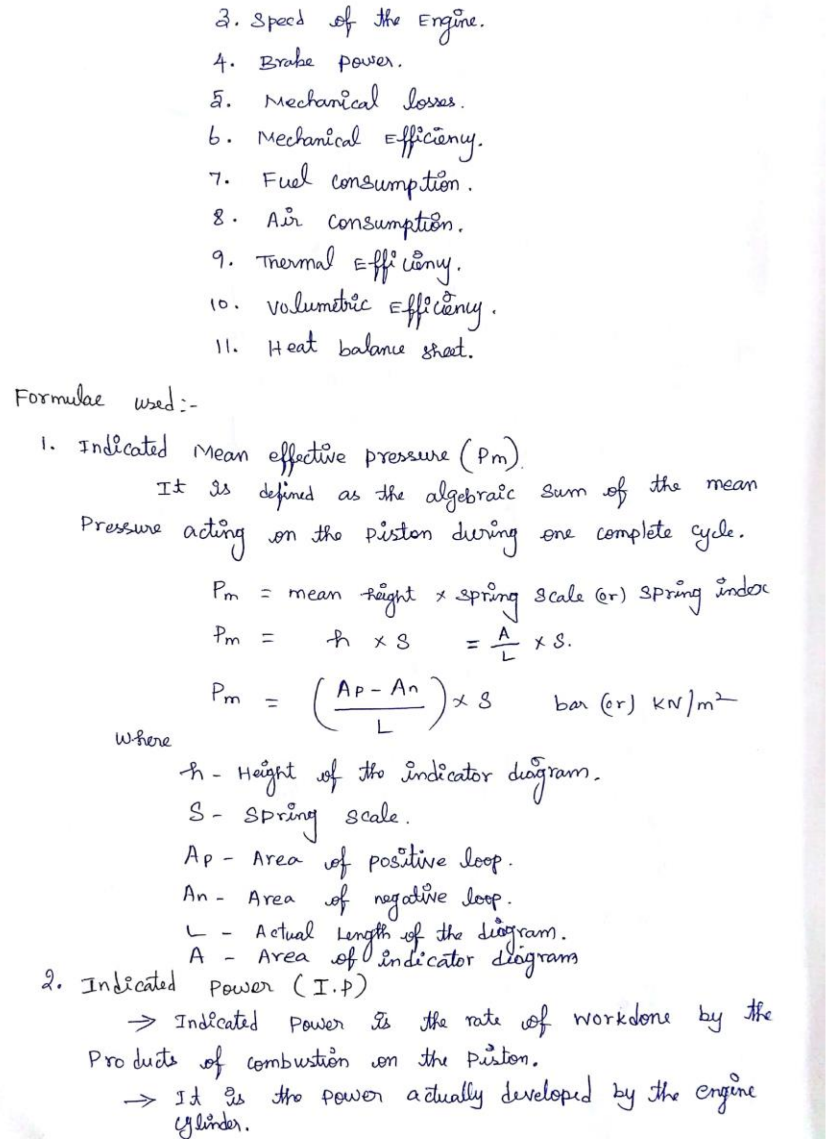
1. Commercial Tests :-
   (a) Lubricating oil consumption.
   (b) valve and port timings
   (c) Cooling water consumption.
   (d) Overload carrying capacity.

2. Thermodynamics Tests :-

These tests are used to compare the actual performance with the theoritical performance.

For comparing the performance, the following quantities should be determined.

1. Indicated mean Effective pressure.
2. Indicated power.



3. Speed of the Engine.
4. Brake power.
5. Mechanical losses.
6. Mechanical Efficiency.
7. Fuel consumption.
8. Air consumption.
9. Thermal Efficiency.
10. Volumetric Efficiency.
11. Heat balance sheet.

Formulae used :-

1. Indicated Mean effective pressure ($P_m$)

It is defined as the algebraic sum of the mean Pressure acting on the piston during one complete cycle.

$P_m$ = mean height × spring scale (or) spring index

$$P_m = h \times S = \frac{A}{L} \times S.$$

$$P_m = \left(\frac{A_P - A_n}{L}\right) \times S \quad \text{bar (or) } KN/m^2$$

Where

$h$ - Height of the indicator diagram.
$S$ - Spring scale.
$A_P$ - Area of positive loop.
$A_n$ - Area of negative loop.
$L$ - Actual Length of the diagram.
$A$ - Area of indicator diagrams

2. Indicated Power (I.P)

→ Indicated Power is the rate of workdone by the Products of combustion on the Piston.

→ It is the power actually developed by the engine cylinder.

$$I.P = P_m \cdot l \cdot a \cdot n \cdot k \quad KW.$$

where

$P_m$ - Indicated mean effective pressure in $kN/m^2$

$l$ - Length of the Stroke in m.

$a$ - Area of the piston in $m^2$

$n$ - Number of working stroke per sec.

$n = \frac{N}{2}$ ( For Four stroke cycle Engine.)

$n = N$ (For Two stroke Engine)

$N$ - Speed of the engine in rps.

$K$ - Number of cylinders.

3. Brake Power:- (BP).

→ Brake Power is the useful power available at the Crank shaft.

→ It is always lesser than indicated power.

Brake Power $BP = 2\pi NT \quad KW.$

Torque T is given by $T = WR \quad kN\text{-}m$

$$BP = 2\pi NWR \quad KW.$$

Where

W - Net Load on Brake (kN)

R - Effective radius of Brake drum (m)

N - Speed of the engine in rps.

Effective Diameter of Brake drum.

$$D = D_1 + d_1 \quad \text{in } m.$$

Effective Brake radius

$$R = \frac{D}{2} = \frac{(D_1 + d_1)}{2} \quad \text{in } m$$

Net Brake Load

$$W = W_1 - W_2 \quad \text{in } kN.$$

Brake Torque

$$T = WR \quad kN\text{-}m.$$

Brake Power

$$B.P = 2\pi NT$$

$$= 2\pi NWR \qquad \therefore 2R = D.$$

$$= \pi DNW \quad \text{in } k$$

Brake mean effective Pressure

Brake Power $BP = P_{mb} \cdot l \cdot a \cdot n \cdot k$ in KW

$P_{mb}$ - Brake mean effective pressure in $kN/m^2$

4. Friction Power:-

Friction Power of an engine may be defined as the Difference between the indicated power and the Brake power.

Friction power = Indicated power - Brake power

$$F.P = I.P - B.P.$$

5. Fuel Consumption rate: (kg/hr)

$$= \frac{\text{Fuel consumed} \times \text{Specific gravity of fuel}}{1000 \times t.}$$

where

t - time per sec.

6. Efficiencies of IC Engine.

The Efficiency of the Engine is defined as the ratio of workdone by the engine to the Energy supplied to an Engine.

(a) Mechanical Efficieny ($\eta_{mech}$):-

It is the ratio of breake power to the Indicated power.

$$\eta_{mech} = \frac{\text{Brake power}}{\text{Indicated power}} = \frac{BP}{IP.}$$

(b) Indicated thermal Efficieny ($\eta_{IT}$).

It is the ratio of indicated power to the heat Supplied to an Engine.

$$\eta_{IT} = \frac{\text{Indicated Power}}{\text{Heat Supplied.}}$$

$$\text{Heat supplied Per sec} = \frac{m_f \times CV}{3600} \text{ in kW.}$$

where

$m_f$ - mass of fuel consumed per hour

CV - calorific value of fuel kJ/kg.

$$\therefore \eta_{IT} = \frac{IP}{\left(\frac{m_f \times CV}{360}\right)} \text{ in kW.}$$

(c) Brake thermal Efficieny ($\eta_{BT}$):-

It is ratio of Brake power to the heat supplied to an engine.

$$\eta_{BT} = \frac{\text{Brake Power}}{\text{Heat Supplied.}}$$

$$\eta_{BT} = \frac{BP}{\left(\frac{m_f \times CV}{3600}\right)}$$

(d) Relative efficieny ($\eta_R$):-

→ It is also known as efficieny ratio.

→ It is the ratio of indicated thermal efficieny is to the air standard Efficieny.

$$\eta_R = \frac{\text{Indicated thermal Efficieny}}{\text{Air Standard Efficieny.}}$$

(e) volumetric efficieny ($\eta_v$):-

It is the ratio of actual volume of the charge admitted during the suction stroke to the swept volume of the piston.

$$\eta_v = \frac{\text{volume of charge admitted during suction stroke}}{\text{Swept volume of the piston.}}$$

7.) Specific fuel consumption (SFC):-

It is defined as the amount of fuel consumed per brake power per hour of work. It is called as brake SFC.

$$BSFC = \frac{m_f}{BP} \text{ kg/ kW-hr.}$$

Brake Efficieny

$$\eta_{BT} = \frac{BP}{\left(\frac{m_f \times CV}{3600}\right)} = \frac{3600}{BSFC \times CV}$$

$$\therefore BSFC = \frac{3600}{\eta_{BT} \times CV}$$

The amount of fuel consumed per indicated power per hour of work is known as indicated SFC.

$$ISFC = \frac{m_f}{I.P.}$$

Indicated thermal Efficiency.

$$\eta_{IT} = \frac{IP}{\left(\frac{m_f \times CV}{3600}\right)} = \frac{3600}{ISFC \times CV}$$

$$\therefore ISFC = \frac{3600}{\eta_{IT} \times CV}$$

Problems:-

1. A single cylinder four stroke Diesel engine, having a swept volume of 750 $cm^3$ is tested at 300 rpm. when a breaking torque of 65 N-m is applied, the mean effective Pressure is 1100 $KN/m^2$. calculate the Brake power and mechanical Efficiency of the Engine.

Given Data:-

Single cylinder Four stroke Engine

$V_s = 750 \ cm^3 = 0.00075 \ m^3 = a \times l.$

$N = 300 \ rpm = 5 \ rps$

$T = 65 \ N\text{-}m$

$P_m = 1100 \ KN/m^2$

To Find:-

$B.P = ?$

$\eta_{mech} = ?$

Solution:-

Brake power

$$BP = 2\pi NT$$

$$= 2\pi \times 5 \times 65$$

$$BP = 2042.04 \text{ watt.}$$

$$\boxed{BP = 2.042 \ KW}$$

Indicated power

$n = \frac{N}{2}$ (Four stroke).

$$I.P = P_m \, l \, a \, n \, k.$$

$$= P_m \times l \times a \times \frac{N}{2} \times k$$

$$= 1100 \times 0.00075 \times \frac{5}{2} \times 1$$

$$\boxed{IP = 2.0625 \ KW.}$$

Mechanical Efficiency

$$\eta_{mech} = \frac{BP}{IP} = \frac{2.042}{2.0625}$$

$$\eta_{mech} = 0.989$$

$$\boxed{\eta_{mech} = 98.9\%}$$

Result:-

Brake power = 2.042 KW

Mechanical Efficiency = 98.9%

2) An engine develops 5.4 KW of power with its indicated thermal efficiency and mechanical Efficiency is 36% and 78%. Respectively. Estimate the fuel consumption of engine, indicated specific fuel consumption and brake specific fuel consumption Assume CV of fuel 41100 KJ/kg.

Given data :-

BP = 5.4 KW

$\eta_{IT}$ = 36 %

$\eta_{mech}$ = 78 %

CV = 41100 KJ/kg.

To Find :-

Fuel consumption = ?

ISFC = ?

BSFC = ?

Solution :-

W.K.T.

$$\eta_{mech} = \frac{BP}{IP.}$$

$$0.78 = \frac{5.4}{IP.}$$

$$IP = 6.92 \text{ KW}$$

Indicated thermal Efficiency

$$\eta_{IT} = \frac{IP \times 3600}{m_f \times CV}$$

$$0.36 = \frac{6.92 \times 3600}{m_f \times 41100}$$

Fuel consumption

$$m_f = 1.683 \text{ kg/hr.}$$

Indicated SFC

$$= \frac{m_f}{IP} = \frac{1.683}{6.92} = 0.243 \text{ kg/KW-hr.}$$

Brake SFC

$$= \frac{m_f}{BP} = \frac{1.683}{5.4}$$

Brake SFC = 0.311 kg/KW-hr.

Result :-

Fuel consumption = 1.683 kg/hr.

Indicated SFC = 0.243 kg/KW-hr.

Brake SFC = 0.311 kg/KW-hr.

3. A Two stroke two cylinder IC Engine develops the indicated power 18 KW at 900 rpm. If the mean effective Pressure is 550 KN/m². calculate the dimensions of the cylinder. Assume the ratio of stroke to diameter as 2.

Given data :-

K = 2

IP = 18 KW

N = 900 rpm = 15 rps.

n = N = 900 rpm = 15 rps.

$P_m$ = 550 KN/m²

$$\frac{l}{d} = 2 \Rightarrow l = 2d$$

To Find :-

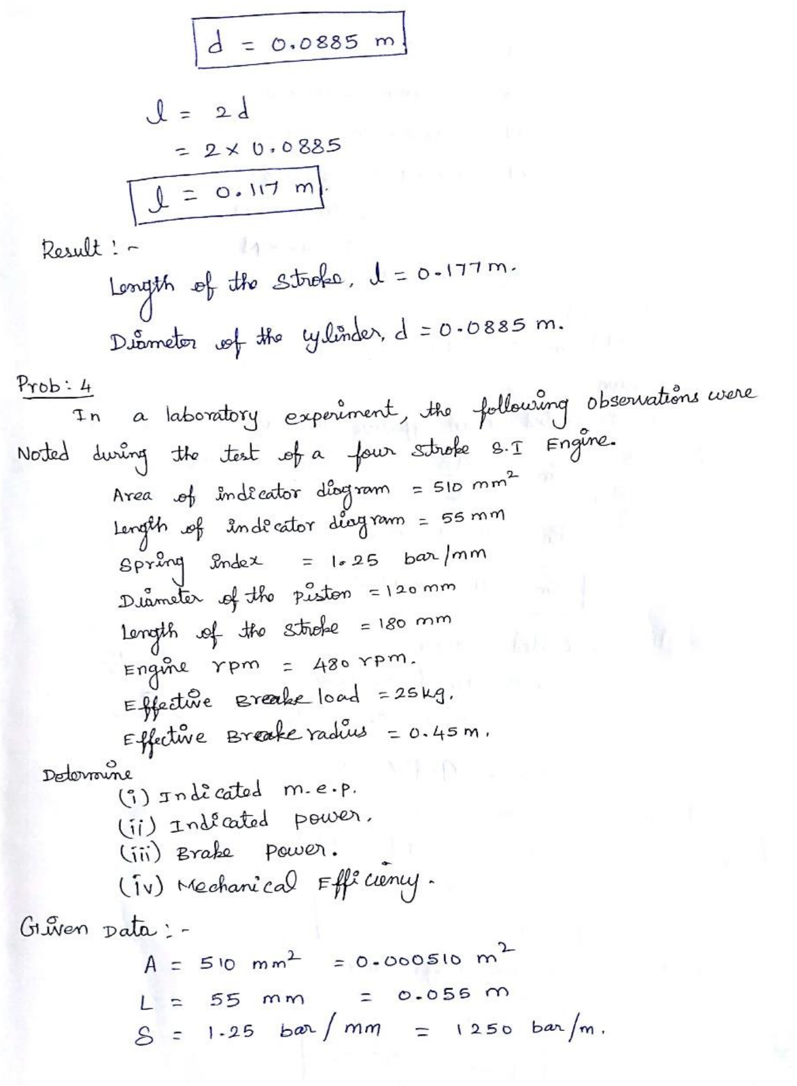
l and radius of cylinder.

Solution :-

$$IP = P_m . l . a . n . k$$

$$18 = 550 \times 2d \times \frac{\pi}{4} d^2 \times 15 \times 2$$

$$d^3 = 6.9449 \times 10^{-4}$$



$$\boxed{d = 0.0885 \text{ m}}$$

$l = 2d$

$= 2 \times 0.0885$

$$\boxed{l = 0.117 \text{ m}}$$

Result :-

Length of the stroke, $l = 0.177$ m.

Diameter of the cylinder, $d = 0.0885$ m.

Prob : 4

In a laboratory experiment, the following observations were Noted during the test of a four stroke S.I Engine.

Area of indicator diagram = 510 $mm^2$

Length of indicator diagram = 55 mm

Spring Index = 1.25 bar/mm

Diameter of the piston = 120 mm

Length of the stroke = 180 mm

Engine rpm = 480 rpm.

Effective Breake load = 25 kg.

Effective Breake radius = 0.45 m.

Determine

(i) Indicated m.e.p.

(ii) Indicated power.

(iii) Brake power.

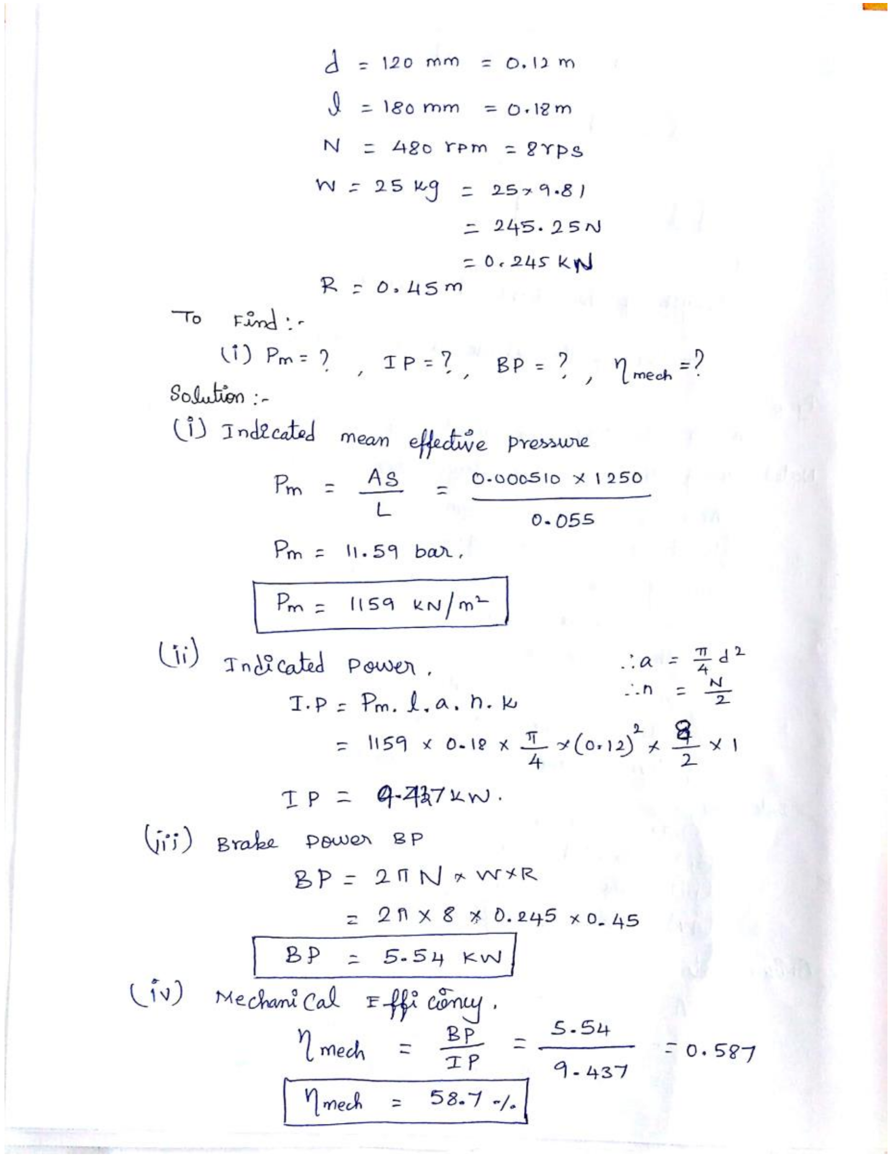
(iv) Mechanical Efficiency.

Given Data :-

$A = 510 \text{ mm}^2 = 0.000510 \text{ m}^2$

$L = 55 \text{ mm} = 0.055 \text{ m}$

$S = 1.25 \text{ bar/mm} = 1250 \text{ bar/m}.$



$d = 120 \text{ mm} = 0.12 \text{ m}$

$l = 180 \text{ mm} = 0.18 \text{ m}$

$N = 480 \text{ rpm} = 8 \text{ rps}$

$W = 25 \text{ kg} = 25 \times 9.81$

$= 245.25 \text{ N}$

$= 0.245 \text{ kN}$

$R = 0.45 \text{ m}$

To Find :-

(i) $P_m = ?$ , $IP = ?$, $BP = ?$ , $\eta_{mech} = ?$

Solution :-

(i) Indicated mean effective pressure

$$P_m = \frac{AS}{L} = \frac{0.000510 \times 1250}{0.055}$$

$P_m = 11.59$ bar.

$$\boxed{P_m = 1159 \text{ kN/m}^2}$$

(ii) Indicated Power.

$\therefore a = \frac{\pi}{4} d^2$

$\therefore n = \frac{N}{2}$

$I.P = P_m . l . a . n . k$

$$= 1159 \times 0.18 \times \frac{\pi}{4} \times (0.12)^2 \times \frac{8}{2} \times 1$$

$IP = 9.437$ kW.

(iii) Brake power BP

$BP = 2\pi N \times W \times R$

$= 2\pi \times 8 \times 0.245 \times 0.45$

$$\boxed{BP = 5.54 \text{ kW}}$$

(iv) Mechanical Efficiency.

$$\eta_{mech} = \frac{BP}{IP} = \frac{5.54}{9.437} = 0.587$$

$$\boxed{\eta_{mech} = 58.7 \%}$$

Results :-

(i) Indicated Mean effective pressure

$P_m = 115 \text{ KN/m}^2$

(ii) Indicated Power

$IP = 9.4377 \text{ KW}.$

(iii) Brake power

$BP = 5.54 \text{ kW}.$

(iv) Mechanical Efficiency

$\eta_{mech} = 58.7\%$

Prob: 5.

A rope Brake has a brake wheel diameter of 750 mm and the diameter of the rope is 8mm. The Dead load on the Brake is 275 N and spring balance reads 35 N. If the engine rpm is 480, Find the Brake power developed.

Given Data :-

$D = 750 \text{ mm} = 0.75 \text{ m}$

$d = 8 \text{ mm} = 0.008 \text{ m}$

$W_1 = 275 \text{ N}$

$W_2 = 35 \text{ N}$

$N = 480 = 8 \text{ rps}.$

To Find :-

Brake power = ?

Solution :-

Effective brake Radius

$$R = \frac{D+d}{2} = \frac{0.75 + 0.008}{2} = 0.379 \text{ m}$$

Net load on Brake drum

$W = W_1 - W_2$

$= 275 - 35$

$= 240 \text{ N}$

$W = 0.24 \text{ KN}$

Brake power

$BP = 2\pi N W R$

$= 2\pi \times 8 \times 0.24 \times 0.379$

$$\boxed{BP = 4.572 \text{ KW}}$$

Result :-

Brake Power (B.P) = 4.572 KW.

Problem : 6

During a test on single cylinder four stroke cycle oil engine, the following Data obtained.

Stroke volume $= 0.0227 \text{ m}^3$

Mean effective pressure = 5 bar

Engine speed = 4000 rpm.

Brake Torque = 67.6 N-m

Fuel used per hour = 37.3 kg.

calorific value of fuel = 43,000 KJ/kg.

Calculate the Individual power, brake power, indicated thermal Efficiency, Brake thermal Efficiency and Mechanical Efficiency.

Given Data :-

Four stroke Engine

$k = 1$

$V_s = 0.0227 \text{ m}^3$

$P_m = 5 \text{ bar} = 500 \text{ kN/m}^2$

$N = 4000 \text{ rpm} = 66.66 \text{ rps.}$

$n = \frac{N}{2} = \frac{66.66}{2} = 33.33$

$T = 676 \text{ Nm} = 0.676 \text{ kN-m.}$

$m_f = 37.3 \text{ kg/hr.}$

$C_V = 43000 \text{ kJ/kg.}$

To Find:-

$IP = ?$ , $BP = ?$ , $\eta_{IT} = ?$ , $\eta_{BT} = ?$ , $\eta_{mech} = ?$

Salution:-

Indicated Power.

$IP = P_m . l . a . n . k .$

$= P_m . V_s . n . k .$

$= 500 \times 0.0227 \times 33.33 \times 1$

$\boxed{IP = 378.29 \text{ kW.}}$

Brake Power

$BP = 2\pi N T$

$= 2\pi \times 66.66 \times 0.676$

$\boxed{BP = 283.13 \text{ kW.}}$

Mechanical Efficieny.

$\eta_{mech} = \frac{BP}{IP.}$

$= \frac{283.13}{378.29}$

$\boxed{\eta_{mech} = 74.8\%}$

Indicated thermal Efficieny.

$\eta_{IT} = \frac{IP \times 3600}{m_f \times CV}$

$= \frac{378.29 \times 3600}{37.3 \times 43000}$

$= 0.848$

$\boxed{\eta_{IT.} = 84.8\%}$

Brake thermal Efficieny.

$\eta_{BT} = \frac{IP \times 3600}{m_f \times C_V}$

$= \frac{283.13 \times 3600}{37.3 \times 43000}$

$= 0.635$

$\boxed{\eta_{BT.} = 63.5\%}$

Result:-

Indicated Power IP = 378.29 kW.

Brake Power BP = 283.13 kW.

Indicated thermal Efficieny = 84.8%

Brake thermal Efficieny = 63.5%

Mechanical Efficieny = 74.8%

Prob : 7

A six cylinder petrol engine has a compression ratio of 5 to 1. The clearance volume for each cylinder is 110 cc. It operates on the four-stroke constant volume cycle and the indicated thermal efficiency ratio refered to the air standard cycle is 0.56. At a speed 2400 rev/min is consumed 10 kg of fuel per hour, the energy of combustion being 44 MJ/kg. Determine the Average indicated mean effective pressure in the cylinder:-

Given Data:-

Four stroke constant volume cycle.

$K = 6.$

$r = 5:1 = 5.$

$V_c = 110 \text{ cc} = 110 \text{ cm}^3$

$= 110 \times 10^{-6} \text{ m}^3$

$$\frac{\eta_{IT}}{\eta_{air\ std}} = 0.56$$

$N = 2400 \text{ rpm} = 40 \text{ rps.}$

$m_f = 10 \text{ kg/hr.}$

$CV = 44 \text{ MJ/kg} = 44000 \text{ kJ/kg.}$

To Find:-

Indicated mean effective pressure: ?

Solution:-

Air standard Efficiency

$$\eta_{air\ std} = 1 - \frac{1}{(r)^{\gamma-1}}$$

$$\eta_{air\ std} = 1 - \frac{1}{(5)^{1.4-1}}$$

$$= 0.4746$$

$$\boxed{\eta_{air\ std} = 47.46\ \%.}$$

W.K.T From given data:-

$$\frac{\eta_{IT}}{\eta_{air\ std}} = 0.56.$$

$$\eta_{IT} = 0.56 \times 0.4746$$

$$\eta_{IT} = 0.2658.$$

W.K.T

$$\eta_{IT} = \frac{IP \times 3600}{m_f \times CV}$$

$$0.2658 = \frac{IP \times 3600}{10 \times 44000}$$

$$\boxed{IP = 32.49 \text{ kW.}}$$

W.K.T

$IP = P_m . l . a . n . K.$

$IP = P_m . V_s . n . K$

Here $n = \frac{N}{2} = \frac{40}{2} = 20 \text{ rps.}$

$$r = \frac{V_c + V_s}{V_c}$$

$$5 = \frac{110 \times 10^{-6} + V_s}{110 \times 10^{-6}}$$

$V_s = 4.4 \times 10^{-4} \, m^3$

$IP = P_m \, V_s \, n . k.$

$32.49 = P_m \times 4.4 \times 10^{-4} \times 20 \times 6.$

$$\boxed{P_m = 615.34 \, kN/m^2}$$

Result :-

Indicated mean effective pressure

$P_m = 615.34 \, kN/m^2$

Prob: 8

A Four cylinder diesel engine works on Four stroke cycle has a cylinder bore of 90 mm and a stroke of 150 mm. The crank speed is 370 rpm, and fuel consumption is 15 kg/hr, having a calorific value of 39000 KJ/kg. The indicated mean effective pressure is 5 bar. If the compression ratio is 14 and cut off ratio is 2.3. Calculate the relative efficiency. Taking $\gamma = 1.4$.

Given Data :-

$K = 4$

$d = 90 \, mm = 0.09 \, m$

$l = 150 \, mm = 0.15 \, m$

Four stroke Diesel Engine.

$N = 370 \, rpm = 6.166 \, rps.$

$m_f = 15 \, kg/hr.$

$CV = 39000 \, kJ/kg.$

$P_m = 5 \, bar = 500 \, KN/m^2$

$\gamma = 1.4$

$\rho = 2.3$

To Find.

Relative Efficiency = ?

Solution :-

$$\text{Relative efficiency} = \frac{\eta_{IT}}{\eta_{Air\,std.}}$$

$IP = P_m \, l . a . n . k$

$$= 500 \times 0.15 \times \frac{\pi}{4} (0.09)^2 \times 3.033 \times 4$$

$$\boxed{IP = 5.82 \, kN.}$$

$$\eta_{IT} = \frac{IP \times 3600}{m_f \times CV}$$

$$= \frac{5.82 \times 3600}{10 \times 39000}$$

$$\boxed{\eta_{IT} = 0.0542}$$

$$\eta_{Air\,std} = 1 - \frac{1}{\gamma (r)^{\gamma-1}} \left[ \frac{e^{\gamma} - 1}{e - 1} \right]$$

$$= 1 - \frac{1}{1.4 \, (14)^{1.4-1}} \left[ \frac{2.3^{1.4} - 1}{2.3 - 1} \right]$$

$\eta_{Air\,std} = 0.5775$

Relative Efficiency $\eta_R = \frac{0.0542}{0.5775} = 0.0938$

$\eta_R = 9.38 \%$

Result :-

Relative Efficiency $\eta_R = 9.38 \%$

### 4.2.9. Heat Balance Sheet:

The complete record of heat supplied and heat rejected during a certain time by an I.C. engine is entered in a tabular form known as *heat balance sheet*.

All the heat energy supplied to an engine can not be converted into useful work some of the heat energy may be lost by means of some sources.

The sources of heat losses are given below.

(i) Heat rejected to the cooling water.
(ii) Heat carried away by exhaust gases.
(iii) Unaccounted heat losses by means of radiation, incomplete combustion and errors in observations etc.

The following values should be calculated for tabulating heat balance sheet.

*(i) Heat supplied by the fuel ($Q_S$):*

$$Q_S = m_f \times CV \text{ KJ/hr}$$

*(ii) Heat absorbed in I.P. produced ($Q_{IP}$):*

Indicated power $Q_{IP} = P_m\, l\, a\, n\, k$ KW

*(iii) Heat rejected to the cooling water ($Q_W$):*

$$Q_W = m_w\, C_w\, (T_2 - T_1) \text{ KJ/hr}$$

Where, $m_w$ – Mass of the cooling water circulated in *kg/hr*.
$C_w$ – Specific heat capacity of water in *KJ/kg k* = 4.19 *KJ/kg k*
$T_1$ – Inlet temperature in *K*
$T_2$ – Outlet temperature in *K*

*(iv) Heat carried away by exhaust gas ($Q_g$):*

$$Q_g = m_g\, C_g\, (T_g - T_a)$$

Where, $m_g$ – Mass of exhaust gases in *kg/hr*
$C_g$ – Specific heat capacity of exhaust gas in *KJ/kg k*= 1.005 *KJ/kgk*

*(v) Unaccounted heat losses ($Q_{ua}$):*

$$Q_{ua} = Q_S - [Q_{IP} + Q_w + Q_g + \dots\ ] \text{ KJ/hr}$$

Find out all the above heat losses into percentage

$$\% \text{ heat loss} = \frac{\text{Heat loss}}{Q_S} \times 100$$

***Heat balance sheet:***

| *Credit* | *KJ* | *%* | *Debit* | *KJ* | *%* |
|---|---|---|---|---|---|
| 1. Heat supplied by the fuel ($Q_S$). | -- | 100 | 1. Heat equivalent to IP or BP. | -- | -- |
| | | | 2. Heat carried away by cooling water ($Q_W$). | -- | -- |
| | | | 3. Heat carried away by exhaust gas ($Q_g$). | -- | -- |
| | | | 4. Unaccounted heat loss ($Q_{ua}$). | -- | -- |
| *Total* | -- | *100* | *Total* | -- | *100* |

**Problem 11** ***A six cylinder four-stroke engine of 340mm bore and 390mm stroke was tested gave the following informations:***

***Engine speed – 360rpm***
***Brake power – 180KW***
***Mean effective pressure – 3.8bar***
***Fuel per minute of C.V. – 45000 KJ/kg = 0.77kg***
***Flow of cooling water – 64kg/min; with a temperature rise of 9 °C; Draw the heat balance sheet for the engine.***

***Given data:***

$k = 6$

Four stroke engine

$d = 340mm = 0.34m$

$l = 390mm = 0.39m$

$N = 360rpm = 6rps$

$$n = \frac{N}{2} = \frac{6}{2} = 3$$

$BP = 180KW$

m.e.p. $p_m = 3.8bar = 380KN/m^2$

$m_f = 0.77kg/min$

$CV = 45000KJ/kg$

$m_w = 64kg/min$

$$T_2 - T_1 = 9°C$$

*To find:*

Draw a heat balance sheet.

☺ ***Solution:***

Indicated power IP = $p_m\, l\, a\, n\, k$

$$= 380 \times 0.39 \times \frac{\pi}{4} \times (0.34)^2 \times 3 \times 6$$

$$IP = 242.19KW = 14531.4KJ/min$$

Heat supplied by the fuel,

$$Q_s = m_f \times CV$$

$$= 0.77 \times 45000 = 34650KJ/min$$

$$\text{IP in \%} = \frac{14531.4}{34650} \times 100 = 41.937\%$$

Heat carried away by cooling water,

$$Q_W = m_w\, C_w\, (T_2 - T_1)$$

$$= 64 \times 4.2 \times 9$$

$$\therefore Q_W = 2419.2KJ/min$$

$$Q_W \text{ in \%} = \frac{2419.2}{34650} \times 100 = 6.982\%$$

Unaccounted heat loss,

$$Q_{ua} = 34650 - (14531.4 + 2419.2)$$

$$Q_{ua} = 17699.4KJ/min$$

$$Q_{ua} \text{ in \%} = \frac{17699.4}{34650} \times 100 = 51.08\%$$

*Result:*

*Heat balance sheet*

| *Credit* | *KJ/min* | *%* | *Debit* | *KJ/min* | *%* |
|---|---|---|---|---|---|
| 1. Heat supplied by the fuel ($Q_S$). | 34560 | 100 | 1. Heat equivalent to IP | 14531.4 | 41.937 |
| | | | 2. Heat carried away by cooling water ($Q_W$). | 2419.2 | 6.982 |
| | | | 3. Unaccounted heat loss ($Q_{ua}$). | 17699.4 | 51.081 |
| *Total* | *34560* | *100* | *Total* | *34560* | *100* |

**Problem 13** ***The following observations were taken during a test on a single cylinder four-stroke cycle engine having a bore of 300mm and stroke of 450mm.***

*Ambient air temperature = 22 °C*

*Fuel consumption = 11[illegible]kg/hr*

*CV of fuel = 42,000 KJ/kg*

*Engine speed = 300rpm*

*Mean effective pressure = 6 bar*

*Net brake load = 1.0KN*

*Brake drum diameter = 2m* diameter of the rope = 2cm

*Quantity of jacket cooling water = 590kg/hr*

*Temperature of entering cooling water = 22 °C*

*Temperature of leaving cooling water = 70 °C*

*Quantity of air as measured = 225kg/hr*

*Specific heat of exhaust gas = 1.005KJ/kgk*

*Exhaust gas temperature = 405 °C*

***Determine Indicated power, Brake power, Mechanical efficiency and draw up a heat balance sheet on hour basis.***

***Given data:***

Single cylinder, four stroke engine

$d = 300mm = 0.3m$

$l = 450mm = 0.45m$

$T_a = 22°C$

$m_f = 11kg/hr$

$CV = 42000KJ/kg$

$N = 300rpm = 5rps$

$$n = \frac{N}{2} = \frac{5}{2} = 2.5$$

$p_m = 6bar = 600KN/m^2$

$W = 1.0KW$

$D = 2m$

$d = 2cm = 0.02m$

$m_w = 590kg/hr$

$T_2 = 70°C$

$T_1 = 22°C$

$m_g = 225kg/hr$

$C_g = 1.005KJ/kgk$

$T_g = 405°C$

***To find:***

IP, BP, $\eta_{mech}$ and draw the heat balance sheet.

☺ ***Solution:***

Indicated power $IP = p_m\, l\, a\, n\, k$

$$= 600 \times 0.45 \times \frac{\pi}{4}(0.3)^2 \times 2.5 \times 1$$

$$= \mathbf{47.7KW} \qquad \textbf{Ans.}$$

Brake power $BP = 2\pi\, N\, W\, R$ $\qquad \left[\because R = \frac{D+d}{2}\right]$

$$= 2\pi \times 5 \times 1 \times \frac{2+0.02}{2}$$

$$= \mathbf{31.73KW} \qquad \textbf{Ans.}$$

Mechanical efficiency

$$\eta_{mech} = \frac{BP}{IP} = \frac{31.73}{47.7} = 0.6652 = \mathbf{66.52\%Ans.}$$

Heat supplied by the fuel

$$Q_S = m_f \times CV = 11 \times 42000$$

$$Q_S = 462000KJ/hr$$

Heat carried away by the cooling water

$$Q_w = m_w\, C_w\, (T_2 - T_1)$$

$$= 590 \times 4.2\ (70 - 22)$$

$$Q_w = 118944KJ/hr$$

$$Q_w \text{ in } \% = \frac{118944}{462000} = 25.745\%$$

Heat carried away by the exhaust gas

$$Q_g = m_g\, C_g\, (T_g - T_a)$$

$$= 225 \times 1.005 \times (405 - 22)$$

$$Q_g = 86605.875 KJ/hr$$

$$Q_g \text{ in } \% = \frac{86605.875}{462000} \times 100 = 18.746\%$$

Unaccounted heat loss

$$Q_{ua} = Q_S - (Q_w + Q_g + Q_{IP})$$

Heat for IP = 47.7$KW$ = 171720$KJ/hr$

$$\text{IP in } \% = \frac{171720}{462000} \times 100 = 37.168\%$$

Problem : 6

During a test

[redacted]

③ **Problem 1** *A test on a single cylinder 4 stroke oil engine having bore of 180mm and stroke of 360mm gave the following results: Speed = 290rpm, brake torque = 392N-m, IMEP = 7.2bar, oil consumption = 3.5kg/h, Coolant flow = 270kg/h, cooling water temperature rise = 36°C, air-fuel ratio by weight = 25, exhaust gas temperature = 415°C, room temperature = 21°C. The fuel has a calorific value 45200KJ/kg and take specific heat of the exhaust gases as 1.0035KJ/kg-k.*

*Calculate (i) Indicated thermal efficiency.*

*(ii) Draw up a heat balance sheet in KJ/min basis.*

*[Anna University-Nov.'2003]*

*Given data:*

Single cylinder, four stroke engine

$d = 180mm = 0.18m$

$l = 360mm = 0.36m$

$$N = 290rpm = \frac{290}{60} = 4.833$$

$$n = \frac{N}{2} = \frac{4.833}{2} = 2.4167$$

Brake torque, $T = 392N\text{-}m$

$$p_m = 7.2\text{bar} = 720KN/m^2$$

$$m_f = 3.5kg/hr = \frac{3.5}{3600}\ kg/sec$$

$$m_w = 270kg/hr = \frac{270}{3600}\ kg/sec$$

$$(T_2 - T_1) = 36°\text{C}$$

Air-fuel ratio A/F = 25:1

$T_e = 415°\text{C}$

$T_a = 21°\text{C}$

CV = 45200$KJ/kg$

$C_g = 1.0035KJ/kg\text{-}k$

***To find:***

$\eta_{IT}$ and heat balance sheet.

☺ ***Solution:***

Indicated power, IP = $p_m\ l\ a\ n\ k$

$$= 720 \times 0.36 \times \frac{\pi}{4}(0.18)^2 \times 2.4167 \times 1$$

$$= 15.94KW$$

Heat supplied by the fuel,

$$Q_s = m_f \times \text{CV}$$

$$= \frac{3.5}{3600} \times 45200$$

$$Q_s = 43.94KW$$

Indicated thermal efficiency,

$$\eta_{IT} = \frac{IP}{Q_s} = \frac{15.94}{43.94} = 0.3627$$

$$\eta_{IT} = \mathbf{36.27\%\ Ans.}$$

$$\text{I.P. in \% of } Q_s = \frac{15.94}{43.94} \times 100 = 36.27\%$$

Heat carried away by the cooling water

$$Q_w = m_w\ C_w\ (T_2 - T_1)$$

$$= \frac{270}{3600} \times 4.2 \times 36$$

$$Q_w = 11.34KW$$

$$Q_w \text{ in \%} = \frac{11.34}{43.94} \times 100 = 25.8\%$$

Heat carried away by exhaust gas,

$$Q_g = m_g\ C_g\ (T_g - T_a)$$

We know that Air-fuel ratio, A/F = 25:1

Mass of air $m_g = 25 \times$ Mass of fuel consumed

$$m_g = 25 \times \frac{3.5}{3600} = 0.0243kg/sec$$

$$\therefore Q_g = 0.0243 \times 1.0035 \times (415 - 21)$$

$$Q_g = 9.61KW$$

$$Q_g \text{ in \% of } Q_s = \frac{9.61}{43.94} \times 100 = 21.87\%$$

$$\text{Brake power, BP} = \frac{2\pi NT}{60} = \frac{2 \times \pi \times 290 \times 392}{60} = 11904W$$

$$\text{BP in \% of } W_s = \frac{11.904}{43.94} \times 100 = 27.1\%$$

Unaccounted heat loss,

$$Q_{ua} = 43.94 - (11.904 + 11.34 + 9.61)$$

$$= 11.086KW$$

$$Q_{ua} \text{ in \% of } Q_s = \frac{11.086}{43.94} \times 100 = 25.23\%$$

*Results:*

Indicated thermal efficiency = 36.27%

*Heat balance sheet*

| *Credit* | *KW* | *%* | *Debit* | *KW* | *%* |
|---|---|---|---|---|---|
| 1. Heat supplied by the fuel ($Q_S$). | 43.94 | 100 | 1. Heat equivalent to BP | 11.904 | 27.1 |
| | | | 2. Heat carried away by cooling water ($Q_W$). | 11.34 | 25.8 |
| | | | 3. Heat carried away by exhaust gas ($Q_g$) | 9.61 | 21.87 |
| | | | 4. Unaccounted heat loss ($Q_{ua}$). | 11.086 | 25.23 |
| ***Total*** | ***43.94*** | ***100*** | ***Total*** | ***43.94*** | ***100*** |

**Problem 3** *The following data related to the testing of a four-cylinder four-stroke diesel engine. Bore = 36cm; stroke = 40cm; speed = 350rpm; BP = 257KW; m.e.p. = 7bar; fuel consumption = 72kg/hr; CV of fuel = 43960 KJ/kg; air consumption 28.2kg/min; rise in temperature of engine cooling water = 41°C; room temperature = 20°C; temperature of exhaust gas = 345°C; specific heat of exhaust gas 1.041KJ/kg; cooling water consumed per hour = 1800kg. Draw up heat balance sheet.* *[MU- Oct. 98]*

*Given data:*

Four cylinder; four stroke

$d = 36cm = 0.36m$

$k = 4$

$l = 40cm = 0.4m$

$N = 350rpm = 5.83rps$

$n = \frac{N}{2} = \frac{5.83}{2} = 2.916rps$

$\text{BP} = 257KW$

$p_m = 7bar = 700KN/m^2$

$m_f = 72kg/hr = \frac{72}{3600}\ kg/sec$

$(T_2 - T_1) = 41°C$

$T_c = 20°C$

$T_g = 345°C$

$C_g = 1.041KJ/kgk$

$m_w = 1800kg/hr = \frac{1800}{3600}\ kg/sec$

*To find:*

Heat balance sheet

☺ *Solution:*

Indicated power

$$\text{IP} = p_m\, l\, a\, n\, k$$

$$= 700 \times 0.4 \times \frac{\pi}{4}(0.36)^2 \times 2.916 \times 4$$

$$\text{IP} = 332.43KW$$

Heat supplied by the fuel

$$Q_S = m_f \times \text{CV}$$

$$= \frac{72}{3600} \times 43960$$

$$Q_S = 879.2KW$$

$$\text{IP in \%} = \frac{332.43}{879.2} \times 100 = 37.81\%$$

Heat carried away by the cooling water

$$Q_w = m_w\, C_w\, (T_2 - T_1)$$

$$= \frac{1800}{3600} \times 4.2 \times 41$$

$$Q_w = 86.1KW$$

$$Q_w \text{ in } \% = \frac{86.1}{879.2} \times 100 = 9.79\%$$

Heat carried away by exhaust gas

$$Q_g = m_g\, C_g\, (T_g - T_a)$$

$$= \frac{28.2}{60} \times 1.041 \times (345 - 20)$$

$$Q_g = 159KW$$

$$Q_g \text{ in } \% = \frac{159}{879.2} \times 100 = 18.086\%$$

Unaccounted heat loss

$$Q_{ua} = 879.2 - (332.43 + 86.1 + 159)$$

$$Q_{ua} = 301.67KW$$

$$Q_{ua} \text{ in } \% = \frac{301.67}{879.2} \times 100 = 34.34\%$$

***Result:***

***Heat balance sheet***

| ***Credit*** | ***KW*** | ***%*** | ***Debit*** | ***KW*** | ***%*** |
|---|---|---|---|---|---|
| 1. Heat supplied by the fuel ($Q_S$). | 879.2 | 100 | 1. Heat equivalent to IP | 332.43 | 37.81 |
| | | | 2. Heat carried away by cooling water ($Q_W$). | 86.1 | 9.79 |
| | | | 3. Heat carried away by exhaust gas ($Q_g$) | 159 | 18.086 |
| | | | 4. Unaccounted heat loss ($Q_{ua}$). | 301.6 | 34.314 |
| ***Total*** | ***879.2*** | ***100*** | ***Total*** | ***879.2*** | ***100*** |

**Problem 15** *The following data were obtained in a test of single cylinder diesel engine.*

*Fuel consumption = 13.5kg/hr*

*Air consumption = 230kg/hr*

*Engine speed = 33rps*

*Net brake torque = 260N-m*

*Exhaust gas temperature = 400 °C*

*Atmospheric temp = 30 °C*

*Cooling water mass flow rate = 1100kg/hr*

*Temperature of cooling water inlet = 25 °C*

*Temperature of cooling water outlet = 65 °C*

*Calorific value of fuel = 44000KJ/kg*

*Specific heat of exhaust gas = 1.005KJ/kgk*

*Calculate brake power, brake thermal efficiency and draw the energy balance chart?* *[BU-April 1996]*

***Given data:***

FC = 13.5*kg/hr*

AC = 230*kg/hr*

$N = 33 rps$

$T = 260 N\text{-}m$

$T_g = 400°C$

$T_a = 30°C$

$m_w = 1100 kg/hr$

$T_1 = 25°C$

$T_2 = 65°C$

CV = 44000*KJ/kg*

$Cp_g = 1.005 KJ/kgk$

***To find:***

BP, $\eta_{BT}$ and energy balance sheet

☺ ***Solution:***

Brake power, BP = $2\pi NT = 2 \times \pi \times 33 \times 260$

$$BP = 53.91 KW$$

Brake thermal efficiency

$$\eta_{BT} = \frac{Brake\ Power}{m_f \times CV}$$

$$= \frac{53.91}{\frac{13.5}{3600} \times 44000} \times 100$$

$$\eta_{BT} = 32.67\%$$

Energy equal to brake power = 53.91*KW*

$$BP \text{ in } \% = \frac{53.91}{165} \times 100 = 32.6\%$$

Energy equal to cooling water,

$$Q_w = \frac{1100}{3600} \times 4.13(65-25)$$

$$Q_w = 51.08 KW$$

$$Q_w \text{ in } \% = \frac{51.08}{165} \times 100 = 30.96\%$$

Energy equal to exhaust gas,

$$Q_g = \frac{230}{3600} \times 1.005(400-30)$$

$$Q_g = 23.76 KW$$

$$Q_g \text{ in } \% = \frac{23.76}{165} \times 100 = 14.4\%$$

Unaccounted heat losses,

$$Q_{ua} = 165 - [53.91 + 51.08 + 23.76]$$

$$Q_{ua} = 36.25 KW$$

$$Q_{ua} \text{ in } \% = \frac{36.25}{165} \times 100 = 21.97\%$$

***Result:***

| *Credit* | *KW* | *%* | *Debit* | *KW* | *%* |
|---|---|---|---|---|---|
| 1. Energy supplied by the fuel ($Q_S$). | 165 | 100 | 1. Energy equal to brake power. | 53.91 | 32.67 |
| | | | 2. Energy equal to cooling water. | 51.08 | 30.96 |
| | | | 3. Energy equal to exhaust gas | 23.76 | 14.4 |
| | | | 4. Unaccounted heat loss ($Q_{ua}$). | 36.25 | 21.97 |
| *Total* | *165* | *100* | *Total* | *165* | *100* |

# UNIT-II

## FUEL:

Fuel may be chemical or nuclear. Here we shall consider briefly chemical fuels only.

A chemical fuel is a substance which releases heat energy on combustion. The principal combustible elements of each fuel are carbon and hydrogen. Though sulphur is a combustible element too but its presence in the fuel is considered to be undesirable.

## CLASSIFICATION OF FUELS:

fuels can be classified according to whether

- They occur in nature called primary fuels or are prepared called secondary fuels.
- They are in solid, liquid or gaseous state.

The detailed classification of fuels can be given in a summary from as follows:

| Type of fuel | Natural(primary) | Prepared(secondary) |
|---|---|---|
| Solid | Wood<br>Peat<br>Lignite coal | Coke<br>Charcoal<br>Briquettes |
| Liquid | Petroleum | Gasoline<br>Kerosene<br>Fuel oil<br>Alcohol<br>Benzol<br>Shale oil |
| Gaseous | Natural gas | Petroleum gas<br>Producer gas<br>Coal gas<br>Coke-oven gas<br>Blast furnace gas<br>Carbureted gas<br>Sewer gas |

## SOLID FUELS:

### Coal:

- Its main constituents are carbon, hydrogen, oxygen, nitrogen, sulphur, moisture and ash. Coal passes through different stages during its formation form vegetation. These stages are enumerated and discussed below: Plant debris-peat-lignite-brown coal-sub-bituminous coal-bituminous coal-semi bituminous coal-semi-anthracite coal-anthracite coal-graphite.

**Peat:**

- It is the first stage in the formation of coal from wood. It contains huge amount of moisture and therefore it is dried for about 1 to 2 months before it is put to use. It is used as a domestic fuel in Europe and for power generation in Russia. In India it does not come in the categories of good fuels.

**Lignite and brown coals**:

- These are intermediate stages between peat and coal. They have a woody or often clay like appearance associated with high moisture, high as and low heat contents. Lignite is usually amorphous in character and impose transport difficulties as they beak easily. They burn with a smoky flame. Some of this type is suitable for local use only.

**Wood charcoal:**

- It is obtained by destructive distillation of wood. During the process the volatile matter and water are expelled. The physical properties of the residue however depend upon the rate of heating and temperature. Briquettes: these are prepared from fine coal or coke by compressing the material under high pressure.

**LIQUID FUELS:**

- The chief source of liquid fuels is petroleum which is obtained from wells under the earth's crust. These fuels have proved more advantageous in comparison to solid fuels in the following respects.

**Advantages:**

1. Require less space for storage.
2. Higher calorific value
3. Easy control of consumption.
4. Staff economy
5. Absence of danger from spontaneous combustion
6. Easy handling and transportation
7. Cleanliness
8. No ash problem
9. Non-deterioration of the oil in storage

**Petroleum:**

- There are different opinions regarding the origin of petroleum. However, now it is accepted that petroleum has originated probably from organic matter like fish and plant life etc., by bacterial action or by their distillation under pressure and heat. It consists of a mixture of gases, liquids and solid hydrocarbons with small amounts of nitrogen and sulphur compounds. In India, the main sources of Petroleum are Assam and Gujarat.

- Heavy fuel oil or crude oil is imported and then refined at different refineries. The refining if crude oil supplies the most important product called petrol. Petrol can also be made by polymerization of refinery gases.
- Other liquid fuels are kerosene, fuels oils, colloidal fuels and alcohol.

## GASEOUS FUELS:

- Natural gas: the main constituents of natural gas are methane ($CH_4$) and ethane ($C_2H_6$). It has calorific value nearly 21000 $KJ/m^3$. Natural gas is used alternately or simultaneously with oil for internal combustion engines.

**Coal gas:**

- Mainly consists of hydrogen, carbon monoxide and hydrocarbons. It is prepared by carbonization of coal. It finds its use in boilers and sometimes used for commercial purposes.

**Coke-oven gas**:

- It is obtained during the production of coke by heating the bituminous coal. The volatile content of coal is driven off by heating and major portion of this gas is utilized in heating the ovens. This gas must be thoroughly filtered before using in gas engines.

**Blast furnace gas**:

- It is obtained from smelting operation in which air is forced through layer of coke and iron ore, the example being that of pig iron manufacture where this gas is produced as by product and contains about 20% carbon monoxide(CO). After filtering it may be blended with richer gas or used in gas engines directly. The heating value of this gas is very low.

**Advantages:**

- Better control of combustion
- Much less excess air is needed for complete combustion
- Economy in fuel and more efficiency of furnace operation
- Easy maintenance of oxidizing or reducing atmosphere
- Cleanliness
- No problem of storage of the supply is available from public supply line
- The distribution of gaseous fuels even over a wide area is easy through the pipelines and as such handling of the fuel is altogether eliminated
- Gaseous fuels give economy of heat and produce higher temperatures (as they can be preheated in regenerative furnaces and thus heat from hot fuel gases can be recovered)

## COMBUSTION OF FUEL AND REACTION:

- We know that all fuels contain carbon, hydrogen and sulphur. These constituents will readily combine with oxygen and release heat while combusting.
- Therefore, the sufficient air is required to make complete combustion of fuels. If the supplied air is insufficient, still some amount of heat will be present in the burnt fuel which is indicated by presence of CO. if any fuel gas contains only $CO_2$ after combustion, it will refer the process is complete combustion. Otherwise the same combustion process is said to be incomplete combustion which is identified by the presence of both $CO_2$ and CO.
- Combustion means burning. The combustion of fuels can be represented by chemical equations.
- $C+O_2= CO_2$ -------------------(1) ----- complete combustion equation
- $2C+O_2=2CO$ ------------------(2)-----incomplete combustion equation
- $4C+3O_2=2CO_2+2CO$---------(3)----- incomplete combustion equation

| Substances | Symbol | Atomic weight | Molecular weight |
|---|---|---|---|
| Carbon | C | 12 | 12 |
| Hydrogen | $H_2$ | 1(1.008) | 2 |
| Nitrogen | $N_2$ | 14 | 28 |
| Oxygen | $O_2$ | 16 | 32 |
| Sulphur | S | 32 | 32 |
| Carbon dioxide | Co | - | 12+32=44 |
| Water vapour | $H_20$ | - | 2+16=28 |
| Sulphur dioxide | $So_2$ | - | 32+32=64 |
| Methane | $Ch_4$ | - | 12+4=16 |

**Combustion of carbon:**

When carbon combines with oxygen, carbon dioxide is formed and a large amount of heat is released. This process is represented by the chemical equation as.

$$C+O_2=CO_2$$

Carbon + oxygen=carbon dioxide

Substituting the values of molecular weight

$$C+O_2=CO_2$$

$$12+2x16=12+(2x16)$$

$$12+32=44$$

Divide by 12   1+32/12=44/12

1kg+8/3kg=11/3k

This means 1Kg of carbon requires 8/3Kg of oxygen for its completes
Combustion and produces 11/3 Kg of carbon dioxide.

**Carbon Monoxide:**

2C+O2=2CO

Carbon +oxygen =carbon monoxide

2c+02=2co

2x12+2x16=(2x12)+(2x16)

24+32=56

Divide by 24 1kg+4/3kg=7/3kg

1Kg of carbon needs 4/3Kg of oxygen to form carbon monoxide.

Carbon monoxide is highly poisonous. It has no smell and hence it should not be allowed to escape.

**Combustion of carbon monoxide:**

Carbon monoxide combine with oxygen to form carbon dioxide and this represented as,

$2CO + O_2 = 2CO_2$

Carbon monoxide + oxygen = carbon dioxide

Molecular weight

2x28 + 2x16=2(12+32)

56+32=88

Divide by 56 1kg+4/7kg=11/7 kg

1 kg of carbon monoxide requires 4/7 kg of oxygen to form 11/7 kg of carbon dioxide

**Combustion of hydrogen:**

Hydrogen burns with oxygen and produce water vapor. It is represented by the following chemical equation.

2H2+O2=2H2)

Hydrogen + oxygen =water vapour

(2x1x2)+ (2x16) =2(2+16)

4kg + 32kg=36kg

Divide by 4 1kg+32/4kg=36/4kg

1kg+8kg=9kg

1kg of hydrogen burns with 8kg of oxygen and produces 9kg of water vapour.

**Combustion of Sulphur:**

Sulphur burns with oxygen and produce sulphur dioxide. This is represented by the following chemical equation

$$S+O2=SO2$$

Sulphur+Oxygen=Sulphur dioxide

$$32+ (16x2) =32+ (16x2)$$

$$32+32=64$$

Divide by 32   1kg+1kg=2kg

1kg of sulphur burns with 1kg of oxygen and forms 2kg of sulphur dioxide.

## PROPERTIES OF FUEL USED IN IC ENGINES

### Flash point:

- It is defined as the lowest temperature at which the lubricating oil will flash when a small flame is passed across its surface.
- The flash point of the oil should be sufficiently high so as to avoid flashing of oil vapours at the temperature occurring in common use.
- High flash point oils are needed in air compressors.

### Fire point:

- It is the lowest temperature at which the oil burns continuously. The fire point also must be high in lubricating oil, so that oil does not burn in service.

### Calorific valve:

- The calorific value or heating value of the fuel is defined as the energy liberated by the complete oxidation of a unit mass or volume of a fuel.
- It is expressed in KJ/Kg for solid and liquid fuels and $KJ/m^3$ for gases.
- If a fuel contains hydrogen water will be formed as one of the products of combustion. If this water is condensed a large amount of heat will be released. Than if the water exists in the vapour phase. For this reason two heating values are defined the higher or gross heating value and the lower or net heating value.

### Gross or higher calorific value:

- Gross calorific value takes into account the heat recovered from the hot fuel gases due to burning of fuel.
- It is defined as the amount of heat obtained by the complete combustion of 1kg of a fuel. When the products of combustion are cooled down to the temperature of the air supplied ($15^oc$) it briefly written as H.C.V or H.H.V
- If the chemical analysis of a fuel is known the higher calorific valve of the fuel can be determined by the following formula known as Dulong's formula

$$H.C.V=33800c+144000H_2+9270s \text{ KJ/Kg}$$

$$H.C.V=33800c+144000[H_2-O_2/8]+9270s\ KJ/Kg$$

**Net or Lower Calorific Value:**

- Lower calorific value comes into picture (this is in actual practice) when the heat absorbed by the products of combustion is not recovered.
- The higher calorific value is known then the lower calorific value may be obtained by subtracting the amount of heat carried away by the combustion products (steam) from H.C.V
- L.C.V=H.C.V-heat of steam formed during combustion

  L.V.C =H.C.V-amount of steam x heat liberated

**In solid fuels:**

L.C.V=H.C.V-($9H_2$ x2466) KJ/kg where $H_2$-amount of hydrogen present in %

**In gaseous fuels:**

L.C.V=H.C.V-(Mc/Vsx2466)KJ/$m^3$

$M_c$-amount of steam condensed (Kg)

$V_s$-volume of gas used at S.T.P ($m^3$)

Water equivalent

$W_e=m_b$ x $s_b$ gm

$W_e$-water equivalent of the substance

$M_b$-mass of the substance

$S_b$-specific heat of the substance

**Solid and liquid fuels:**

The heat released by the fuel on combustion is absorbed by the surrounding water and the calorimeter.

Heat released by the fuel sample=$W_f$ x C

$W_f$-weight of fuel sample (Kg)

C-calorific value (higher) of the fuel (KJ/Kg)

Heat received by water and calorimeter

$= (w_m+w_e)$ xC x $[(t_2-t_1)+t_c]$

$W_m$-weight of water (Kg)

$W_e$-water equivalent of calorimeter (kg)

$t_2$- final temperature of water and c.m

$t_1$- Initial temperature of water and c.m

c- Specific heat of water

tc- radiation corrections

Heat lost =heat gained

$$w_f \text{ x } c=(w_w+w_e) \text{ x } c \text{ x}[(t_2-t_1)+t_c]$$

$$C= \frac{(w_w+w_e) \text{ x} c \text{ x}[(t_2-t_1)+t_c]}{w_f}$$

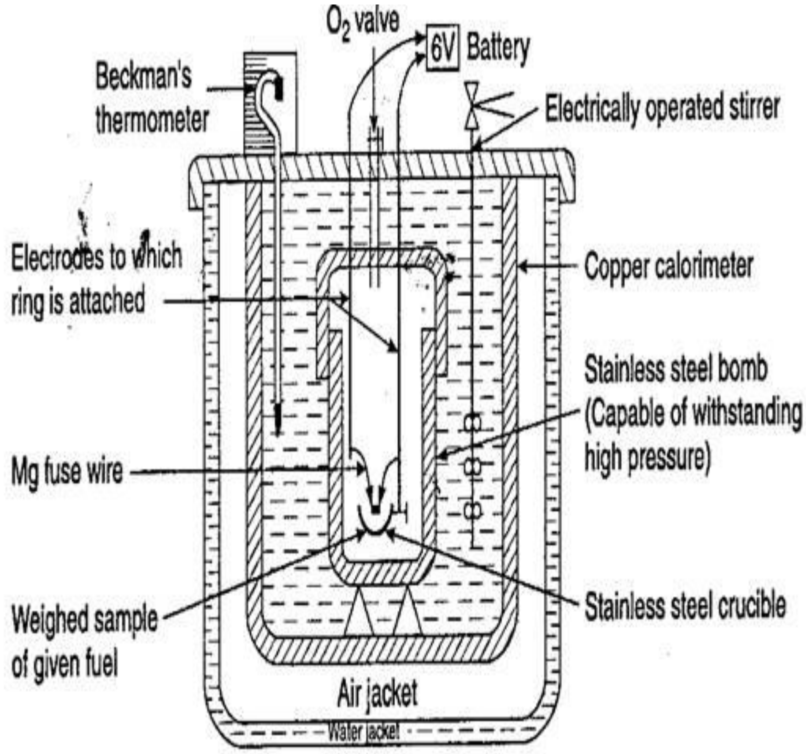
## SOLID AND LIQUED FUEL:

### SOLID FUEL

**Bomb calorimeter:**

- The calorific value of solid and liquid fuels is determined in the laboratory by Bomb calorimeter.
- It is so named because its shape resembles that of a bomb fig shown the schematic sketch of a bomb calorimeter.
- The calorimeter is made of austenitic steel which provides considerable resistance to corrosion and enables it to withstand high pressure.
- In the calorimeter is a strong cylindrical bomb in which combustion occurs.
- The bomb has two values at the top. One supplies oxygen to the bomb and other releases the exhaust gases.
- A crucible in which a weighted quantity of fuel sample is burnt is arranged between the two electrodes as shown in figure. The calorimeter is fitted with water jacket which surrounds the bomb.
- To reduce the losses due to radiation Calorimeter is further provided with a jacket of water and air.

A stirrer for keeping the temperature of water uniform and a thermometer to measure the temperature up to accuracy of 0.001°c is fitted through the lid of the calorimeter.



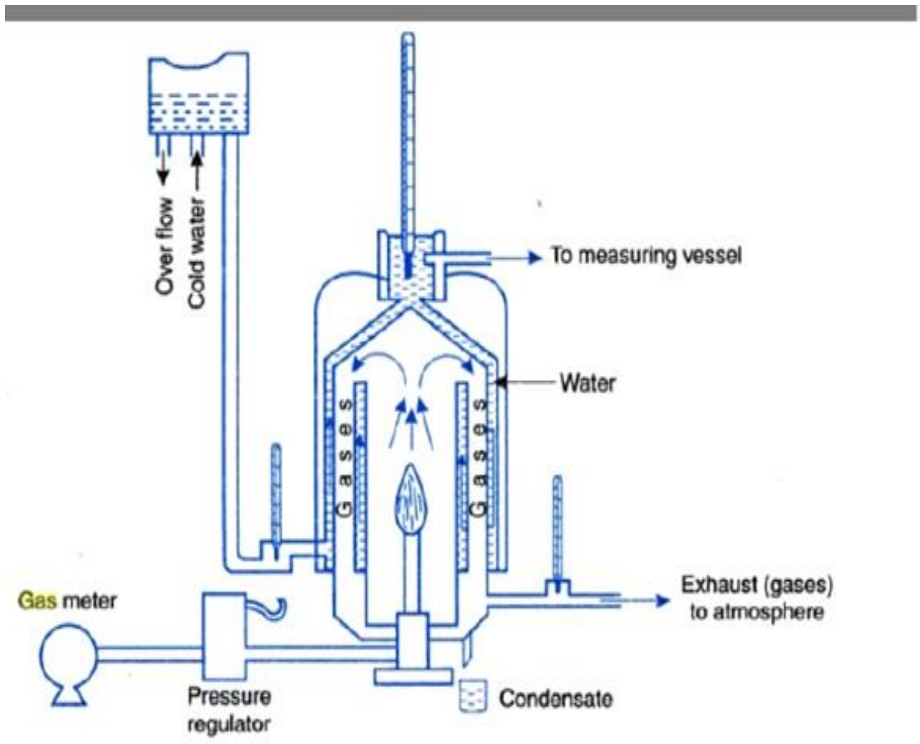
### Procedure:

- To start with about 1gm of fuel sample is accurately weight into the crucible and a fuse wire (whose weight is known) is stretched between the electrodes.
- It should be ensured that wire is in close contact with the fuel.
- To absorb the combustion products of sulphur and nitrogen 2ml of water is poured in the bomb.
- Bomb is then supplied with pure oxygen through the value to an amount of 25 atmospheres.
- Bomb is then placed in the weighted quantity of water in the calorimeter.
- The stirring is started after making necessary electrical connections and when the thermometer indicates a steady temperature fuel is fired and temperature fuel is fired and temperature readings are recorded after ½ minute intervals until maximum temperature is attained.
- The bomb is then removed the pressure slowly released through the exhaust valve and the contents of the bomb are carefully weight for further analysis.

## GASEOUS FUELS:

### Junker's gas calorimeter:

- The calorific value of gaseous fuels can be determined by Junker's gas calorimeter.
- Junkers gas calorimeter is shown in figure its principle is somewhat similar to bomb calorimeter in respect that heat evolved by burning the gas is taken away by the water
- In its simplest construction it consists of a combustion chamber in which the gas is burnt (in a gas burner). A water jacket through which a set of tubes called flues pass surrounds this chamber.
- Thermometers are incorporated at different places (as shown in figure) to measure the temperatures.



**Procedure:**

- A metered quantity of gas whose calorific value is to be determined is supplied to the gas burner via a gas meter which records its volume and pressure regulator which measures the pressure of the gas by means of a manometer.
- When the gas burns the hot products of combustion travel upwards in the chamber and then downwards through the flues and finally a space to the atmosphere through the outlet.
- The temperature of the escaping gas is recorded by the thermometer fitted at the exist and this temperature should be as close to room temperature as possible so that entire heat of combustion is absorbed by water. The cold water enters the calorimeter near the bottom and leaves near the top. Water which is formed by condensation of steam is collected in a pot.
- The quantity of gas used during the experiment is accurately measured by the meter and temperature of ingoing and outgoing water is indicated by the thermometers. From the above data the calorific value of the gas can be calculated.

### A/F RATIO OR STOCHIOMETRIC RATIO

- The mixture that contains just enough air for the complete combustion of all the fuel is called as the air fuel ratio or stoichiometric fuel ratio.

### RICH MIXTURE

- A mixture having more fuel than a chemically correct mixture is known as rich mixture.

### LEAN MIXTURE

- A mixture having less fuel than a chemically correct mixture is known as lean mixture.

## COMBUSTION PROCESS IN IC ENGINES

- Combustion in SI engine
- Combustion in CI engine

## COMBUSTION PHENOMENONIN S.I. ENGINES:

**Normal combustion**:

Fuel is completely vaporized and homogeneously premixed with air and residual gas, when combustion occurs means is called by normal combustion.

The combustion is **normal** when 2 conditions are respected:

1) The ignition of combustible mixture is controlled by the spark plug, with a present timing .
2) After the ignition, the flame propagates regularly to the whole mixture, without any sudden increase of velocity.

- The combustion process in SI engine can be divided into three broad categories.

1. Ignition delay or ignition lag.
2. Flame propagation.
3. After burning.

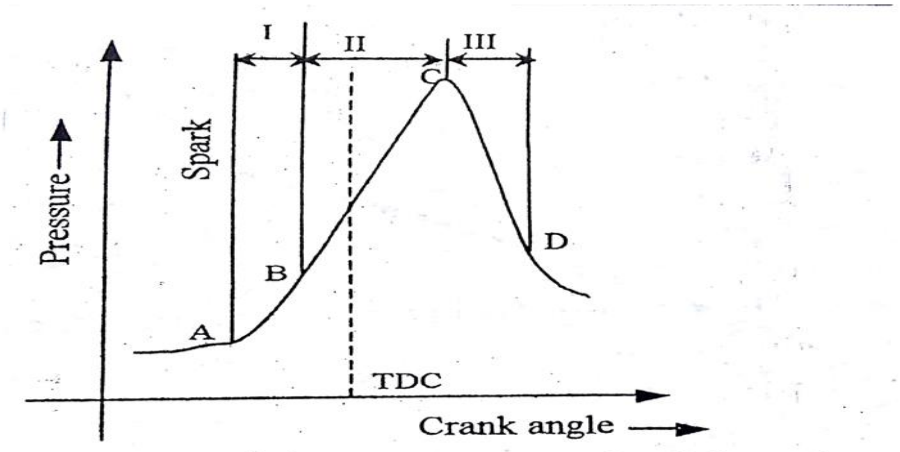
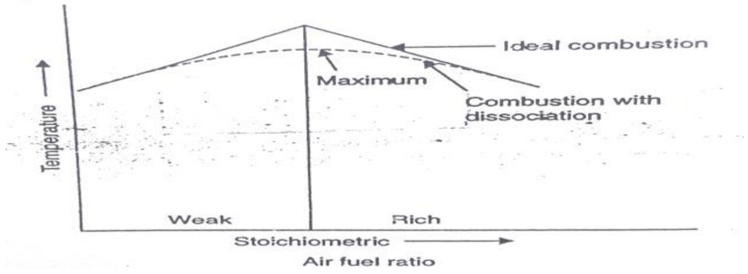


*Fig. 4.1.2. Actual p-θ diagram for S.I. engines*

*I – Ignition delay*
*II – Flame propagation*
*III – After burning*

1. **Ignition delay(A-B)**
- Point A refers the ignition of spark.
- But burning of air fuel mixture commences only at point B.
- interval between A and B is known as ignition delay or lag.
2. **Flame propagation (B-C)**
- The turbulent flame rapidly spreads over the main part of the combustion chamber, whose volume remains almost constant, since in this phase the piston moves slowly around TDC.
- This is the main phase characterized by a rapid burning. It begins when 4-8% of the mixture volume has been ignited and it ends when the flame front comes near the walls and the max pressure in the cylinder is reached.
3. **After burning(C-D)**
- During which the mixture completes its oxidation processes behind the flame front, when the expansion stroke is occurring.
- The burning of fuel after the maximum pressure is known as after burning.
- The flame velocity decreases during this stage.
- The rate of combustion becomes low due to lower flame velocity and reduced flame front surface.

**Factors affecting normal combustion are S.I engines:**



The factors which affect normal combustion in S.I engines are brief described below:

1. **Induction pressure:** As the pressure falls delay period increases and the ignition must be earlier at low pressures. A vacuum control may be incorporated.
2. **Engine speed:** As speed increases the constant time delay period needs more crank angle and ignition must be earlier. A centrifugal control may be employed.
3. **Ignition timing:** if ignition is too early the peak pressure will occur too early and work transfer falls. If ignition is too late the peak pressure will be low and work transfer falls. Combustion may be complete by the time the exhaust valve opens and the valve may burn.
4. **Mixture strength:** although the stoichiometric ratio should give the best results, the effect of dissociation is to make a slightly rich mixture necessary for maximum work transfer.
5. **Compression ratio:** an increase in compression ratio increases the maximum pressure and the work transfer.
6. **Fuel choice**:
   - The induction period of the fuel will affect the delay period.
   - The calorific value and the enthalpy of vaporization will affect the temperatures achieved

**Abnormal combustion**

When the 2 conditions of normal combustion do not occur, the combustion process is **abnormal**, causing engine **damages** or simply performance worsening and **noise**.

There are two forms of abnormal combustion in SI engines:

- The first of these is pre ignition of the mixture by carbon particles in the chamber. This will have the effect of reducing the work transfer.
- The second abnormality is generally known as knock and is a complex condition with many facets.

**Pre-ignition**

- It is defined as the phenomenon of ignition of the charge before the ignition spark occurs.
- This ignition is caused when some parts of combustion space like spark plug, exhaust valve, carbon particles in the combustion chamber are over heated under certain operating conditions.

**Knocking or detonation**

- If the temperature of the air fuel mixture is raised high enough, the mixture will self-ignite without the need of spark plug. This phenomenon is called as **self-ignition or auto ignition**.
- The temperature above which self-ignition occurs called the **self-ignition temperature.**

- If the temperature of the unburnt mixture exceeds the self-ignition temperature during the ignition delay period, auto ignition occurs at various locations in the cylinder.
- This will generate the pressure pulses. This high pressure pulses can cause damage to the engine and quite often are in the audible frequency range .This phenomenon is often called **knocking or detonation.**

**Effects of detonation:**

- Noise and roughness
- Mechanical damage
- Carbon deposits
- Increase in heat.
- Decrease in power output and efficiency

**Control of detonation:**

The detonation can be controlled or even stopped by the following methods

- Increase engine r.p.m
- Uniform spark production.
- Reducing pressure in the inlet manifold by throttling
- Making the ratio too lean or too rich.

**Factors affecting knock:**

The likelihood of knock is increased by any reduction in the induction period of combustion and any reduction in the progressive explosion flame velocity. Particular factors are listed below

1. **Fuel choice**: a low self ignition temperature promotes knock.
2. **Induction pressure**: increase of pressure decreases the self ignition temperature and the induction period. Knock will tend to occur at full throttle.
3. **Engine speed**: low engine speeds will give low turbulence and low flame velocities and knock may occur at low speed.
4. **Ignition timing:** advanced ignition timing increases peak pressures and promotes knock.
5. **Mixture strength**: optimum mixture strength gives high pressures and promotes knock.
6. **Compression ratio**: high compression ratios increase the cylinder pressures and promote knock
7. **Combustion chamber design**: poor design gives long flame paths, poor turbulence and insufficient cooling all of which promote knock.
8. **Cylinder cooling:** poor cooling raises the mixture temperature and promotes knock

**Desirable characteristics of combustion chamber for S.I. engines:**

From the point of view of attaining highest resistance to detonation under a given set of service conditions, and also promote high power output; high thermal efficiency, and smooth operation, the following would appear to be desirable characteristics for S.I. engines:

1. Short combustion time
2. Short ratio of flame path to bore
3. Absence of hot surfaces in the end gas region
4. Use of squish areas particularly in the end gas region
5. High velocity through the inlet valve
6. Large surface to volume ratio for the end gas
7. Cooling of hot spots

**COMBUSTION CHAMBER DESIGN-SI ENGINES:**

Combustion chambers are usually designed with every possible attempt made to meet the following objectives:

- To regulate the rate of pressure rise such that the greatest force is applied to the piston as closely after T.D.C on the power stroke as possible, with a gradual decrease in the force on the piston during the power stroke. The forces must be applied to the piston smoothly, however, thus placing a limit on the rate of the pressure rise, as well as the position of the peak pressure with respect to T.D.C
- To prevent the possibility of detonation at all times

To obtain these objectives attempt is made to design the combustion chambers with the following factors in mind:

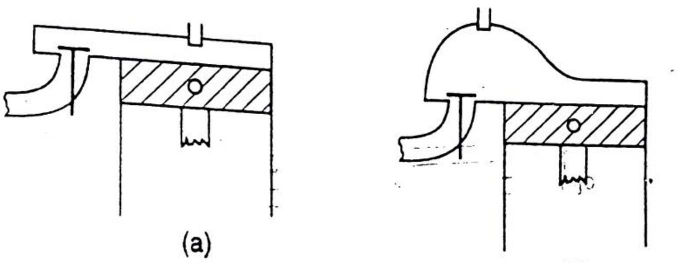
i) To achieve the highest possible flame front velocity through the creation of high turbulence of the minute “swirl” type.

ii) To burn the largest mass of charge as soon as possible after ignition with progressive reduction in the mass of the charge burnt toward the end of the combustion

iii) To reduce the possibility of detonation by:

*reducing the temperature of the last portion of the charge to burn, through the application of a high surface to volume ratio in that part of the combustion chamber where this portion burns. Such a ratio increases the temperature of the final unburned charge

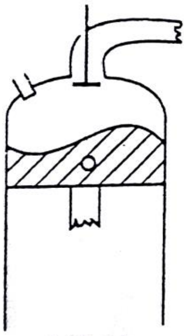
* reducing the distance for the flame to travel by centrally locating the spark plug, or in some engines, by using dual spark plugs.



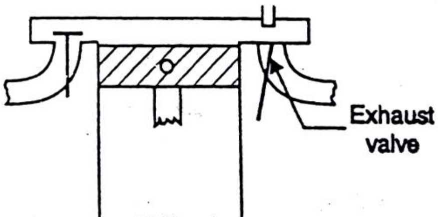
(a)

(b)

L-Head Types



I-Head
(c)



T-Head
(d)

It may be noted that these chambers are designed to obtain the objectives outlined above, namely:

1. A high combustion rate at the start
2. A high surface to volume ratio near the end of burning
3. A rather centrally located spark plug.

**TURBULENCE IN S.I ENGINES:**

1. Turbulence plays a very important role in combustion phenomenon in S.I engines. The flame speed is very low in non-turbulent mixtures. A turbulent motion of the mixture intensifies the processes of heat transfer and mixing of the burned and unburned portions in the flame front. These two factors cause the velocity of turbulent flame to increase practically in proportion to the turbulent velocity. The turbulence of the mixture is due to admission of fuel-air mixture through comparatively narrow sections of the intake pipe, valves etc. in the suction stroke. The turbulence can be increased at the end of the compression stroke by suitable design of combustion chamber which involves the geometry of cylinder head and piston crown.

2. The degree of turbulence increases directly with the piston speed.

The effects of turbulence can be summed up as follows:

i) Turbulence accelerates chemical action by intimate mixing of fuel and oxygen. Thus weak mixtures can be burnt.

ii) The increase of flame speed due to turbulence reduces the combustion time and hence minimizes the tendency to detonate.

iii) Turbulence increase the heat flow to the cylinder wall and in the limit excessive turbulence may extinguish the flame.

iv) Excessive turbulence results in the more rapid pressure rise and the high pressure rise causes the crankshaft to spring and rest of the engine to vibrate with high periodicity, resulting in rough and noisy running of the engine.

**The following points are worth noting:**

**Swirl:** the main macro mass motion within the cylinder is rotational motion called swirl. It is generated by constructing the intake system to give a tangential component to the intake flow as it enters the cylinder. This is done by shaping and contouring the intake manifold, valve ports and even the piston face.

Swirl greatly enhances the mixing of air and fuel to give a homogeneous mixture in the very short time available for this in modern high speed engines. It is also a main mechanism for very rapid spreading of the flame front during the combustion process.

**Squish and Tumble:**

As the piston approaches T.D.C at the end of compression stroke, the volume around the outer edges of the combustion chamber is suddenly reduced to a very small value. Many modern combustion chamber designs have most of the clearance volume near the centerline of the cylinder. As the piston approaches T.D.C the gas mixture occupying the volume at the outer radius of the cylinder is forced radially inward as this outer volume is reduced to near zero. This radial inward motion of the gas mixture is called squish. It adds to other mass motions within the cylinder to mix the air and fuel and to quickly spread the flame front. Maximum squish velocity usually occurs about $10^o$ before T.D.C

As the piston nears T.D.C squish motion generates a secondary rotational flow called tumble. This rotation occurs about a circumferential axis near the outer edge of the piston bowl.

## FIAME PROPAGATION:

- Typical flame propagation velocities range from something like 15to 70 m/s.
- This would relate to the combustion flame velocity increasing from about 15 m/s at an idle speed of About 1000r.p.m to roughly 70 m/s at a maximum speed of 6000 r.p.m.
- When ignition occurs the nucleus of the flame spreads with the whiling or rotating vortices in the Form of ragged burning crust from the initial spark plug ignition site.
- The speed of the flame propagation is roughly proportional to the velocity at the periphery of the Vortices.

### Effect of engine variable on flame propagation:

1. **Fuel air ratio:** when the mixture is made leaner or is enriched and still more velocity of flame diminishes.
2. **Compression ratio:** the speed of combustion increase of compression ratio .the increase in compression ratio results in increase in temperature which increases the tendency of the engine to detonate.

3. **Intake temperature and pressure:** increase in intake temperature and pressure increases the flame speed.
4. **Engine load:** as the load on the engine increases the cycle pressures increase and hence the flame speed increases.
5. **Turbulence:** the flame speed is very low in non-turbulent mixture. A turbulent motion of the mixture intensifies the processes of heat transfer and mixing of the burned and unburned portions in the flame front these two factors cause the velocity of turbulent flame to increase practically in proportion to the turbulent velocity.
6. **Engine speed:** the flame speed increases almost linearly with engine speed. The crank angle required for flame propagation, which is the main phase of combustion will remain constant at all speed.
7. **Engine size:** the number of crank degrees required for flame travel be about the same irrespective of engine size provided the engine are similar.

## COMBUSTION PHENOMENON IN C.I ENGINES:

### Normal combustion

- The process of combustion in the compression ignition (C.I) engine is fundamentally different from that in a spark-ignition engine. C.I engine combustion occurs by the high temperature produced by the compression of the air, i.e., it is an auto-ignition.

**According to Sir Ricardo, the combustion process in C.I. engine can be divided into four broad regions as:**

(i) Ignition-delay period.
(ii) Uncontrolled combustion period
(iii) Controlled combustion period
(iv) After-burning period.

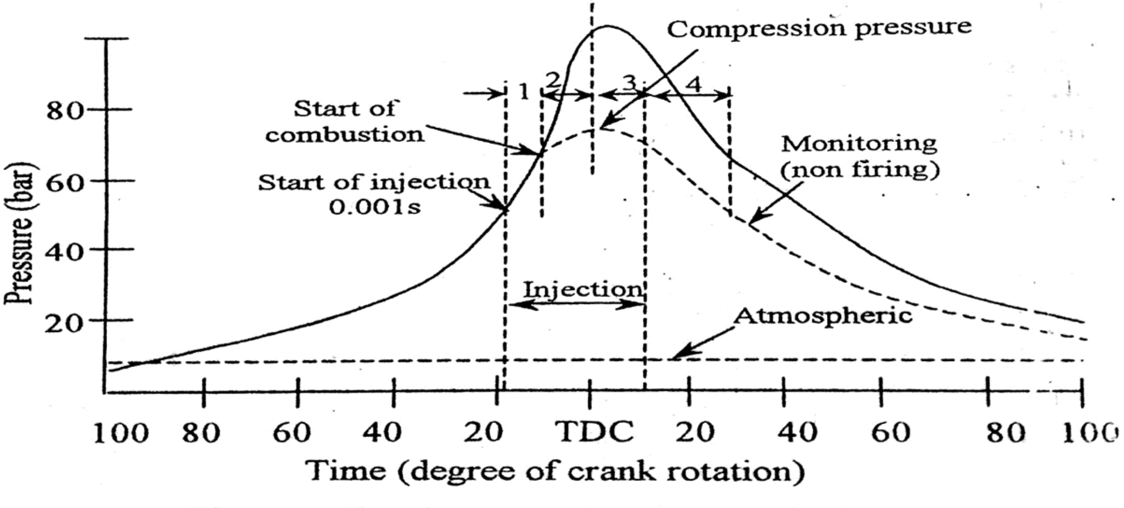


*Fig 4.1.4 Combustion phenomenon in C.I. engine*

The third phase is followed by after burning, which may be called the fourth phase of combustion.

1. **Ignition delay period:**
   - The delay period is counted from the start of injection to the point where the p-ɵ combustion curve departs from air compression curve. The delay period can be roughly sub-divided into physical delay and chemical delay.

- The period of physical delay is the time between the beginning of injection and the attainment of chemical reaction conditions. In the physical delay period, the fuel is atomized, vaporized, mixed with air, and raised in temperature.
- In the chemical delay period reaction starts slowly and the accelerated until inflammation or ignition takes place.
- The delay period exerts a great influence in the C.I engine combustion phenomenon. It is clear that the pressure reached during the second stage will depend upon the duration of the delay period; the longer the delay, the more rapid and higher the pressure rise, since more fuel will be present in the cylinder before the rate of burning comes under control.
- This causes rough running and may cause diesel knock. Therefore we must aim to keep the delay period as short as possible, both for the sake of smooth running and in order to maintain control over the pressure changes.
- But some delay period is necessary otherwise the droplets would not be dispersed in the air for complete combustion. However, the delay period imposed upon is greater than what is needed and the designer's efforts are to shorten it as much as possible.

2. **Period of rapid or uncontrolled combustion:**
    - The second stage of combustion C.I engines, after the delay period, is the period of rapid or uncontrolled combustion. This period is counted from the end of the delay period to the point of maximum pressure on the indicator diagram.
    - In this second stage of combustion, the rise of pressure is rapid because during the delay period the droplets of fuel have had time to spread themselves out over a wide area and they have fresh air all around them. About one-third of head is evolved during this process.
    - The rate of pressure rise depends on the amount of fuel present at the end of delay period, degree of turbulence, fineness of atomization and spray pattern.
3. **Period of controlled combustion**:
    - At the end of second stage of combustion, the temperature and pressure are so high that the fuel droplets injected in the third stage burn almost as they enter and any further pressure rise can be controlled by purely mechanical means, i.e. by the injection rate.
    - The period of controlled combustion is assumed to end at maximum cycle temperature. The heat evolved by the end of controlled combustion is about 70 to 80 per cent.
4. **After burning**:
    - The combustion continues even after the fuel injection is over, because of poor distribution of fuel particles. This burning may continue in the expansion stroke upto $70^o$ to $80^o$ of crank travel from T.D.C this continued burning, called the after burning, may be considered as the fourth stage of the combustion. The total heat evolved by the end of entire combustion process is 95 to 97%; 3 to 5% of heat goes as unburned fuel in exhaust.
    - In the p-V diagram, the stages of combustion are not seen because of little movement of piston with crank angle at the end and reversal of stroke. So for studying the combustion stages, therefore, a pressure-crank angle or time, p-ɵ or p-t diagram is invariably used. In the actual diagram, the various stages of combustion look merged, yet the individual stage is distinguishable.

## Factors affecting combustion in C.I engines:

- In compression ignition combustion, the length of the delay period plays a vital role. This period serves a useful purpose in that it allows the fuel jet to penetrate well into the combustion space. If there were no delay the fuel would burn at the injector resulting in incomplete combustion. If delay is too long the amount of fuel available for simultaneous explosion is too great and the resulting pressure rise is too rapid:

**Delay is reduced by the following:**

1. High charge temperature
2. High fuel temperature
3. Good turbulence;
4. A fuel with a short induction period

**Fuel choice**:

- The choice of fuel has a large effect on combustion since induction period of the fuel contributes a major part of the delay period.

**Combustion chamber design**:

- The shape of the chamber is critical so that the fuel is introduced with swirl and the pistons are also shaped to produce a squish effect, the whole concept of design being to give controlled motion of the air and fuel rather than rely on haphazard turbulence.

**Engine load**:

- In most engines injection starts at a fixed crank angle and continues as long as the rack setting allows. At light loads all the fuel will be injected into the delay and simultaneous explosion periods, or at very light loads when temperatures are lower and the delay period longer injection may be completed in the delay period.

**Engine speed:**

- The effect of speed is allied to the design of the combustion chamber and varies with individual designs. Delay may be reduced or increased.

### Abnormal combustion in C.I engines:

In C.I engines abnormal combustion is not as great a problem as in S.I engines. The only abnormal is diesel knock. This occurs when the delay period is excessively long so that there is a large amount of fuel in the cylinder for the simultaneous explosion phase. The rate of pressure rise per degree of crank angle is then so great than an audible knocking sound occurs. Turning is rough and if allowed to become extreme the increase in mechanical and thermal stresses may damage the engine. Knock is thus a function of the fuel chooser and may be avoided choosing a fuel with characteristics that do not give too long a delay period.

### Delay period (or ignition lag) in C.I engines:

In C.I engine, the fuel which is in atomized form is considerably colder than the hot compressed air in the cylinder. Although the actual ignition is almost instantaneous, an appreciable time elapses before the combustion is in full progress. This time occupied is called the delay period or ignition lag. It is the time immediately following injection of the fuel during which the ignition process is being initiated and the pressure does not rise beyond the value it would have due to compression of air: The delay period extends for about $13^o$, movement of the crank. The time for which it occurs decreases with increase in engine speed.

**The delay period depends upon the following:**

i) Temperature and pressure in the cylinder at the time of injection.
ii) Nature of the fuel mixture strength
iii) Relative velocity between the fuel injection and air turbulence.
iv) Presence of residual gases.
v) Rate of fuel injection.
vi) To small extent the finess of the fuel spray

The delay period increase with load but is not much affected by injection pressure

The delay period should be as short as possible since a long delay period gives a more rapid rise in pressure and thus causes knocking.

**Diesel knocks:**

If the delay period in C.I engines is long a large amount of fuel will be injected and accumulated in the chamber. The auto-ignition of this large amount of fuel may cause high rate of pressure rise and high maximum pressure which may cause knocking in diesel engines. A long delay period not only increases the amount of fuel injected by the moment of ignition but also improves the homogeneity of the fuel-air mixture and its chemical preparedness for explosion type self-ignition similar to detonation is S.I engines.

The following are the differences in the knocking phenomena of the S.I and C.I engines:

1. In the S.I engine, the detonation occurs near the end of combustion whereas in the C.I engine detonation occurs near the beginning of combustion.
2. The detonation in the S.I engine is of a homogeneous charge causing very high rate of pressure rise and very high maximum pressure. In the C.I engine the fuel and air are imperfectly mixed and hence the rate of pressure rise is normally lower than that in the detonating part of the charge in the S.I engine.
3. In the C.I engine the fuel is injected into the cylinder only at the end of the compression stroke, there is no question of pre-ignition as in S.I engine.
4. In the S.I engine it is relatively easy to distinguish between knocking and non-knocking operation as the human ear easily finds the distinction.

## BASIC DESIGNS OF C.I ENGINES COMBUSTION CHAMBERS:

In C.I engines several types of combustion chambers are used. Each of these has its own peculiarities, and desirable, as well as undesirable features. Any one of these combustion chambers amy produce good results in one field of application, but less desirable, or even poor results in another. No one combustion chamber design has yet been developed which will produce the best result is all types of engines. The particular design chose, then, must be that which accomplishes the best performance fot the application desired.

Four specific designs which find wide use in C.I engines are discussed below:

1. The non-turbulent type
    i) Open combustion chamber

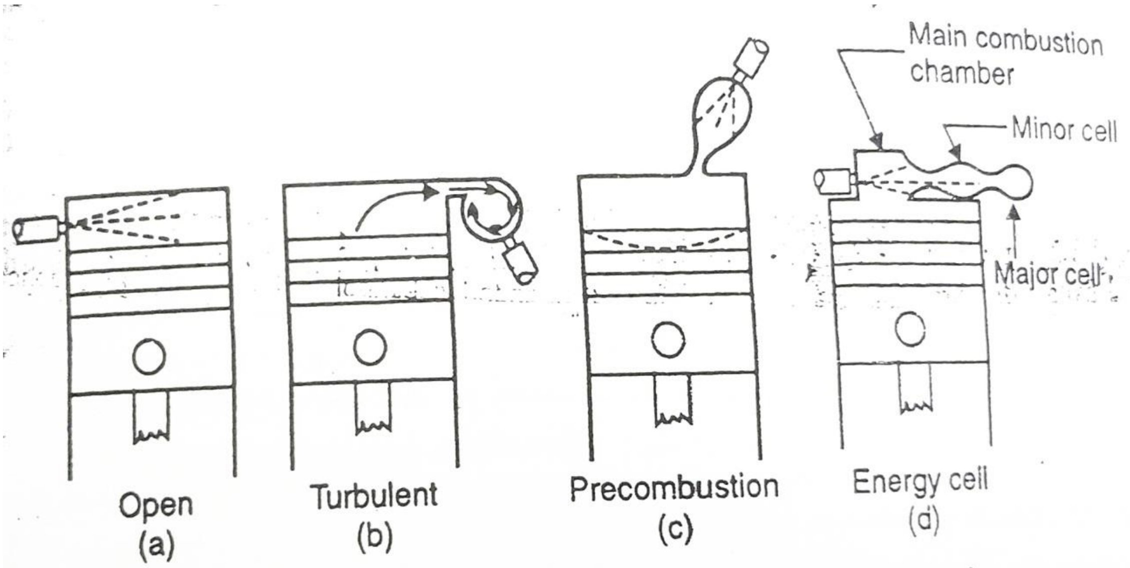
2. The turbulent type
    i) Turbulent chamber
    ii) Precombustion chamber
    iii) Energy cell.

**1. Non-turbulent type:**

- Fig illustrates the usual design of open combustion chamber, which is representative of non-turbulent type. The fuel is injected directly into the upper portion of the cylinder which acts as the combustion chamber. This type depends little on turbulence to perform the mixing. Consequently, the heat loss to the chamber walls is relatively low and easier starting results.
- In order to obtain proper penetration and dispersal of the fuel necessary for mixing with the air, however, high injection pressures and multi-orifice nozzles are required. This necessitates small nozzle openings and results in more frequent clogging or diversion of the fuel spray by accumulated carbon particles, with consequent higher maintenance costs.
- This type of chamber is ordinarily used on low speed engines, where injection is spread through a greater period of time and thus ignition delay is a relatively less important factor. Consequently, less costly fuels with longer ignition delay may be used.

**2. Turbulent type:**

- The turbulent chamber, precombustion chamber and energy cell are variations of the turbulent type of chamber and are illustrated in fig.



- In the turbulent chamber fig(b) the upward moving piston forces a flow of air into a small antechamber, thus imparting a rotary motion to the air passing the pintle type nozzle. As the fuel is injected into the rotating air, it is partially mixed with his air, and commences to burn. The pressure built up in the antechamber by the expanding burning gases force the burning and unburned fuel and air mixtures back into the main chamber, again imparting high turbulence and further assisting combustion.
- In the precombustion chamber fig(c) the upward moving piston forces part of the air into a side chamber, called the precombustion chamber. Fuel is injected into the air in the precombustion

chamber by a pintle type nozzle. The combustion of the fuel and air produces high pressures in the precombustion chamber, thus creating high turbulence and producing good mixing and combustion.

- The energy cell is more complex than the pre combustion chamber. It is illustrated in fig (d). As the piston moves up on the compression stroke, some of the air is forced into the major and minor chambers of the energy cell. When the fuel is injected through the pintle type nozzle, part of the fuel passes across the main combustion chamber and enters the minor cell, where it is mixed with the entering air. Combustion first commences in the main combustion chamber where the temperature is higher but the rate of burning is slower in this location, due to insufficient mixing of the fuel and air.
- Summarily it may be said that a particular combustion chamber design must be chosen to perform a given job. No one combustion chamber can produce an ultimate of performance in all tasks. As most engineering work, the design of the chamber must be based on a compromise, after full considerations of the following factors: i) heat lost to combustion chamber walls ii)injection pressure iii) nozzle design iv)maintenance, v)ease of starting, vi) fuel requirement vii)utilization of air viii) weight relation of engine to power output ix)capacity for variable speed operation.

## RATING OF FUELS

- Fuels are rated for their antiknock qualities.
  - ☐ Gasoline : Octane number
  - ☐ Diesel : Cetane number

Resistance to knock depends upon the chemical composition.

- Other operating parameters:
  - ☐ F-A ratio
  - ☐ Ignition timing
  - ☐ Engine speed
  - ☐ Shape of combustion chamber
  - ☐ Compression ratio etc.

## SI ENGINE FUELS (OCTANE NUMBER)

Antiknock property is compared with reference to

- Iso-octane ($C_8H_{18}$) Þ 100 Octane No. Very good antiknock fuel
- Heptane ($C_7H_{16}$) Þ Zero Octane No. Very poor antiknock fuel

- Fuel with Octane Number of 70 indicates
  - ☐ 70 % octane, and
  - ☐ 30 % heptane

- **Definition:** It indicates the % by volume of iso-octane in a mixture of iso-octane and heptane which exhibit the same characteristics of the fuel in a standard engine under a set of operating conditions.

- Common octane numbers for gasoline fuels used in automobile range from 87 to 95, with higher values for special high performance and racing cars.

Tests for Rating Octane Number (ON)

- Two most common methods of rating gasoline and other SI engine fuels are the Motor Method and the Research Method. These give the motor octane number (MON) and research octane number (RON).
- Another less common methodis the Aviation Method used for aircraft fuel, and this gives an Aviation Octane Number (AON).

**CI ENGINE FUELS (CETANE NUMBER)**

Rating of CI Engine Fuel

- Reference Fuels
  - Cetane ($C_{16}H_{34}$) Þ 100 Cetane No.
  - A-methyl naphthalene ($C_{11}H_{10}$) Þ Zero Cetane No.
- Fuel with Cetane Number of 60 indicates
  - 60 % $C_{16}H_{34}$
  - 40 % $C_{11}H_{10}$

**Definition:** It indicates the % by volumeof normal Cetane in a mixture of Cetane ($C_{16}H_{34}$) and a-methyl naphthalene ($C_{11}H_{10}$) which exhibit the same ignition characteristics (ID) as the test fuel when combustion is carried out under specified operating conditions.

- Cetane number is a measure of its ability to auto-ignite quickly when the fuel is injected into the combustion chamber.
- Higher the CN, lesser is the tendency to knock. Further, too high a Cetane number may induce pre-ignition.
- Diesel usually has a Cetane number between 40-60, whereas gasoline has a Cetane number of 10-20. This is why it is not suitable as diesel fuel due to its poor auto- ignition quality. A good diesel engine fuel is a bad gasoline engine fuel.

**Qualities of CI Engine Fuel**

- It should have good antiknock quality. Must have short ignition delay.
- Must be sufficiently volatile in the operating range to ensure proper mixing and complete combustion.
- Should not promote smoke in the exhaust.
- Should not cause corrosion / wear in the engine components.
- Easy handling / availability.

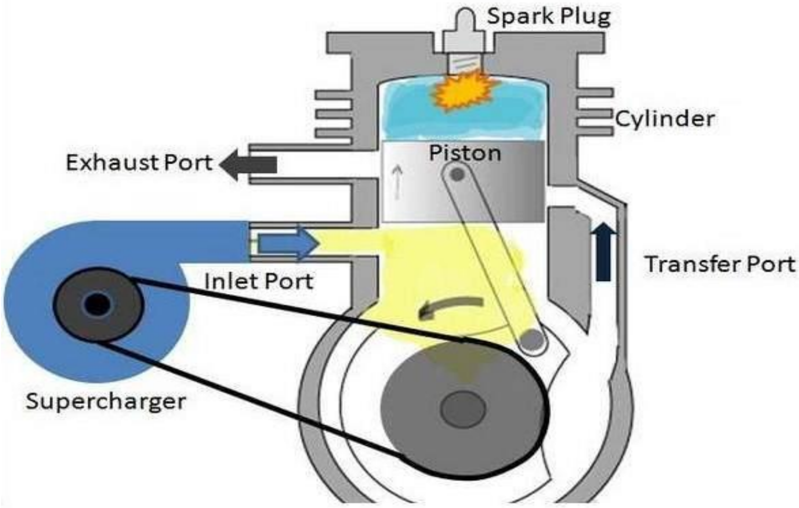
## SUPERCHARGING AND TURBOCHARGING:

The most efficient method of increasing the power of an engine is by supercharging, i.e. increasing the flow of air into the engine to enable more fuel to be burnt.

- A Supercharger is run by the mechanical drive, powered by engine power.
- A turbocharger uses the otherwise unused energy in the exhaust gases to drive a turbine directly connected by a co-axial shaft to a rotary compressor in the air intake system.

### Need of turbocharger and super charger

- For ground installations, it is used to produce a gain in the power output of the engine.
- For aircraft installations, in addition to produce a gain in the power output at sea-level, it also enables the engine to maintain a higher power output as altitude is increased.



Supercharger

### Type of Compressor used in super charger

Compressors used are of the following three types:

1. **Positive displacement type** used with many reciprocating engines in stationary plants, vehicles and marine installations.

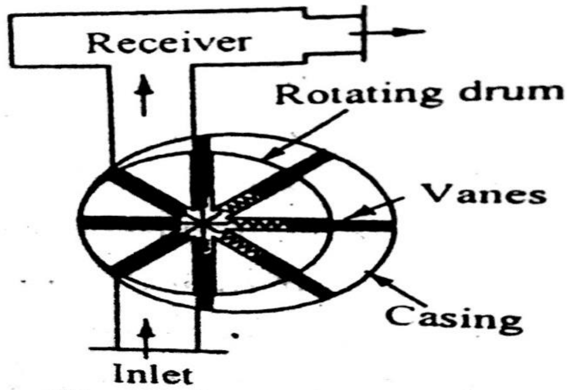


*Fig 4.1.9 Vane blower*

2. **Axial flow type** seldom used to supercharge reciprocating engines, it is widely used as the compressor unit of the gas turbines.

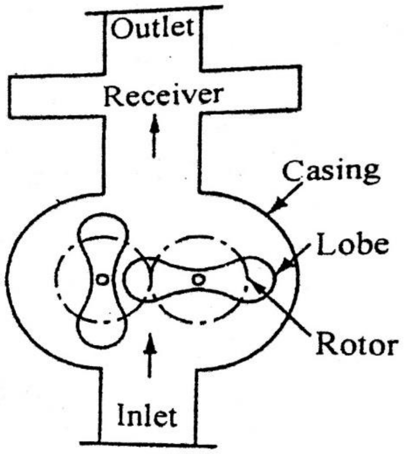


*Fig 4.1.8 Roots blower*

3. **Centrifugal type** widely used as the supercharger reciprocating engines, as well as compressor for gas turbines. It is almost exclusively used as the supercharger with reciprocating power plants for aircraft because it is relatively light and compact, and produces continuous flow rather than pulsating flow as in some positive displacement types.

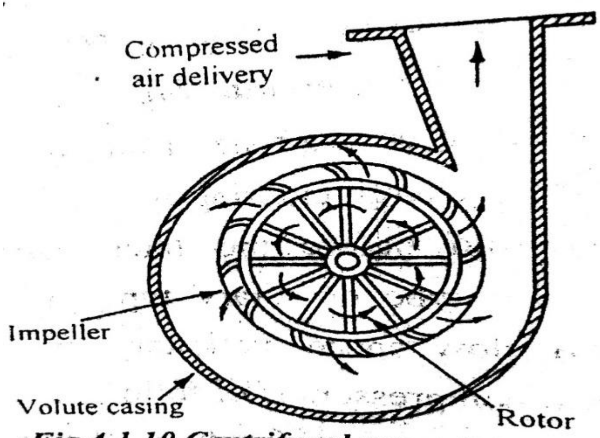


*Fig 4.1.10 Centrifugal superchargers*

**Application of supercharging**

- It is used in aircrafts and stationary installations in mountains to maintain the power output at higher attitudes.
- It is used in locomotives and marine engines to reduce the bulk of the engines to fit into a limited space.
- It is used to increase the power of an existing engine when a greater power is needed.
- It is used in racing car engines and aircrafts because of reduced weight of the engine per power output.

**WORKING PRINCIPLE OF A TURBOCHARGER**

- A turbocharger is a small radial fan pump driven by the energy of the exhaust gases of an engine.
- A turbocharger consists of a turbine and a compressor on a shared shaft.
- The turbine converts exhaust to rotational force, which is in turn used to drive the compressor.
- The compressor draws in ambient air and pumps it in to the intake manifold at increased pressure, resulting in a greater mass of air entering the cylinders on each intake stroke.

**Components of turbocharger**

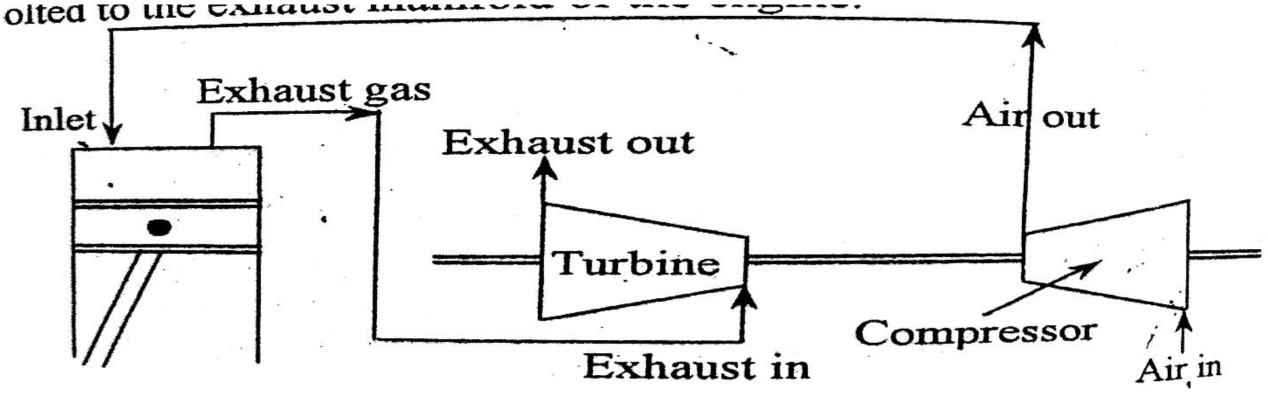
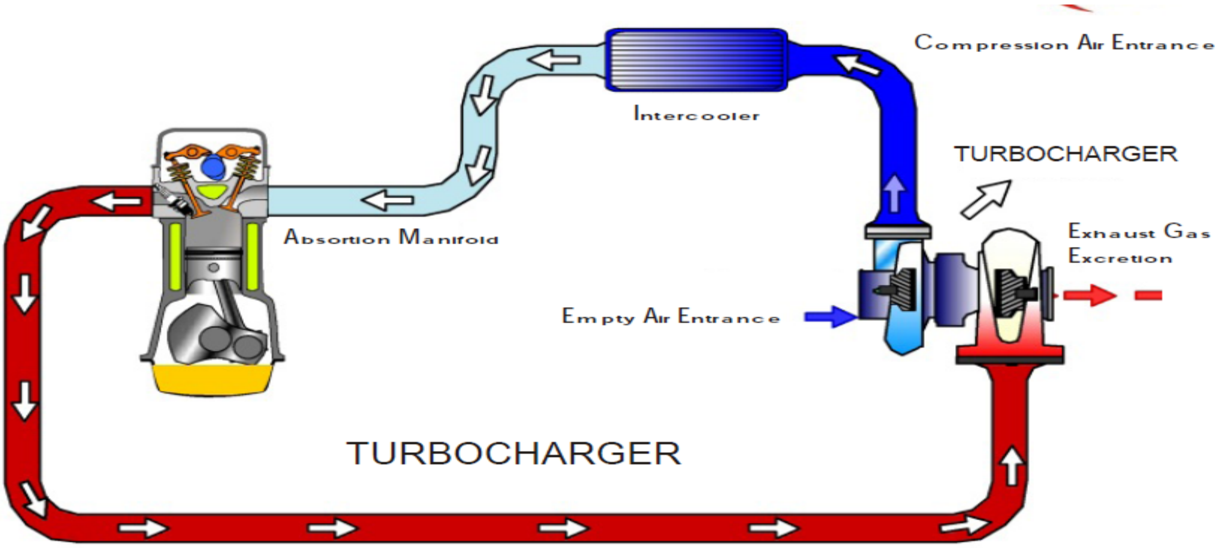


*Fig 4.1.11 Turbocharger*



**Application of turbo charger**

- Turbo charger is very common in diesel engines, both conventional automobile and also for trucks, marine etc.
- It is most commonly used on petrol engine in high performance automobiles, particularly in race cars which has less space for fitting engine.
- Turbo chargers are suitable for engines running at high attitudes where air is less dense.

## ENGINE EMISSION

- IC engine fuels contain hydrogen and carbon in various combinations.
- During combustion, oxygen combines with hydrogen and carbon to form water ($H_2O$), carbon monoxide (CO) and carbon dioxide ($CO_2$).
- The nitrogen in the fuel combines with oxygen and forms nitrogen oxides ($NO_2$).
- Remaining fuel goes unburnt resulting in smoke and ash.
- Exhaust gas constituents consist of partly burned petrol, carbon monoxide, nitrogen oxides and if sulphur in petrol, sulphur oxides, pollute the air.
- There are two types of emissions will be in IC engine. (1) Diesel engine emission. (2) SI engine emission.

### Diesel engine emission

- Well maintained diesel engine will emits negligible amount of carbon monoxide and hydro carbons through considerable amount of nitrogen oxides are emitted.
- Diesel smoke is another pollutant in case of diesel engines.
- Two types (1) white smoke (2) black smoke.
- The white smoke normally arises due to, too operating temperature and too long delay between the start of fuel injection and beginning of combustion.
- The white smoke appears during starting and warming up.
- The black smoke appears after the engine has fully warmed up and accelerating or pulling under load.
- The black smoke results incomplete combustion of fuel.
- Blue smoke occurs due to excessive lubricating oil consumption.
- Its emission indicates very poor condition of the engine.
- The blue smoke is not considered as a serious problem.

### SI engine emission

There are three main sources of air pollution due to petrol engine.

1. Evaporative emission.
2. Crankcase blow by.
3. Exhaust emission.

### Evaporative emission.

- Evaporative emission takes place from the fuel supply system.
- The main reason is hydrocarbon evaporation at high temperature.
- About 30% of total hydrocarbon emission occurring from the fuel tank, fuel line and carburator.

**Crankcase blow by.**

- Crank case blow by means of the leakage past the piston and piston rings from the cylinder to the crankcase.

**Exhaust emission.**

- During combustion, oxygen combines with hydrogen and carbon to form water ($H_2O$), carbon monoxide (CO) and carbon dioxide ($CO_2$).
- The nitrogen in the fuel combines with oxygen and forms nitrogen oxides ($NO_2$).
- Remaining fuel goes unburnt resulting in smoke and ash.
- Exhaust gas constituents consist of partly burned petrol, carbon monoxide, nitrogen oxides and if sulphur in petrol, sulphur oxides, pollute the air.

## AIR POLLUTION AND POLLUTION CONTROL

### 1. Hydrocarbons.

- Hydro carbon is produced due to incomplete combustion.
- Hydrocarbon produces smog. This affects vision. Smog is the mixture of fog and smoke.

Methods for Control of hydro carbon

- Reducing the compression ratio.
- Changing the design of combustion chamber.
- Changing the design of piston.
- By supplying lean mixture.
- By maintenance of piston and piston rings.

Methods for Destroying hydro carbons

- By supplying air to the inlet manifold.
- By using after burner.
- By using catalytic convertor.

### 2. Carbon monoxide

- it is produced because of insufficient supply of air for combustion.

- CO has more affinity than oxygen for haemoglobin in our blood.
- It reduces the ability of haemoglobin to carry oxygen to body tissues.
- Hence it will affects the nervous system, vision and finally affects hearts .

Methods for reducing CO

- By using closed loop control.
- By supplying lean mixture.
- Providing suitable overlap of valves.

Methods for destroying CO

- By using reactor in exhaust manifold.
- By using after burner.
- By using catalytic convertor.

**3. Oxides of nitrogen**

- In high temperature, nitrogen react with the oxygen and produces nitric oxide and nitrogen oxide.
- They affect living organisms. They affect blood purification system.
- It may be mixed with moisture and produces dilute nitric acid in the heart and affects hearts.

Control of oxides of nitrogen.

- By supplying exhaust again to the inlet manifold.
- By spraying water in the inlet manifold to add moisture to the mixture.
- By using catalytic convertor in the exhaust, the oxides of nitrogen can be destroyed.

**4. Photo chemical smog**

- Some hydro carbons and oxides of nitrogen in the exhaust react with the atmospheric air in the presence of sun light and produces photochemical smog.
- It damages the plant's life. It reduces visibility.
- It produces irritation and affects the respiratory system of human beings.

**5. Smoke**

- Smoke is produced because of insufficient mixing of fuel and air.it contains CO and CO2
- When cold starting, blue white smoke is produced when more carbon particles are mixed with exhaust.

- Smog is produced by means of smoke.
- it causes irritation of eyes,coughing,headache and vomiting.

Control of smoke and smog

- Running the engine with limited load.
- Maintaining the engine well.
- By adding barium salt in the fuel.
- By using catalytic muffler.

**6. Lead**

- Lead is poisonous. It is a toxic air pollutant.
- It is produced from the combustion of gasoline.it affects liver and kidneys. It causes mental effects to children.

**7. Particulate**

- Particulates are minute separate particles found in the air. They may solid or liquid particles.
- The dust soot and  fly ash are included in it.
- It cause respiratory diseases like bronchitis and lung cancer and allergic diseases.

**8. Sulphur oxide**

- Sulphur oxide is produced if the fuel has sulphur. It may damage the plants.
- It causes irritation to eye and throat and gives respiratory troubles to children. It corrodes materials.

**EXHAUST GAS ANALYSIS**

1. Exhaust gas recirculation system.(EGR)
2. Treating the exhaust gas.
    1. Air Injection System.
    2. Catalytic Convertor.

## 2.26. EXHAUST GAS ANAYLSIS

### 2.26.1. Exhaust Gas Recirculation (EGR) system

Excessive nitrogen oxides ($NO_2$) form when peak combustion temperature exceeds 1950°C. To lower the combustion temperature, many engines have EGR system. It recirculates about 10% of the inert gas back into the intake manifold.The cooler exhaust gas absorbs heat from the much hotter combustion process. It reduces peak combustion temperature and lowers the formation of $NO_2$. The EGR system provides a passage between exhaust manifold and inlet manifold. An EGR valve provided on this line opens and closes the passage. Fig.2.83. shows a simplified diagram of a conventional EGR valve.

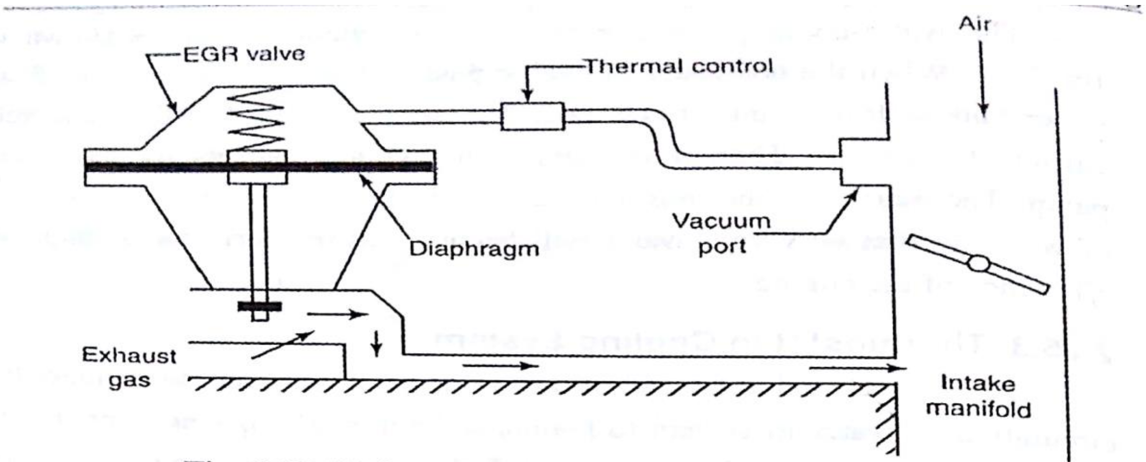


*Fig. 2.83. Exhaust gas recirculation (EGR) system*

It consists of a spring-loaded vacuum diaphragm linked to a tapered valve. A vacuum chamber is there at the top of the valve. This valve controls the passage for the exhaust gas. The chamber is connected by a tube to a vacuum port in the throttle body. When there is no vacuum at this port, the spring will push the diaphragm down and keep the passage closed. Therefore, no exhaust gas recirculates. It happens during idle, when $NO_2$ formation is at a minimum. Also EGR could stall an idling engine.

When the throttle is opened from the idle position, vacuum applied will gradually open the tapered valve. It causes the exhaust gas to flow into the intake manifold. At wide-open throttle, the intake manifold vacuum is low and the EGR valve is closed by the spring. Thus, EGR valve systems do not affect full power operation. It is thus seen that exhaust gas is recirculated only in this system when the engine operating conditions are from $NO_2$.

Many engines have thermal vacuum switch. It prevents EGR until engine temperature reaches 38°C. This switch is mounted in the engine water jacket where it consciences coolant temperature. The switch closes when the engine is cold. It prevents EGR just after a cold engine starts. After the engine warms up, the switch is opened.

### 2.26.2. Treating the exhaust gas

Treating the exhaust gas means some cleaning or reducing the percentage of pollutants in it. It takes place after leaving the exhaust gas from the engine cylinders and before it exits the tailpipe and enters the atmosphere. It reduces the amount of HC, CO and $NO_2$ in the exhaust gas. The exhaust gas is treated in two ways. One is by injecting fresh air into the exhaust system. The other is by sending the exhaust gas through catalytic converter.

***(1) Air Injection System:***

The main parts of an air-injection system are the air pump, one way check valve and piping into the exhaust manifold. The air pump is driven by a belt from the crank shaft pulley. When the engine is cold, air is supplied to exhaust manifold. This fresh air supplies excess oxygen to oxide of HC and CO into $CO_2$ and water vapour.

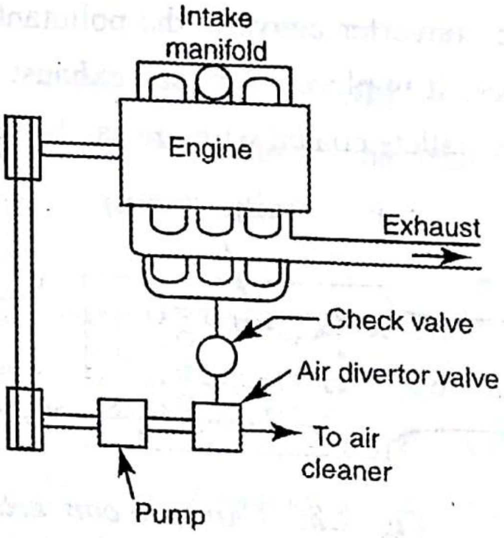


*Fig. 2.84. Air injection system*

The diverter valve prevents back-firing in the exhaust system. The diverter valve is operated by a vacuum diaphragm. When the fuel injection system is in idling or in part or full throttle conditions, the manifold vacuum is not enough to move the diverter valve diaphragm. But, on sudden decelerations manifold vacuum will increase. This will lift the diaphragm and send the air to the atmosphere or air cleaner.

High manifold vacuum draws rich mixture from the carburetor. It is too difficult to ignite in the combustion chamber due to which reasons, it goes out in the exhaust manifold.

When air is pumped into exhaust manifold, the mixture becomes lean. It would ignite when hot exhaust gases are released from the next cylinder. This may result in back firing. To prevent this, the diverter valve diverts the fresh air from exhaust manifold during deceleration. Check valve is meant to prevent back flow whenever exhaust pressure is greater than air pump pressure.

***(2) Catalytic converter:***

The catalytic converter converts the pollutants like HC, CO and $NO_2$ into harmless gases. It is placed between exhaust manifold and silencer. It contains the plastic pallets coated with the catalyst.

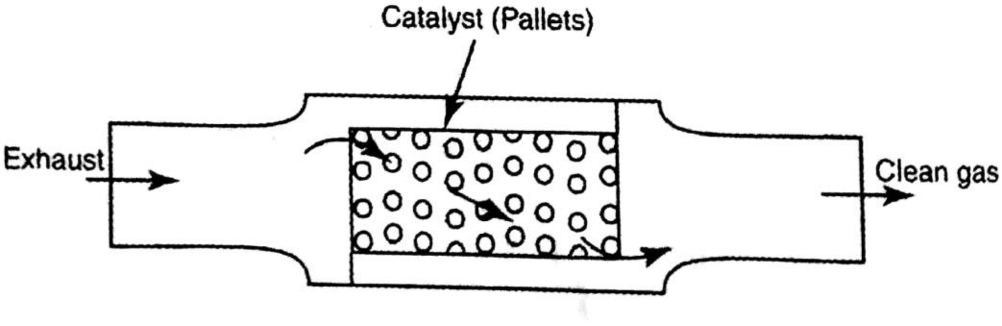
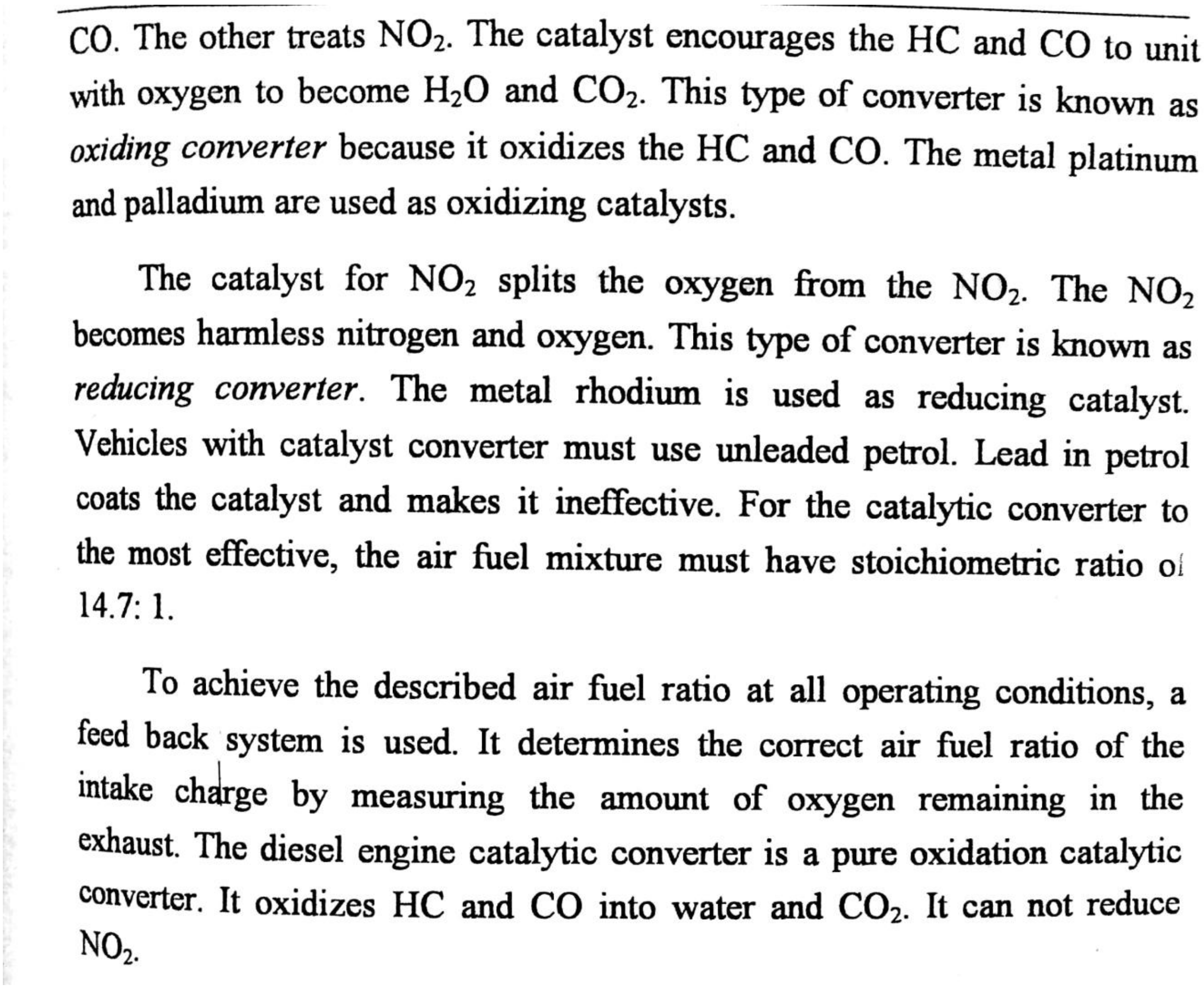


***Fig. 2.85. Catalytic converter***

All exhaust gas must flow through it. The catalyst causes a chemical change without being a part of the chemical reaction. The catalytic converter may have two different catalysts. One catalyst treats the HC and

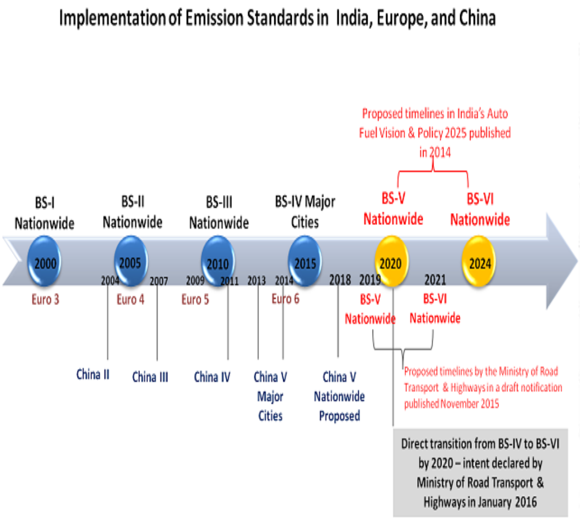


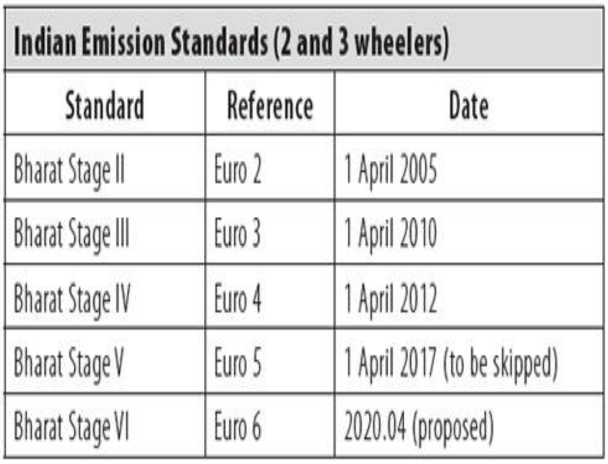
CO. The other treats $NO_2$. The catalyst encourages the HC and CO to unit with oxygen to become $H_2O$ and $CO_2$. This type of converter is known as *oxiding converter* because it oxidizes the HC and CO. The metal platinum and palladium are used as oxidizing catalysts.

The catalyst for $NO_2$ splits the oxygen from the $NO_2$. The $NO_2$ becomes harmless nitrogen and oxygen. This type of converter is known as *reducing converter*. The metal rhodium is used as reducing catalyst. Vehicles with catalyst converter must use unleaded petrol. Lead in petrol coats the catalyst and makes it ineffective. For the catalytic converter to the most effective, the air fuel mixture must have stoichiometric ratio of 14.7: 1.

To achieve the described air fuel ratio at all operating conditions, a feed back system is used. It determines the correct air fuel ratio of the intake charge by measuring the amount of oxygen remaining in the exhaust. The diesel engine catalytic converter is a pure oxidation catalytic converter. It oxidizes HC and CO into water and $CO_2$. It can not reduce $NO_2$.

## EMISSION STANDARDS





**Indian Emission Standards (2 and 3 wheelers)**

| Standard | Reference | Date |
|---|---|---|
| Bharat Stage II | Euro 2 | 1 April 2005 |
| Bharat Stage III | Euro 3 | 1 April 2010 |
| Bharat Stage IV | Euro 4 | 1 April 2012 |
| Bharat Stage V | Euro 5 | 1 April 2017 (to be skipped) |
| Bharat Stage VI | Euro 6 | 2020.04 (proposed) |

*Source: Auto Fuel Vison & Policy 2025*

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# 1

# COMPRESSIBLE FLOW – FUNDAMENTALS

## 1.1. INTRODUCTION

A fluid is defined as a substance which continuously deforms und
the action of shearing forces. Liquids and gases are termed as fluids.

The study of fluids at rest is known as **fluid statics.**

The study of fluids in motion where pressure forces are n
considered, is known as **fluid kinematics.**

The study of fluids in motion where pressure forces are al
considered, is known as **fluid dynamics.**

## 1.2. GAS DYNAMICS

Gas dynamics deals with the study of motion of gases and its effec
It differs from fluid dynamics. Gas dynamics considers thermal or chemi
effects while fluid dynamics these effects are generally excluded.

## 1.3. TYPES OF FLUID FLOW

The fluid flow is classified as follows

1. Steady and Unsteady flows
2. Uniform and Nonuniform flows
3. Laminar and Turbulent flows
4. Compressible and Incompressible flows
5. Rotational and Irrotational flows
6. One dimensional flow, Two dimensional flow and Th
   dimensional flow.

### 1. Steady and Unsteady flows

Steady flow is that type of flow, in which the fluid characterisitcs like velocity, pressure and density at a point do not change with time.

Unsteady flow is that type of flow, in which the fluid characteristics like velocity, pressure and density at a point changes with respect to time.

### 2. Uniform and Non–uniform flows

Uniform flow is that type of flow, in which the velocity of fluid particles at all sections are equal.

Non–uniform flow is that type of flow, in which the velocity of fluid particles at all sections are not equal.

### 3. Laminar and Turbulant flows

Laminar flow is sometimes called stream line flow. In this type of flow, the fluid moves in layers and each fluid particle follows a smooth and continuous path.

In turbulant flow, the fluid particles move in very irregular paths.

### 4. Compressible and incompressible flows

Compressible flow is that type of flow in which the density of the fluid changes from point to point, i.e. density is not constant for the fluid.

Density, $\rho \neq$ constant

*Examples :* Gases, vapours

Incompressible flow is that type of flow in which the density of the fluid is constant

Density, $\rho =$ constant

*Example* : Liquids

### 5. Rotational and irrotational flows

Rotational flow is that type of flow in which the fluid particles flowing along stream lines and also rotate about their own axis.

Irrotational flow is that type of flow in which the fluid particles flowing along stream lines but do not rotate about their own axis.

### 6. One dimensional, Two dimensional and Three dimensional flows.

One dimensional flow is that type of flow in which the flow parameter such as velocity is a function of time and one space co–ordinate ($x$) only. In this type of flow the stream lines may be represented by straight lines.

Two dimensional flow is that type of flow in which the flow parameter such as velocity is a function of time and two space co–ordinates ($x,y$) only. In this type of flow the stream lines may be represented by a curve.

The flow of liquid whose stream lines may be represented in space along three mutually perpendicular axis ($x$, $y$ *and* $z$) is called three dimensional flow.

## 1.4 STEADY FLOW ENERGY EQUATION

From first law of Thermodynamics, we know that the total energy entering the system is equal to total energy leaving the system. This law is applicable to the steady flow systems.

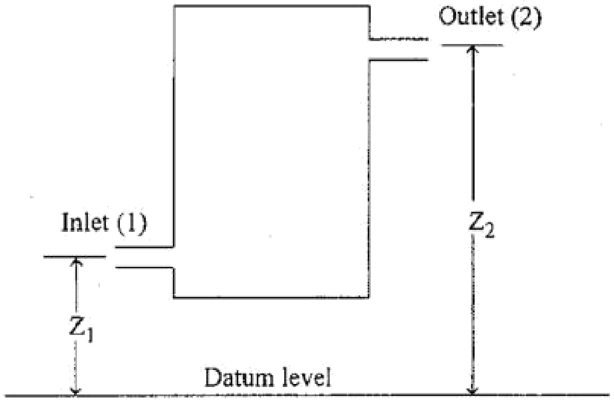


*Fig 1.1*

Consider an open system through which the working substance flows as a steady rate as shown in fig.1.1. The working substance entering the system at (1) and leaves the system at (2).

Let,

$p_1$ – Pressure of the working substance entering the system ($N/m^2$)

$v_1$ – Specific volume of the working substance entering the system ($m^3/kg$)

$c_1$ – Velocity of the working substance entering the system (m/s)

$U_1$ – Specific internal energy of the working substance entering the system (J/kg)

$Z_1$ – Height above the datum level for inlet in (m).

$p_2$, $v_2$, $c_2$, $U_2$ and $Z_2$ – Corresponding values for the working substance leaving the system.

Q – Heat supplied to the system (J/kg)

W – Work delivered by the system (J/kg).

Total energy entering the system

= Potential energy ($gZ_1$)

\+

Kinetic energy $\left(\frac{c_1^2}{2}\right)$

\+

Internal energy ($U_1$)

\+

Flow energy ($p_1 v_1$)

\+

Heat (Q)

Total energy leaving the system

= Potential energy ($gZ_2$)

\+

Kinetic energy $\left(\frac{c_2^2}{2}\right)$

\+

Internal energy ($U_2$)

\+

Flow energy ($p_2 v_2$)

\+

Work (W)

From first law of Thermodynamics,

Total energy entering the system = Total energy leaving the systei

$$\Rightarrow gZ_1 + \left(\frac{c_1^2}{2}\right) + U_1 + p_1 v_1 + Q$$

$$= gZ_2 + \left(\frac{c_2^2}{2}\right) + U_2 + p_2 v_2 + W$$

$$\boxed{\Rightarrow gZ_1 + \frac{c_1^2}{2} + h_1 + Q = gZ_2 + \frac{c_2^2}{2} + h_2 + W} \quad \text{----(1.}$$

$[\because h = U + p$

The above equation is known as steady flow energy equation.

## 1.5 STEADY FLOW ENERGY EQUATION FOR TURBO MACHINES

Most of the compressible flow turbomachines such as turbines a compressors are classified as adiabatic machines. In these machines th is no heat transfer takes place and change in potential energy ($Z_1 - Z_2$ also negligible.

So, apply $Q = 0$,

$Z_1 - Z_2 = 0$ in Equation (1.1).

$$(1.1) \Rightarrow \boxed{\frac{c_1^2}{2} + h_1 = \frac{c_2^2}{2} + h_2 + W} \qquad ----(1.2)$$

## 1.6 STEADY FLOW ENERGY EQUATION FOR NOZZLE AND DIFFUSER

Nozzle is a device which increases the velocity and decreases the pressure of working substance.

Diffuser is a device which increases the pressure and decreases the velocity of the working substance.

In these systems

1. There is no work is done by the system i.e. W = 0
2. There is no heat transfer takes place

   i.e. Q = 0
3. Change in potential energy is negligible i.e. $gZ_1 = gZ_2$

Apply these conditions in Equation (1.1).

$$(1.1) \Rightarrow \boxed{\frac{c_1^2}{2} + h_1 = \frac{c_2^2}{2} + h_2} \qquad ----(1.3)$$

## 1.7 VELOCITY OF SOUND (a)

The velocity with which sound waves propagate in a medium is called velocity of sound (a).

Sound waves are generated due to infinitesimally small pressure disturbances.

The velocity of sound is given by

$a = \sqrt{\gamma R T}$

## 1.8 DERIVATION OF ACCOUSTIC VELOCITY (or) SOUND VELOCITY (*a*)

Sound waves are infinitely small pressure disturbances. The speed with which sound propagates in a medium is called speed of sound and is denoted by '*a*'.

If an infinitesimal disturbance is created by the piston, as shown in fig 1.2, the wave propagates through the gas at the velocity of sound relative to the gas into which the disturbance is moving.

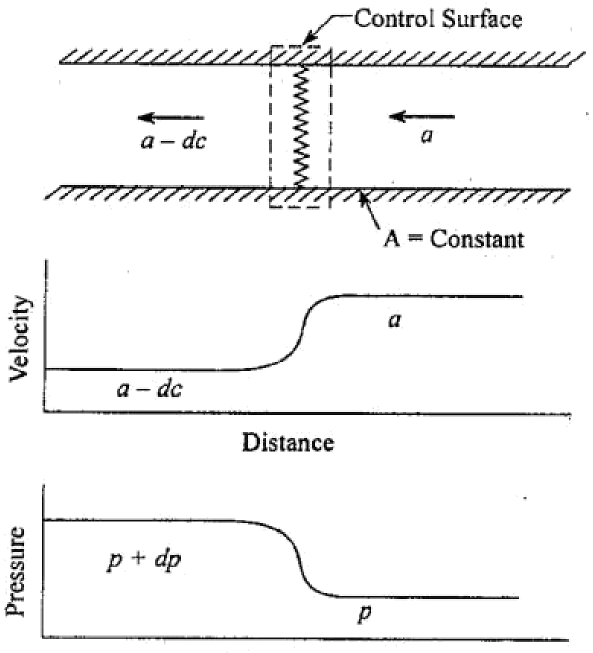


*Fig. 1.2 Propagation of an infinitesimal wave in a constant area duct*

In the case the stagnant gas at pressure 'p' on the right appears to flow towards the left with a velocity *a*. When the flow has passed through the wave to the left its pressure is raised to p + dp and the velocity is lowered to *a* – dc.

Apply momentum equation for this process.

$$A[p - (p+dp)] = m\,[(a-dc)-a]$$

$$\Rightarrow A[p - p - dp] = \rho A a\,[a-dc-a] \qquad [\because m = \rho A c,\ \text{Here } c = a \Rightarrow m = \rho A a]$$

$$\Rightarrow -Adp = \rho A a\,[-dc]$$

$$\Rightarrow \boxed{dp = \rho\, a\, dc} \qquad ----(A)$$

From continuity equation for the two sides of the wave

$$m = \rho A a = (\rho + d\rho)(a-dc)A$$

$$\Rightarrow \rho A a = A\,[a\rho - \rho dc + a d\rho - d\rho dc]$$

$$\Rightarrow \rho a = [\rho a - \rho dc + a d\rho - d\rho dc]$$

The term $d\rho dc$ is negligible

$$\Rightarrow \rho a = \rho a - \rho dc + a d\rho$$

$$\Rightarrow \boxed{\rho dc = a d\rho} \qquad ----(B)$$

Substituting Equation (B) in Equation (A)

$$(A) \Rightarrow dp = (a d\rho) \times a$$

$$= a^2 d\rho$$

$$\frac{dp}{d\rho} = a^2$$

$$\Rightarrow \sqrt{\frac{dp}{d\rho}} = a$$

For Isentropic flow

$$\frac{dp}{d\rho} = \gamma RT$$

$$\Rightarrow a = \sqrt{\frac{dp}{d\rho}} = \sqrt{\gamma RT}$$

Sound velocity (or) accoustic velocity (a) = $\sqrt{\gamma RT}$

## 1.9 MACH NUMBER

The Mach number is an index of the ratio between inertia force and elastic force

$$M^2 = \frac{\text{Inertia force}}{\text{Elastic force}}$$

$$= \frac{\rho A c^2}{KA}$$

$$M^2 = \frac{\rho c^2}{K}$$

where

ρ = Density of the fluid

c – Velocity of the fluid

A – Flow area

K – Bulk Modulus

$$\Rightarrow M^2 = \frac{\rho c^2}{K}$$

$$= \frac{\rho c^2}{\rho a^2} \qquad [\because K = \rho a^2]$$

$$M^2 = \frac{c^2}{a^2}$$

$$\boxed{M = \frac{c}{a}}$$

This relation gives another important definition of Mach numbe i.e., the Mach number is defined as the ratio of the fluid velocity (c) to th velocity of sound (a).

We know that

Velocity of sound, a = $\sqrt{\gamma RT}$

$$\Rightarrow \boxed{M = \frac{c}{a} = \frac{c}{\sqrt{\gamma RT}}} \qquad ---- (1.4)$$

## 1.10 STAGNATION STATE

Stagnation state is obtained by decelerating a gas isentropically to zero velocity at zero elevation. The stagnation state of a gas is often used as a reference state.

## 1.11 STAGNATION ENTHALPY ($h_0$)

Stagnation enthalpy can be defined as the enthalpy of a gas when it is isentropically decelerated to zero velocity at zero elevation.

Put

$h_1 = h \quad c_1 = c$ for the initial state,

$h_2 = h_0 \quad c_2 = 0$ for the final state, in Equation (1.3).

$$(1.3) \Rightarrow \frac{c^2}{2} + h = h_0$$

$$\Rightarrow \boxed{h_0 = h + \frac{c^2}{2}} \qquad ----(1.5)$$

where

$h_0$ – Stagnation enthalpy

$h$ – Static enthalpy

c – Fluid velocity

## 1.12 STAGNATION TEMPERATURE ($T_0$)

Stagnation temperature is the temperature of the gas when it is isentropically decelerated to zero velocity at zero elevation.

We know that

Stagnation enthalpy

$$h_0 = h + \frac{c^2}{2}$$

$$\Rightarrow c_pT_0 = c_pT + \frac{c^2}{2} \qquad [\because h = c_pT, \; h_0 = c_pT_0]$$

Divided by $c_p$

$$\Rightarrow T_0 = T + \frac{c^2}{2c_p}$$

$$\boxed{\text{Stagnation Temperature, } T_0 = T + \frac{c^2}{2c_p}} \qquad ----(1.6)$$

Divided by T

$$\Rightarrow \frac{T_0}{T} = 1 + \frac{c^2}{2c_pT}$$

$$\Rightarrow \frac{T_0}{T} = 1 + \frac{c^2}{2 \times \frac{\gamma R}{\gamma-1} \times T} \qquad [\because c_p = \frac{\gamma R}{\gamma-1}]$$

$$\Rightarrow \frac{T_0}{T} = 1 + \frac{c^2}{2 \times \frac{a^2}{\gamma-1}} \qquad [\because a = \sqrt{\gamma RT}]$$

$$\Rightarrow \frac{T_0}{T} = 1 + \frac{\gamma-1}{2} \times \frac{c^2}{a^2}$$

$$\Rightarrow \frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2 \qquad [\because \text{Mach Number } M = {}^{c}/_{a}]$$

$$\boxed{\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2} \qquad ----(1.7)$$

where,

$T_0$ – Stagnation temperature

T – Static temperature

M – Mach Number

## 1.13 STAGNATION PRESSURE ($p_0$)

Stagnation pressure is the pressure of the gas when it is isentropically decelerated to zero velocity at zero elevation.

For isentropic flow

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/\gamma-1}$$

For stagnation condition,

Put

$$p_2 = p_0 \qquad T_2 = T_0$$

$$p_1 = p \qquad T_1 = T$$

$$\Rightarrow \frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\gamma/\gamma-1}$$

$$\boxed{\frac{p_0}{p} = \left[1 + \frac{\gamma-1}{2}M^2\right]^{\gamma/\gamma-1}} \qquad ----(1.8)$$

$$\left[\because \frac{T_0}{T} = 1 + \frac{\gamma-1}{2}M^2\right]$$

Where

$p_0$ – Stagnation pressure

p – Static pressure

M – Mach Number

## 1.14 STAGNATION DENSITY ($\rho_0$)

Stagnation density is the density of the gas when it is isentropically decelerated to zero velocity at zero elevation

For isentropic flow

$$\frac{\rho_2}{\rho_1} = \left(\frac{p_2}{p_1}\right)^{1/\gamma}$$

For stagnation condition,

Put

$$\rho_2 = \rho_0 \qquad p_2 = p_0$$

$$\rho_1 = \rho \qquad p_1 = p$$

$$\Rightarrow \frac{\rho_0}{\rho} = \left(\frac{p_0}{p}\right)^{1/\gamma}$$

$$\frac{\rho_0}{\rho} = \left[1 + \frac{\gamma-1}{2}M^2\right]^{\gamma/\gamma-1 \times 1/\gamma}$$

$$\left[\because \frac{p_0}{p} = \left[1 + \frac{\gamma-1}{2}M^2\right]^{\gamma/\gamma-1}\right]$$

$$\Rightarrow \boxed{\frac{\rho_0}{\rho} = \left[1 + \frac{\gamma-1}{2}M^2\right]^{\frac{1}{\gamma-1}}} \qquad ----(1.9)$$

Where

$\rho_0$ – Stagnation density

$\rho$ – Static density

## 1.15 STAGNATION VELOCITY OF SOUND ($a_0$)

Velocity of sound, $a = \sqrt{\gamma RT}$

For stagnation condition,

Put $a = a_0$

$T = T_0$

$$\Rightarrow a_0 = \sqrt{\gamma RT_0}$$

Gas constant, $R = \frac{\gamma-1}{\gamma} \times c_p$

$$\Rightarrow a_0 = \sqrt{\gamma \times \frac{\gamma-1}{\gamma} \times c_p \times T_0}$$

$$\Rightarrow a_0 = \sqrt{(\gamma-1)\, c_p\, T_0}$$

$$\boxed{a_0 = \sqrt{(\gamma-1)\, h_0}} \qquad ----(1.10)$$

$[\because h_0 = c_p\, T_0]$

where,

$a_0$ - Stagnation velocity of sound

$h_0$ - Stagnation enthalpy.

## 1.16 VARIOUS REGIONS OF FLOW

The adiabatic energy equation for a perfect gas is derived in terms of velocity of fluid (c) and velocity of sound (a). This is then discussed graphically by differentiating various regions of flow on the c – a plot.

We know that

Stagnation enthalpy, $h_0 = h + {}^1/_2\, c^2$ ----(1.11)

Static enthalpy, $h = c_p T$

$$= \frac{\gamma R}{\gamma-1} \times T \qquad [\because c_p = \frac{\gamma R}{\gamma - 1}]$$

$$\Rightarrow \boxed{h = \frac{\gamma RT}{\gamma-1}} \qquad ----(1.12)$$

Velocity of sound, $a = \sqrt{\gamma RT}$

$$\boxed{a^2 = \gamma RT}$$

Substitute $\gamma RT$ value in Equation (1.12)

$$(1.12) \Rightarrow \boxed{h = \frac{a^2}{\gamma-1}}$$

Substitute $h$ value in stagnation enthalpy equation

$$(1.11) \Rightarrow \boxed{h_0 = \frac{a^2}{\gamma-1} + {}^1/_2\, c^2} \qquad ----(1.13)$$

***Case (i)***

At T = 0, $h$ = 0 and c = $c_{max}$ substitute these values in stagnation enthalpy equation.

$$(1.11) \Rightarrow \boxed{h_0 = {}^1/_2\, c_{max}^2} \qquad ----(1.14)$$

***Case (ii)***

At c = 0, a = $a_0$ substitute these values in stagnation enthalpy equation

$$(1.11) \Rightarrow h_0 = h$$

$$= \frac{a^2}{\gamma-1} \qquad \left[\because h = \frac{a^2}{\gamma-1}\right]$$

$$= \frac{a_0^2}{\gamma-1} \qquad [\because a = a_0]$$

$$\boxed{h_0 = \frac{a_0^2}{\gamma-1}} \qquad ----(1.15)$$

From Equation (1.13), (1.14) and (1.15)

We know that

$$\boxed{h_0 = \frac{a^2}{\gamma-1} + {}^1/_2\, c^2 = {}^1/_2\, c_{max}^2 = \frac{a_0^2}{\gamma-1}} \qquad ----(1.16)$$

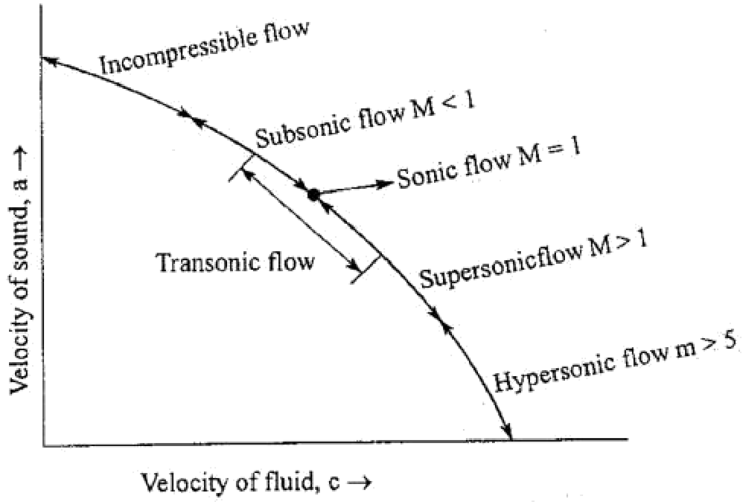


*Fig. 1.3 c – a curve*

There are six different regions on this curve.

*1. Incompressible flow region*

In incompressible flow region, fluid velocity (c) is much smaller than the sound velocity (a). Therefore the Mach number ($M = c/a$) is very very low.

*2. Subsonic flow region*

The subsonic flow region is on the right of the incompressible flow region. In subsonic flow, fluid velocity (c) is less than the sound velocity (a) and the Mach number in this region is always less than unity. i.e. $M = c/a < 1$.

*3. Sonic flow region*

If the fluid velocity (c) is equal to the sound velocity (a), that type of flow is known as sonic flow. In sonic flow Mach number value is unity. $M = c/a = 1 \Rightarrow c = a$.

*4. Transonic flow region*

If the fluid velocity close to the speed of sound, that type of flow is known as transonic flow. In transonic flow, Mach number value is in between 0.8 and 1.2. i.e., $0.8 < M < 1.2$.

*5. Supersonic flow region*

The supersonic region is on the right of the transonic flow region. In supersonic flow, fluid velocity (c) is more than the sound velocity (a) and the Mach number in this region is always greater than unity.

i.e. $M = c/a > 1$.

*6. Hypersonic flow region*

In hypersonic flow region, fluid velocity (c) is much greater than sound velocity (a). In this flow, Mach number value is always greater than 5.

i.e. $M = c/a > 5$.

## 1.17 REFERENCE VELOCITIES

In compressible fluid flow analysis, the following velocities are used.

*1. Local velocity of sound (a)*

$$\text{Mach Number, } M = \frac{\text{Fluid velocity (c)}}{\text{Sound velocity (a)}}$$

$$\Rightarrow \quad M = c/a$$

$$\Rightarrow \quad \boxed{a = c/M}$$

*2. Stagnation velocity of sound ($a_0$)*

Velocity of sound, $a = \sqrt{\gamma RT}$

For stagnation condition,

Put $a = a_0 : T = T_0$

$$\Rightarrow \boxed{a_0 = \sqrt{\gamma R T_0}}$$

### 3. Maximum velocity of fluid, $c_{max}$

From equation (1.16),

We know that

$$\frac{a_0^2}{\gamma - 1} = {}^1/_2\, c_{max}^2$$

$$\Rightarrow \quad \frac{2}{\gamma - 1} = \frac{c_{max}^2}{a_0^2}$$

$$\boxed{\frac{c_{max}}{a_0} = \sqrt{\frac{2}{\gamma - 1}}} \qquad ----(1.17)$$

### 4. Critical velocity of fluid (or) Critical velocity of sound $c^* = a^*$.

At critical state, Mach number value is unity. i.e., $M_{critical} = M^* = 1$. The velocity at critical state is known as critical velocity

$$M_{critical} = \frac{c^*}{a^*} = 1$$

$$\Rightarrow \boxed{c^* = a^* = \sqrt{\gamma R T^*}}$$

Stagnation enthalpy, $h_0 = h + \frac{c^2}{2}$ [From equation (1.5)]

At critical state,

$c = c^* ; \quad h = h^*$

$$\Rightarrow h_0 = h^* + \frac{c^{*2}}{2}$$

Stagnation Temperature, $T_0 = T + \frac{c^2}{2c_p}$ [From equation (1.6)]

At critical state

$T = T^* ; \; c = c^*$

$$\Rightarrow T_0 = T^* + \frac{c^{*2}}{2c_p}$$

$$\Rightarrow T_0 - T^* = \frac{c^{*2}}{2c_p}$$

$$\Rightarrow c^{*2} = 2c_p\,[T_0 - T^*]$$

$$\Rightarrow c^* = \sqrt{2c_p\,[T_0 - T^*]}$$

$$\Rightarrow \boxed{c^* = a^* = \sqrt{2c_p\,[T_0 - T^*]}} \qquad ----(1.18)$$

Stagnation Temperature – Mach number relation

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 \qquad \text{[From Equation no.(1.7)]}$$

At critical state

$M = M^* = 1 ; \quad T = T^*$

$$\Rightarrow \frac{T_0}{T^*} = 1 + \frac{\gamma - 1}{2}$$

$$\frac{T_0}{T^*} = \frac{2 + \gamma - 1}{2}$$

$$\frac{T_0}{T^*} = \frac{\gamma + 1}{2}$$

$$\boxed{T^* = \frac{2T_0}{\gamma + 1}} \qquad \text{----(1.19)}$$

Substitute T* value in Equation (1.18)

$$c^* = \sqrt{2\, c_p \left(T_0 - \frac{2T_0}{\gamma + 1}\right)}$$

$$= \sqrt{2 \times \frac{\gamma R}{\gamma - 1}\left(T_0 - \frac{2T_0}{\gamma + 1}\right)} \qquad \left[\because c_p = \frac{\gamma R}{\gamma - 1}\right]$$

$$= \sqrt{2 \times \frac{\gamma R T_0}{\gamma - 1}\left(1 - \frac{2}{\gamma + 1}\right)}$$

$$= \sqrt{2 \times \frac{a_0^2}{\gamma - 1}\left(\frac{\gamma + 1 - 2}{\gamma + 1}\right)} \qquad [\because a_0 = \sqrt{\gamma R T_0}]$$

$$= \sqrt{2 \times \frac{a_0^2}{\gamma - 1} \times \frac{\gamma - 1}{(\gamma + 1)}}$$

$$c^* = \sqrt{\frac{2a_0^2}{\gamma + 1}}$$

$$c^* = a_0 \sqrt{\frac{2}{\gamma + 1}}$$

At critical state

$$c^* = a^*$$

$$\Rightarrow c^* = a^* = a_0 \sqrt{\frac{2}{\gamma + 1}}$$

$$\Rightarrow \boxed{\frac{c^*}{a_0} = \frac{a^*}{a_0} = \sqrt{\frac{2}{\gamma + 1}} = 0.913} \qquad \text{----(1.20)}$$

$$[\because \gamma = 1.4]$$

From Equation 1.19, we know that

$$T^* = \frac{2T_0}{\gamma + 1}$$

$$\boxed{\frac{T_0}{T^*} = \frac{\gamma + 1}{2}} \qquad \text{----(1.21)}$$

From equation (1.7), we know

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 \qquad \text{----(1.22)}$$

(1.21) ÷ (1.22)

$$\frac{\dfrac{T_0}{T^*}}{\dfrac{T_0}{T}} = \frac{\dfrac{\gamma + 1}{2}}{1 + \dfrac{\gamma - 1}{2} M^2}$$

$$\frac{T}{T^*} = \frac{\dfrac{\gamma + 1}{2}}{1 + \dfrac{\gamma - 1}{2} M^2}$$

$$\Rightarrow \frac{T^*}{T} = \frac{1 + \dfrac{\gamma - 1}{2} M^2}{\dfrac{\gamma + 1}{2}}$$

$$= \left[1 + \frac{\gamma - 1}{2} M^2\right] \times \frac{2}{\gamma + 1}$$

$$= \frac{2}{\gamma + 1} + \frac{\gamma - 1}{\gamma + 1} M^2$$

$$\boxed{\frac{T^*}{T} = \frac{2}{\gamma + 1} + \frac{\gamma - 1}{\gamma + 1} M^2} \qquad \text{----(1.23)}$$

From equation (1.17), we know that

$$\frac{c_{max}}{a_0} = \sqrt{\frac{2}{\gamma-1}}$$

From equation (1.20), we know that

$$\frac{c^*}{a_0} = \sqrt{\frac{2}{\gamma+1}}$$

$$\Rightarrow \frac{c_{max}}{c^*} = \frac{\frac{c_{max}}{a_0}}{\frac{c^*}{a_0}} = \frac{\sqrt{\frac{2}{\gamma-1}}}{\sqrt{\frac{2}{\gamma+1}}}$$

$$\frac{c_{max}}{c^*} = \sqrt{\frac{2}{\gamma-1}} \times \sqrt{\frac{\gamma+1}{2}}$$

$$\frac{c_{max}}{c^*} = \sqrt{\frac{2}{\gamma-1} \times \frac{\gamma+1}{2}}$$

$$\frac{c_{max}}{c^*} = \sqrt{\frac{\gamma+1}{\gamma-1}}$$

$$\frac{c_{max}}{c^*} = \frac{c_{max}}{a^*} = \sqrt{\frac{\gamma+1}{\gamma-1}} \qquad \text{----(1.24)}$$

$$\Rightarrow \frac{c_{max}}{c^*} = \sqrt{\frac{\gamma+1}{\gamma-1}}$$

$$\Rightarrow c_{max} = c^* \sqrt{\frac{\gamma+1}{\gamma-1}}$$

$$\boxed{c_{max}^2 = c^{*2} \left(\frac{\gamma+1}{\gamma-1}\right)}$$

From equation (1.16), we know that

Stagnation enthalpy, $h_0 = \frac{a^2}{\gamma-1} + \frac{1}{2} c^2 = \frac{1}{2} c_{max}^2 = \frac{a_0^2}{\gamma-1}$

Substitute $c_{max}^2$ value in stagnation enthalpy equation.

$$\boxed{\begin{aligned} \Rightarrow h_0 = \frac{a^2}{\gamma-1} + \frac{1}{2} c^2 = \frac{1}{2} c_{max}^2 = \frac{1}{2} c^{*2} \left(\frac{\gamma+1}{\gamma-1}\right) \\ = \frac{1}{2} a^{*2} \left(\frac{\gamma+1}{\gamma-1}\right) = \frac{a_0^2}{\gamma-1} \end{aligned}} \qquad \text{----(1.25)}$$

$$[\because c^* = a^*]$$

## 1.18 CHARACTERISTIC MACH NUMBER (M*) (OR) CRITICAL MACH NUMBER

It is defined as the ratio between fluid velocity (c) and critical velocity of sound (a*).

$$M^* = \frac{c}{a^*} \qquad [\because c^* = a^*]$$

$$= \frac{c}{c^*}$$

$$\Rightarrow \quad M^{*2} = \frac{c^2}{c^{*2}}$$

$$\Rightarrow \quad M^{*2} = \frac{c^2}{a^2} \times \frac{a^2}{c^{*2}}$$

$$[\because M = {}^{c}/_{a}]$$

$$M^{*2} = M^2 \times \frac{a^2}{c^{*2}}$$

$$\boxed{M^{*2} = M^2 \times \frac{a^2}{a^{*2}}} \qquad \text{----(1.26)}$$

$$[\because c^* = a^*]$$

From equation (1.25), we know that

$$\frac{a^2}{\gamma-1} + \frac{1}{2} c^2 = \frac{1}{2} a^{*2} \left(\frac{\gamma+1}{\gamma-1}\right)$$

Multiplying by 2

$$\frac{2a^2}{\gamma-1} + c^2 = a^{*2}\left(\frac{\gamma+1}{\gamma-1}\right)$$

Divided by $a^{*2}$

$$\frac{2}{\gamma-1}\,\frac{a^2}{a^{*2}} + \frac{c^2}{a^{*2}} = \left(\frac{\gamma+1}{\gamma-1}\right) \qquad ----(1.27)$$

From equation (1.26), we know that

$$M^{*2} = M^2 \times \frac{a^2}{a^{*2}}$$

$$\Rightarrow \boxed{\frac{a^2}{a^{*2}} = \frac{M^{*2}}{M^2}}$$

Substitute $\frac{a^2}{a^{*2}}$ value in equation (1.27)

$$(1.27) \Rightarrow \frac{2}{\gamma-1}\,\frac{M^{*2}}{M^2} + \frac{c^2}{a^{*2}} = \left(\frac{\gamma+1}{\gamma-1}\right)$$

$$\Rightarrow \frac{2}{\gamma-1}\,\frac{M^{*2}}{M^2} + M^{*2} = \left(\frac{\gamma+1}{\gamma-1}\right) \qquad [\because M^* = \frac{c}{a^*}]$$

Multiply by $(\gamma-1)$

$$\Rightarrow \quad 2 \times \left(\frac{M^{*2}}{M^2}\right) + M^{*2}(\gamma-1) = (\gamma+1)$$

$$\Rightarrow \quad M^{*2}\left[\frac{2}{M^2} + (\gamma-1)\right] = (\gamma+1)$$

$$\Rightarrow \quad M^{*2}\left[\frac{2+M^2(\gamma-1)}{M^2}\right] = (\gamma+1)$$

$$\Rightarrow \quad M^{*2} = \frac{\gamma+1}{\dfrac{2+M^2(\gamma-1)}{M^2}}$$

$$\Rightarrow \quad M^{*2} = \frac{M^2(\gamma+1)}{2+M^2(\gamma-1)}$$

Divided by 2 (right side terms)

$$M^{*2} = \frac{\dfrac{M^2}{2}(\gamma+1)}{1+\dfrac{M^2}{2}(\gamma-1)}$$

$$\Rightarrow \quad M^{*2} = \frac{\left(\dfrac{\gamma+1}{2}\right)M^2}{1+\left(\dfrac{\gamma-1}{2}\right)M^2} \qquad ----(1.28)$$

This equation gives the relationship between M* and M.

From equation (1.28), we came to know

$M^* = 1$ if $M = 1$

$M^* < 1$ if $M < 1$

$M^* > 1$ if $M > 1$

$M^* = \sqrt{\frac{\gamma+1}{\gamma-1}}$ if $M \rightarrow \infty$

From that we know M* behaves in the same manner as M, except when M goes to infinity.

It is more inconvenient to use M* instead of M due to the following reasons.

1. At high velocities M approaches infinity but M* gives a finite value $\left(M^* = \sqrt{\frac{\gamma+1}{\gamma-1}}\right)$

2. M is proportional to the fluid velocity (c) and sound velocity (a), but M* is proportional to the fluid velocity alone.

$$M \propto \frac{c}{a}$$

$$M^* \propto \frac{c}{a^*}$$

$$M^* \propto c \qquad [\because a^* = \text{constant}]$$

## 1.19 CROCCO NUMBER ($c_r$)

It is defined as the ratio between fluid velocity (c) and its maximum fluid velocity ($c_{max}$).

$$c_r = \frac{c}{c_{max}}$$

$$= \frac{c}{c^*} \times \frac{c^*}{c_{max}}$$

$$= \frac{c}{a^*} \times \frac{c^*}{c_{max}} \qquad [\because c^* = a^*]$$

$$\boxed{c_r = M^* \times \frac{c^*}{c_{max}}} \qquad \text{----}(1.29)$$

$$\left[\because M^* = \frac{c}{a^*}\right]$$

From Equation (1.24), we know that

$$\frac{c_{max}}{c^*} = \sqrt{\frac{\gamma+1}{\gamma-1}}$$

$$\Rightarrow \boxed{\frac{c^*}{c_{max}} = \sqrt{\frac{\gamma-1}{\gamma+1}}}$$

Substitute this value in Equation (1.29)

$$(1.29) \Rightarrow c_r = M^* \times \sqrt{\frac{\gamma-1}{\gamma+1}}$$

$$\boxed{c_r^2 = M^{*2} \times \frac{\gamma-1}{\gamma+1}} \qquad \text{----}(1.30)$$

From Equation (1.28), we know that

$$M^{*2} = \frac{\left(\frac{\gamma+1}{2}\right) M^2}{1 + \left(\frac{\gamma-1}{2}\right) M^2}$$

Multiply by 2 (right side terms)

$$M^{*2} = \frac{(\gamma+1) M^2}{2 + (\gamma-1) M^2}$$

Substitute $M^{*2}$ value in Equation (1.30)

$$c_r^2 = \frac{(\gamma+1) M^2}{2 + (\gamma-1) M^2} \times \left(\frac{\gamma-1}{\gamma+1}\right)$$

$$= \frac{M^2}{2 + (\gamma-1) M^2} \times (\gamma-1)$$

$$c_r^2 = \frac{M^2 \ (\gamma-1)}{2 + (\gamma-1) M^2}$$

$$c_r^2 \ [2 + (\gamma-1) M^2] = M^2 (\gamma-1)$$

$$2c_r^2 + c_r^2(\gamma-1) M^2 = M^2 (\gamma-1)$$

$$2c_r^2 = M^2 (\gamma-1) - c_r^2(\gamma-1) M^2$$

$$2c_r^2 = M^2 (\gamma-1) [1 - c_r^2]$$

$$\Rightarrow M^2 = \frac{2 c_r^2}{(\gamma-1) [1 - c_r^2]}$$

$$\boxed{M = \sqrt{\frac{2\,c_r^2}{(\gamma-1)\,[\,1-c_r^2]}}} \qquad ------(1.31)$$

This equation gives the relationhip between Mach number and Crocco number.

Stagnation temperature – Mach number relation

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2}\,M^2 \qquad \text{[From Equation no. (1.7)]}$$

From Equation (1.31), we know that

$$M^2 = \frac{2\,c_r^2}{(\gamma-1)\,[\,1-c_r^2]}$$

Substitue $M^2$ value in stagnation temperature equation

$$\Rightarrow \frac{T_0}{T} = 1 + \frac{\gamma-1}{2} \times \frac{2\,c_r^2}{(\gamma-1)\,[\,1-c_r^2]}$$

$$= 1 + \frac{c_r^2}{1-c_r^2}$$

$$= \frac{(1-c_r^2) + c_r^2}{1-c_r^2}$$

$$= \frac{1}{1-c_r^2}$$

$$\boxed{\frac{T_0}{T} = \frac{1}{1-c_r^2}} \qquad ----(1.32)$$

This equation gives the relationship between stagnation temperature and Crocco number.

## 1.20 BERNOULLI EQUATION

From stagnation enthalpy equation, we know that

$$h_0 = h + \frac{c^2}{2} \qquad \text{[From Equation (1.5)]}$$

$$\Rightarrow \qquad h + \frac{c^2}{2} = h_0 = \text{constant}$$

Differentiating this equation

$$dh + \frac{2cdc}{2} = 0$$

$$\boxed{dh + cdc = 0} \qquad ----(1.33)$$

For isentropic flow

$$dh = \frac{dp}{\rho}$$

$$(1.33) \Rightarrow \quad \frac{dp}{\rho} + c\,dc = 0 \qquad ----(1.34)$$

Assuming flow is incompressible. So, $\rho$ = constant.

Integrating Equation (1.34)

$$\Rightarrow \int \frac{dp}{\rho} + \int cdc = \int 0$$

$$\Rightarrow \quad \frac{p}{\rho} + \frac{c^2}{2} = K_{(\text{constant})} \qquad ----(1.35)$$

Stagnation pressure is the pressure of the gas when it is isentropically decelerated to zero velocity at zero elevation. Therefore when $c = 0$,

$$p = p_0 \text{ and } \rho = \rho_0.$$

Substitute these values in equation (1.35).

$$(1.35) \Rightarrow \boxed{\frac{p_0}{\rho_0} = K}$$

Substitute K value in Equation (1.35)

$$(1.35) \Rightarrow \frac{p}{\rho} + \frac{c^2}{2} = \frac{p_0}{\rho_0} \quad \text{-----(1.36)}$$

For incompressible flow,

$$\rho = \text{constant}$$

$$\Rightarrow \boxed{\rho = \rho_0}$$

$$(1.36) \Rightarrow \frac{p}{\rho} + \frac{c^2}{2} = \frac{p_0}{\rho}$$

$$p + \frac{\rho c^2}{2} = p_0$$

$$\Rightarrow \boxed{p_0 = p + \tfrac{1}{2}\rho c^2} \quad \text{----(1.37)}$$

**This is Bernoulli equation and it is valid only when the flow is isentropic and incompressible.**

For Compressible flow,

Stagnation enthalpy, $h_0 = c_p T_0$

$$= \frac{\gamma R}{\gamma - 1} T_0 \qquad [\because c_p = \frac{\gamma R}{\gamma - 1}]$$

$$\boxed{h_0 = \frac{\gamma}{\gamma - 1} \times RT_0}$$

We know that

$$\rho_0 = \frac{p_0}{RT_0}$$

$$\Rightarrow \quad RT_0 = \frac{p_0}{\rho_0}$$

$$\boxed{h_0 = \frac{\gamma}{\gamma - 1} \times \frac{p_0}{\rho_0}} \quad \text{----(1.38)}$$

Substituting $h_0$ value in stagnation enthalpy equation

$$h_0 = h + \frac{c^2}{2}$$

$$\Rightarrow \quad \frac{\gamma}{\gamma - 1} \times \frac{p_0}{\rho_0} = h + \frac{c^2}{2}$$

$$\Rightarrow \quad \frac{\gamma}{\gamma - 1} \times \frac{p_0}{\rho_0} = \frac{\gamma RT}{\gamma - 1} + \frac{c^2}{2} \qquad [\because h = \frac{\gamma RT}{\gamma - 1}$$

[From Equation (1.12)]

$$\Rightarrow \quad \frac{\gamma}{\gamma - 1} \frac{p_0}{\rho_0} = \frac{\gamma}{\gamma - 1} \frac{p}{\rho} + \frac{c^2}{2} \qquad [\because \rho = \frac{p}{RT} \Rightarrow RT = p/\rho]$$

$$\Rightarrow \quad \boxed{\frac{\gamma}{\gamma - 1} \frac{p_0}{\rho_0} = \frac{\gamma}{\gamma - 1} \frac{p}{\rho} + \tfrac{1}{2} c^2} \quad \text{----(1.39)}$$

For insetropic flow

$$\frac{\rho}{\rho_0} = \left(\frac{p}{p_0}\right)^{\frac{1}{\gamma}}$$

$$\Rightarrow \quad \boxed{\rho = \rho_0 \left(\frac{p}{p_0}\right)^{\frac{1}{\gamma}}}$$

$$\Rightarrow \quad \frac{p}{\rho} = \frac{p}{\rho_0 \times \left(\frac{p}{p_0}\right)^{\frac{1}{\gamma}}}$$

$$= \frac{p}{\rho_0} \times \left(\frac{p}{p_0}\right)^{\frac{-1}{\gamma}}$$

$$\frac{p}{\rho} = \frac{p}{\rho_0} \times \frac{p_0}{p_0} \times \left(\frac{p}{p_0}\right)^{\frac{-1}{\gamma}}$$

$$= \frac{p_0}{\rho_0} \times \left(\frac{p}{p_0}\right) \left(\frac{p}{p_0}\right)^{\frac{-1}{\gamma}}$$

$$= \frac{p_0}{\rho_0} \times \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\boxed{\frac{p}{\rho} = \frac{p_0}{\rho_0} \times \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}}$$

Substitute $\frac{p}{\rho}$ value in Equation (1.39)

$$\boxed{\frac{\gamma}{\gamma-1} \frac{p_0}{\rho_0} = \frac{\gamma}{\gamma-1} \times \frac{p_0}{\rho_0} \times \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}} + \tfrac{1}{2} c^2} \qquad \text{----(1.40)}$$

**This is the Bernoulli Equation for isentropic compressible flow.**

## 1.21 EFFECT OF MACH NUMBER ON COMPRESSIBILITY

From Bernoulli equation, we know that stagnation pressure for incompressible flow is

$$p_0 = p + \tfrac{1}{2} \rho c^2$$

$$\Rightarrow \qquad \frac{p_0 - p}{\tfrac{1}{2} \rho c^2} = 1 \qquad \text{----(1.41)}$$

This equation shows the value of pressure co-efficient, $c_p$ (sometimes referred to as compressibility factor) is unity.

But for compressible flow the value of the pressure co-efficient deviates from unity.

For isentropic compressible flow, the relationship between stagnation pressure and stagnation temperature is given by

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\Rightarrow \qquad \frac{p_0}{p} = \left[1 + \frac{\gamma-1}{2} M^2\right]^{\frac{\gamma}{\gamma-1}} \qquad \text{----(1.42)}$$

[From Equation no.(1.8)]

Expanding this equation as Taylor series i.e.,

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + \ldots\ldots$$

Here $x = \frac{\gamma-1}{2} M^2$ $\qquad n = \frac{\gamma}{\gamma-1}$

$$\Rightarrow \left[1 + \frac{\gamma-1}{2} M^2\right]^{\frac{\gamma}{\gamma-1}} = 1 + \frac{\gamma}{\gamma-1}\left[\frac{\gamma-1}{2} M^2\right] + \frac{\frac{\gamma}{\gamma-1}\left[\frac{\gamma}{\gamma-1} - 1\right]}{2!} \times \left[\frac{\gamma-1}{2} M^2\right]^2 + \frac{\frac{\gamma}{\gamma-1}\left[\frac{\gamma}{\gamma-1} - 1\right]\left[\frac{\gamma}{\gamma-1} - 2\right]}{3!} \times \left[\frac{\gamma-1}{2} M^2\right]^3$$

$$= 1 + \frac{\gamma}{2} M^2 + \frac{\frac{\gamma}{\gamma-1}\left[\frac{\gamma-(\gamma-1)}{\gamma-1}\right]}{2} \times \left[\frac{(\gamma-1)^2}{4} M^4\right] + \frac{\frac{\gamma}{\gamma-1}\left[\frac{\gamma-(\gamma-1)}{\gamma-1}\right]\left[\frac{\gamma-2(\gamma-1)}{\gamma-1}\right]}{6} \times \left[\frac{(\gamma-1)^3}{8} \times M^6\right]$$

$$= 1 + \frac{\gamma}{2} M^2 + \frac{\frac{\gamma}{\gamma-1}\left[\frac{1}{\gamma-1}\right]}{2} \times \left[\frac{(\gamma-1)^2}{4} M^4\right]$$

$$+ \frac{\frac{\gamma}{\gamma-1}\left[\frac{1}{\gamma-1}\right]\left[\frac{2-\gamma}{\gamma-1}\right]}{6} \times \left[\frac{(\gamma-1)^3}{8} \times M^6\right]$$

$$= 1 + \frac{\gamma}{2} M^2 + \frac{\gamma}{8} M^4 + \frac{\gamma(2-\gamma)}{48} M^6$$

$$\Rightarrow \left[1 + \frac{\gamma-1}{2} M^2\right]^{\frac{\gamma}{\gamma-1}} = 1 + \frac{\gamma}{2} M^2 + \frac{\gamma}{8} M^4 + \frac{\gamma(2-\gamma)}{48} M^6$$

Substitute this value in equation (1.42)

$$\frac{p_0}{p} = 1 + \frac{\gamma}{2} M^2 + \frac{\gamma}{8} M^4 + \frac{\gamma(2-\gamma)}{48} M^6$$

$$\frac{p_0}{p} - 1 = \frac{\gamma}{2} M^2 + \frac{\gamma}{8} M^4 + \frac{\gamma(2-\gamma)}{48} M^6$$

$$\frac{p_0 - p}{p} = \frac{\gamma}{2} M^2 + \frac{\gamma}{8} M^4 + \frac{\gamma(2-\gamma)}{48} M^6$$

Divided by $\frac{\gamma}{2} M^2$

$$\frac{p_0 - p}{p \times \frac{\gamma}{2} M^2} = 1 + \frac{M^2}{4} + \frac{(2-\gamma)}{24} M^4 \qquad ---- (1.43)$$

we know that,

Mach number, $M = \frac{c}{a}$

$$\Rightarrow M^2 = \frac{c^2}{a^2}$$

$$\boxed{M^2 = \frac{c^2}{\gamma RT}} \qquad [\because a = \sqrt{\gamma RT}]$$

$$\frac{\gamma}{2} M^2 = \frac{\gamma}{2} \times \frac{c^2}{\gamma RT}$$

$$\boxed{\frac{\gamma}{2} M^2 = \frac{c^2}{2RT}}$$

$$p \times \frac{\gamma}{2} M^2 = \frac{pc^2}{2RT}$$

$$= \frac{\rho RT\, c^2}{2RT} \qquad \left[\because pv = RT \Rightarrow p = \frac{RT}{v} \Rightarrow p = \rho RT\right]$$

$$\boxed{p \times \frac{\gamma}{2} M^2 = \frac{\rho c^2}{2}} \qquad ---- (1.44)$$

Substitute $p \times \frac{\gamma}{2} M^2$ value in Equation (1.43)

$$(1.43) \Rightarrow \frac{p_0 - p}{\frac{\rho c^2}{2}} = 1 + \frac{M^2}{4} + \frac{(2-\gamma)}{24} M^4 + -----$$

Substitute $\gamma = 1.4$

$$\Rightarrow \boxed{\frac{p_0 - p}{\frac{1}{2} \rho c^2} = 1 + \frac{M^2}{4} + \frac{M^4}{40} + -----} \qquad ---- (1.45)$$

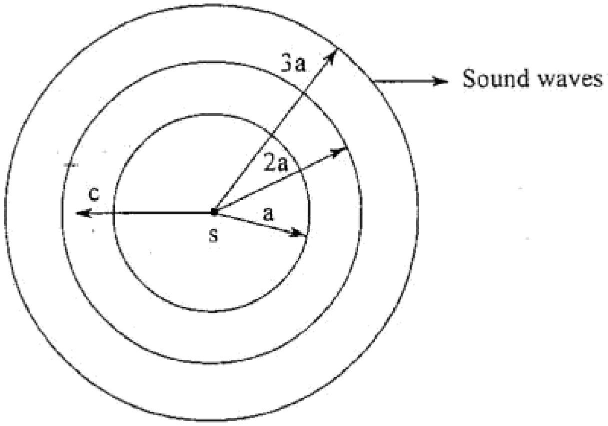
This is the pressure co-efficient equation for compressible flow.

## 1.22 MACH CONE, MACH ANGLE, MACH WAVES

Let us consider a solid body move in a straight line through a fluid which is stationary. The movement of the body will generate pressure waves in the fluid. These pressure waves are transmitted to other particles of the fluid in all the direction with a velocity of sound (a). These sound pulses forms a spherical wave front as shown in figures 1.4, 1.5, 1.6 and 1.7.

### Incompressible flow

In an incompressible flow, the source of disturbance velocity (c) is negligibly small compared to the sound velocity (a). In this case infinitesimal sound waves (spherical waves) are generated as shown in fig.1.4 and travelled at a velocity (a) in all directions.



Where

s → source of disturbance

a → Sound velocity

c → Fluid velocity

***Fig.1.4 Incompressible flow***

### Subsonic flow

In subsonic flow, the source of disturbance velocity (c) is less than the sound velocity (a). The spherical waves are generated as shown in fig 1.5. It is observed that the sound waves move ahead of the source of disturbance and the intensity is not symmetrical.

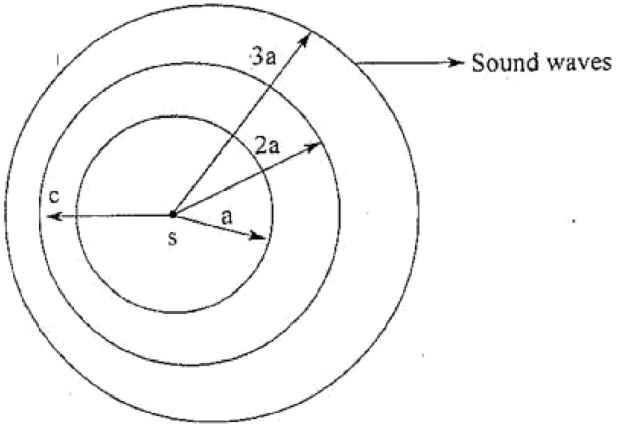


***Fig 1.5 Subsonic flow***

### Sonic flow

In sonic flow, the source of disturbance velocity (c) is equal to the sound velocity (a). Under this condition the sound waves always exit at the present position of the point source and cannot move ahead of it. Therefore, the zone lying on the left of the source of disturbance (s) is called zone of silence because the waves do not reach this zone.

The zone lying on the right of the source of disturbance (s) is called zone of action. Because the waves reaches this zone.

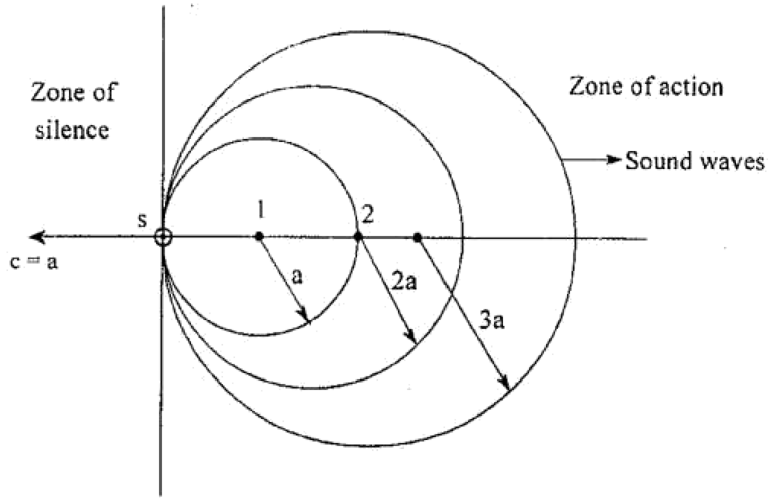


*Fig.1.6 Sonic flow*

### Supersonic flow

In supersonic flow, the source of disturbance velocity (c) is greater than the sound velocity (a). The spherical waves are generated as shown in fig.1.7. It is observed that the point source is always ahead of the wave fronts.

### Mach Cone

Tangents drawn from the point S (Source of disturbance) on the spheres define a conical surface referred to as Mach Cone. The region inside the cone is called the zone of action, and the region outisde the cone is termed as the zone of silence.

### Mach waves or Mach lines

The lines at which the pressure difference is concentrated and which generate the cone are called Mach lines or Mach waves.

### Mach Angle

The angle between the Mach line and the direction of motion of the body (Flow direction) is called the Mach angle.

Mach angle $\alpha = \sin^{-1}\left(\frac{1}{M}\right)$

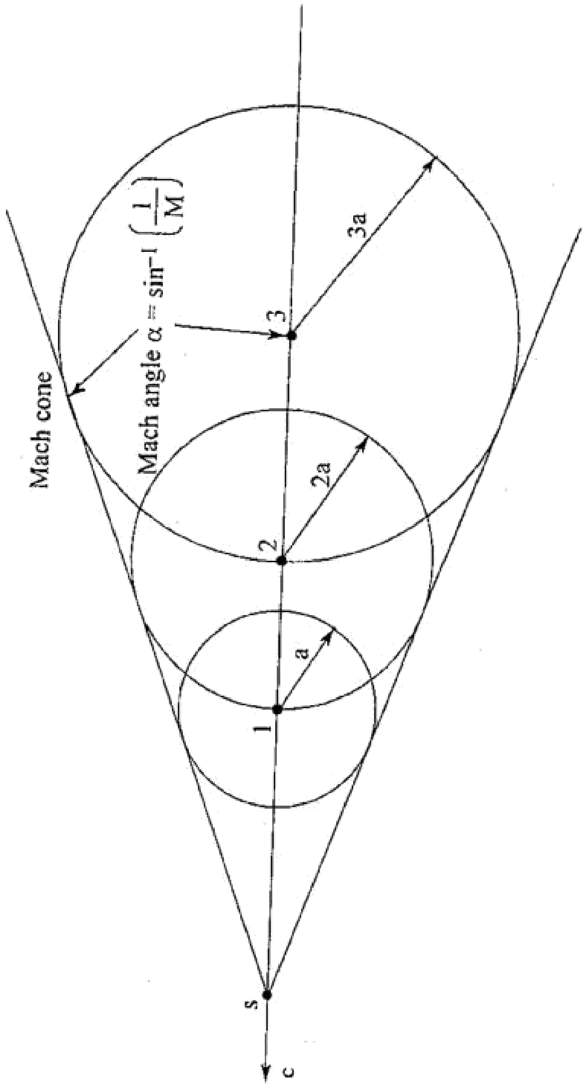


*Fig. 1.7 Supersonic flow*

## 1.23 FORMULAE USED

1. Mach number, $M = \frac{c}{a}$

2. Velocity of sound, $a = \sqrt{\gamma RT}$ m/s

3. Stagnation temperature – Mach number relation

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

4. Stagnation pressure – Stagnation temperature relation

$$\Rightarrow \quad \frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\gamma/\gamma-1}$$

$$\frac{p_0}{p} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\gamma/\gamma-1}$$

5. Stagnation density – stagnation pressure relation

$$\frac{\rho_0}{\rho} = \left(\frac{p_0}{p}\right)^{1/\gamma}$$

6. Static enthalpy, $h = c_p T$

7. Stagnation enthalpy, $h_0 = c_p T_0$

8. Stagnation enthalpy equation

$$h_0 = \frac{a^2}{\gamma - 1} + \frac{1}{2} c^2 = \frac{1}{2} c_{max}^2 = \frac{a_0^2}{\gamma - 1}$$

9. Crocco Number

$$c_r = \frac{c}{c_{max}}$$

10. Mass flow rate

$$m = \rho A c = \rho_1 A_1 c_1 = \rho_2 A_2 c_2 \ldots\ldots$$

11. Critical temperature, $T^* = \frac{2T_0}{\gamma + 1}$

12. Gas constant, $R = \left[\frac{\gamma - 1}{\gamma}\right] c_p$

For air $\gamma = 1.4$ $c_p = 1005$ J/kg K

$R = 287$ J/kgK

$$c_p = \frac{\gamma R}{\gamma - 1}$$

13. Mach angle, $\alpha = \sin^{-1} ({}^1/_m)$

14. From Bernoulli equation, (For incompressible flow)

Stagnation pressure, $p_0 = p + \frac{1}{2} \rho c^2$

15. For isentropic flow,

Stagnation temperature remains constant, i.e., $T_0 = T_{01} = T_{02}$

Stagnation pressure remains constant, i.e., $p_0 = p_{01} = p_{02}$

## 1.24 SOLVED PROLEMS

$\boxed{1}$ ***Calculate the velocity of sound and stagnation temperature of Jet at 300 K. Assume Mach number = 1.2.***

***Given***

$T = 300$ K

$M = 1.2$

***To find***

1. Velocity of sound (a)

2. Stagnation temperature ($T_0$)

***Solution***

We know that

Velocity of sound, $a = \sqrt{\gamma RT}$

$$= \sqrt{1.4 \times 287 \times 300} \qquad [\because \gamma = 1.4, \ R = 287 \text{ J/kg K}]$$

$$\boxed{a = 347.18 \text{ m/s}}$$

Stagnation temperature – Mach number relation,

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 \qquad \text{[From equation no.1.7]}$$

$$\Rightarrow \quad \frac{T_0}{300} = 1 + \frac{1.4 - 1}{2} \times (1.2)^2$$

$$\boxed{T_0 = 386.4 \text{ K}}$$

**Result:** 1. Velocity of sound, $a = 347.18$ m/s

2. Stagnation temperature, $T_0 = 386.4$ K

**2** ***An air jet at 300 K has sonic velocity. Determine the following:***

1. ***Velocity of sound at 300 K***
2. ***Velocity of sound at stagnation conditions***
3. ***Maximum velocity of the jet***
4. ***Stagnation enthalpy***
5. ***Crocco number***

***Take $\gamma = 1.4$, $R = 287$ J/kgK.***

***Given:*** $T = 300$ K

At sonic condition,

Mach number, $M = 1$

$$\Rightarrow \quad M = \frac{c}{a} = 1$$

$$\Rightarrow \quad \boxed{c = a}$$

$$\gamma = 1.4, \quad R = 287 \text{ J/kgK}$$

***To find***

1. Velocity of sound, a
2. Velocity of sound at stagnation condition, $a_0$
3. Maximum velocity of the jet, $c_{max}$
4. Stagnation enthalpy, $h_0$
5. Crocco number, $c_r$.

***Solution***

1. We know that

Velocity of sound, $a = \sqrt{\gamma RT}$

$$= \sqrt{1.4 \times 287 \times 300}$$

$$\boxed{a = 347.18 \text{ m/s}}$$

2. Stagnation temperature – Mach number relation,

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

At sonic condition, $M = 1$

$$\Rightarrow \quad \frac{T_0}{T} = 1 + \frac{\gamma - 1}{2}$$

$$\Rightarrow \quad \frac{T_0}{300} = 1 + \frac{1.4 - 1}{2}$$

$$\Rightarrow \quad \frac{T_0}{300} = 1.2$$

$$\boxed{T_0 = 360 \text{ K.}}$$

Velocity of sound at stagnation condition

$$a_0 = \sqrt{\gamma R T_0}$$

$$= \sqrt{1.4 \times 287 \times 360}$$

$$\boxed{a_0 = 380.32 \text{ m/s}}$$

3. We know. stagnation enthalpy equation

$$h_0 = \frac{a^2}{\gamma - 1} + \tfrac{1}{2} c^2 = \tfrac{1}{2} c_{max}^2 = \frac{a_0^2}{\gamma - 1}$$

[From Equation no.(1.16)]

$$\Rightarrow \quad \tfrac{1}{2} c_{max}^2 = \frac{a_0^2}{\gamma - 1}$$

$$\Rightarrow \quad c_{max}^2 = \frac{2a_0^2}{\gamma - 1}$$

$$c_{max} = \sqrt{\frac{2a_0^2}{\gamma - 1}}$$

$$= \sqrt{\frac{2 \times (380.32)^2}{1.4 - 1}}$$

$$\boxed{\text{Maximum velocity } c_{max} = 850.42 \text{ m/s.}}$$

4. From Stagnation enthalpy equation, we know that

$$h_0 = \tfrac{1}{2} c_{max}^2$$

$$h_0 = \tfrac{1}{2} (850.42)^2$$

$$\boxed{h_0 = 361.6 \times 10^3 \text{ J/kg}}$$

5. Crocco number, $c_r = \frac{c}{c_{max}}$

$$= \frac{a}{c_{max}}$$

[At sonic condition c = a]

$$= \frac{347.18}{850.42}$$

$$\boxed{c_r = 0.408}$$

***Result***

1. $a = 347.18$ m/s
2. $a_0 = 380.32$ m/s
3. $c_{max} = 850.42$ m/s
4. $h_0 = 361.6 \times 10^3$ J/kg
5. $c_r = 0.408$

**3** ***The jet of a gas at 500 K has a Mach number of 1.2. Determine the following***

***1. Local velocity of sound***

***2. Stagnation velocity of sound***

***3. Static enthalpy***

***4. Stagnation enthalpy***

***5. Maximum attainable velocity of this jet***

***Take $\gamma = 1.3$, $R = 469$ J/kg K.***

***Given***

$T = 500$ K

$M = 1.2$

$\gamma = 1.3$

$R = 469$ J/kg K.

***To find***

1. Local velocity, a

2. Stagnation velocity, $a_0$

3. Static enthalpy, $h$

4. Stagnation enthalpy, $h_0$

5. Maximum attainable velocity, $c_{max}$

***Solution***

1. Velocity of sound, $a = \sqrt{\gamma RT}$

$$= \sqrt{1.3 \times 469 \times 500}$$

$$\boxed{a = 552.13 \text{ m/s}}$$

2. Stagnation temperature – Mach number relation,

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

$$\Rightarrow \quad \frac{T_0}{500} = 1 + \frac{1.3-1}{2}(1.2)^2$$

$$\Rightarrow \quad \frac{T_0}{500} = 1.216$$

$$T_0 = 608 \text{ K.}$$

Stagnation velocity of sound

$$a_0 = \sqrt{\gamma RT_0}$$

$$= \sqrt{1.3 \times 469 \times 608}$$

$$\boxed{a_0 = 608.84 \text{ m/s}}$$

3. Static enthalpy, $h = c_pT$

$$= \frac{\gamma R}{\gamma - 1} \times T \qquad [\because c_p = \frac{\gamma R}{\gamma - 1}]$$

$$= \frac{1.3 \times 469}{1.3 - 1} \times 500$$

$$\boxed{h = 10.16 \times 10^5 \text{ J/kg}}$$

4. Stagnation enthalpy, $h_0 = c_pT_0$

$$= \frac{\gamma R}{\gamma - 1} \times T_0$$

$$= \frac{1.3 \times 469}{1.3 - 1} \times 608$$

$$\boxed{h_0 = 12.3 \times 10^5 \text{ J/kg}}$$

5. Stagnation enthalpy equation

$$h_0 = \frac{a^2}{\gamma - 1} + \tfrac{1}{2}c^2 = \tfrac{1}{2}c_{max}^2 = \frac{a_0^2}{\gamma - 1}$$

[From Equation no.(1.16)]

$$\Rightarrow \quad h_0 = \tfrac{1}{2}c_{max}^2$$

$$\Rightarrow \quad 12.3 \times 10^5 = \tfrac{1}{2}c_{max}^2$$

$$\Rightarrow \quad \boxed{c_{max} = 1568.44 \text{ m/s}}$$

***Result :***

1. $a = 552.13$ m/s

2. $a_0 = 608.84$ m/s

3. $h = 10.16 \times 10^5$ J/kg

4. $h_0 = 12.3 \times 10^5$ J/kg

5. $c_{max} = 1568.44$ m/s.

**4** *An air stream enters in to a duct at a pressure of 1.2 bar, temperature of 300 K and Mach number of 1.25. If the Mach number at exit of the duct is 0.6, determine the following*

*1. Temperature of air at the duct exit*

*2. Velocity of air at the duct exit*

*Assuming adiabatic flow.*

### Given

$p_1 = 1.2 \text{ bar} = 1.2 \times 10^5 \text{ N/m}^2$

$T_1 = 300 \text{ K}$

$M_1 = 1.25$

$M_2 = 0.6$

[Note for air

$\gamma = 1.4,$

$R = 287 \text{ J/kg K},$

$c_p = 1005 \text{ J/kg K}$]

### To find

1. Temperature of air at exit ($T_2$)

2. Velocity of air at exit ($c_2$)

**This problem can be solved by using gas tables**

***Solution :***

**At inlet**

Refer Isentropic flow table. for $\gamma = 1.4$ and $M_1 = 1.25$

$$\frac{T_1}{T_{0_1}} = 0.762$$

*[ From Gas tables, (S.M. Yahya Fifth Edition) - page no.32]*

$$\Rightarrow \quad T_{0_1} = \frac{T_1}{0.762}$$

$$\Rightarrow \quad = \frac{300}{0.762} = 393.7 \text{ K}$$

$$\boxed{\text{Stagnation temperature, } T_{0_1} = 393.7 \text{ K}}$$

We know that for Isentropic flow, stagnation temperature remains constant

$$\Rightarrow \quad \boxed{T_0 = T_{0_1} = T_{0_2} = 393.7 \text{ K}}$$

**At outlet**

From Isentropic flow table for $\gamma = 1.4$ and $M_2 = 0.6$

$$\frac{T_2}{T_{0_2}} = 0.933$$

[ From Gas tables page no.29]

$$\Rightarrow \quad T_2 = T_{0_2} \times 0.933$$

$$= 393.7 \times 0.933$$

$$\boxed{\text{Exit temperature, } T_2 = 367.32 \text{ K}}$$

Sound velocity at exit, $a_2 = \sqrt{\gamma R T_2}$

$$= \sqrt{1.4 \times 287 \times 367.32}$$

$$\boxed{a_2 = 384.17 \text{ m/s}}$$

Mach number at exit, $M_2 = \frac{c_2}{a_2}$

$$\Rightarrow \quad 0.6 = \frac{c_2}{384.17}$$

$$\Rightarrow \quad \boxed{c_2 = 230.50 \text{ m/s}}$$

**Result :**

1. $T_2 = 367.32$ K

2. $c_2 = 230.50$ m/s.

**5** ***Air enters a straight duct at 2.5 bar and 30° C. The inlet Mach number is 1.5 and exit Mach number is 2.4. Assuming adiabatic flow, determine***

***a) stagnation temperature***

***b) Temperature and velocity of air at exit.***

***c) The flow rate per square metre of the inlet cross section.***

***Take $\gamma = 1.4$ $R = 287$ J/kg K.***

**Given :**

$p_1 = 2.5 \text{ bar} = 2.5 \times 10^5 \text{ N/m}^2$

$T_1 = 30\,°\text{C} + 273 = 303 \text{ K}$

$M_1 = 1.5$

$M_2 = 2.4$

$\gamma = 1.4, \quad R = 287$ J/kg K.

**To find :**

a. Stagnation temperature, $T_0$

b. Temperature and velocity of air at exit $T_2$, $c_2$

c. The flow rate per square metre of the inlet cross section, $\frac{m}{A_1}$

**Solution :**

**At inlet**

Refer Isentropic flow table for $M_1 = 1.5$ and $\gamma = 1.4$

$$\frac{T_1}{T_{0_1}} = 0.689$$

*[From Gas tables (S.M.Yahya, Fifth Edition) - page no.32]*

$$\Rightarrow \quad T_{0_1} = \frac{T_1}{0.689}$$

$$\Rightarrow \quad = \frac{303}{0.689}$$

$$\boxed{\text{Stagnation temperature, } T_{0_1} = 439.76 \text{ K}}$$

For Isentropic flow stagnation temperature remains constant

$$\Rightarrow \quad \boxed{T_0 = T_{0_1} = T_{0_2} = 439.76 \text{ K}}$$

**At outlet**

Refer Isentropic flow table for $M_2 = 2.4$ and $\gamma = 1.4$

$$\frac{T_2}{T_{0_2}} = 0.464$$

*[From Gas tables page no.35]*

$$\Rightarrow \quad T_2 = T_{0_2} \times 0.464$$

$$= 439.76 \times 0.464$$

$$\boxed{\text{Exit temperature, } T_2 = 204.04 \text{ K}}$$

Mach number at exit, $M_2 = \dfrac{c_2}{a_2}$

$$= \frac{c_2}{\sqrt{\gamma R T_2}} \qquad [\because a = \sqrt{\gamma RT}]$$

$$= \frac{c_2}{\sqrt{1.4 \times 287 \times 204.04}}$$

$$2.4 = \frac{c_2}{286.33}$$

$$\Rightarrow \boxed{\text{Velocity of air at exit, } c_2 = 687.18 \text{ m/s}}$$

Mass flow rate, $m = \rho A c = \rho_1 A_1 c_1 = \rho_2 A_2 c_2$

$$\Rightarrow \quad \dot{m} = \rho_1 A_1 c_1$$

$$\dot{m} = \frac{p_1}{RT_1} \times A_1 c_1$$

$$\Rightarrow \quad \frac{m}{A_1} = \frac{p_1}{RT_1} \times c_1$$

$$= \frac{2.5 \times 10^5}{287 \times 303} \times M_1 \times a_1 \qquad [\because M = \frac{c}{a}]$$

$$= 2.87 \times M_1 \times \sqrt{\gamma R T_1} \qquad [\because a = \sqrt{\gamma RT}]$$

$$= 2.87 \times 1.5 \times \sqrt{1.4 \times 287 \times 303}$$

$$\boxed{\frac{m}{A_1} = 1502.10 \text{ kg/s-m}^2}$$

***Result :***

a) $T_0 = T_{0_1} = T_{0_2} = 439.76$ K

b) $T_2 = 204.04$ K

$c_2 = 687.18$ m/s

c) $\dfrac{m}{A_1} = 1502.10$ kg/s–m$^2$

$\boxed{6}$ ***The pressure, temperature and fluid velocity of air at the entry of a flow passage are 3 bar, 280 K and 140 m/s. The pressure, temperature and velocity at the exit of a flow passage are 2 bar, 260 K and 250 m/s. The area of cross section at entry is 600 cm². Determine for adiabatic flow,***

- ***a) Stagnation temperature***
- ***b) Maximum velocity***
- ***c) Mass flow rate***
- ***d) Area of cross section at exit.***

***Take $\gamma = 1.4$, $R = 287$ J/kgK.***

***Given***

$p_1 = 3 \text{ bar} = 3 \times 10^5 \text{ N/m}^2$

$T_1 = 280$ K

$c_1 = 140$ m/s

$p_2 = 2 \text{bar} = 2 \times 10^5 \text{ N/m}^2$

$T_2 = 260$ K

$c_2 = 250$ m/s

$A_1 = 600 \text{ cm}^2 = 600 \times 10^{-4} \text{ m}^2$

$\gamma = 1.4, \; R = 287$ J/kg K.

***To find***

a. Stagnation temperature $T_0$

b. Maximum velocity, $c_{max}$

c. mass flow rate, m

d. Area of cross section at exit, $A_2$

***Solution***

Inlet Mach number, $M_1 = \frac{c_1}{a_1}$

$$= \frac{140}{\sqrt{1.4 \times 287 \times 280}}$$

$$\boxed{M_1 = 0.417}$$

**At inlet**

Refer Isentropic flow table for $\gamma = 1.4$, and $M_1 = 0.417 \approx 0.42$

$$\frac{T_1}{T_{0_1}} = 0.966 \qquad \frac{p_1}{p_{0_1}} = 0.886$$

$$\frac{A_1}{A^*} = 1.529$$

*[From Gas tables page no.29]*

$$\Rightarrow \qquad \frac{T_1}{T_{0_1}} = 0.966$$

$$T_{0_1} = \frac{T_1}{0.966}$$

$$= \frac{280}{0.966}$$

$$\boxed{T_{0_1} = 289.85 \text{ K}}$$

For isentropic flow, stagnation temperature remains constant.

$$\Rightarrow \qquad \boxed{T_0 = T_{0_1} = T_{0_2} = 289.85 \text{ K}}$$

From Stagnation enthalpy equation, we know

$$h_0 = \frac{a^2}{\gamma - 1} + \tfrac{1}{2} c^2 = \tfrac{1}{2} c_{max}^2 = \frac{a_0^2}{\gamma - 1}$$

$$\Rightarrow \qquad \tfrac{1}{2} c_{max}^2 = \frac{a_0^2}{\gamma - 1}$$

$$c_{max}^2 = \frac{2a_0^2}{\gamma - 1}$$

$$\Rightarrow \qquad c_{max} = \sqrt{\frac{2a_0^2}{\gamma - 1}} \qquad \text{----- (A)}$$

$$a_0 = \sqrt{\gamma R T_0}$$
$$= \sqrt{1.4 \times 287 \times 289.85}$$
$$\boxed{a_0 = 341.26 \text{ m/s}}$$

Substitute $a_0$ value in Equation (A)

$$c_{max} = \sqrt{\frac{2 \times (341.26)^2}{1.4 - 1}}$$
$$\boxed{c_{max} = 763.08 \text{ m/s.}}$$

Mass flow rate, $m = \rho A c = \rho_1 A_1 c_1 = \rho_2 A_2 c_2$

$$\Rightarrow \quad m = \rho_1 A_1 c_1$$
$$= \frac{p_1}{RT_1} \times A_1 \times c_1$$
$$= \frac{3 \times 10^5}{287 \times 280} \times 600 \times 10^{-4} \times 140$$
$$\boxed{m = 31.36 \text{ kg/s}}$$

From gas tables, we know that

$$\frac{A_1}{A^*} = 1.529$$
$$\Rightarrow \quad A^* = \frac{A_1}{1.529}$$
$$A^* = \frac{600 \times 10^{-4}}{1.529}$$
$$\boxed{A^* = 0.039 \text{ m}^2}$$

Mach number at exit, $M_2 = \dfrac{c_2}{a_2}$

$$= \frac{250}{\sqrt{\gamma R T_2}}$$
$$= \frac{250}{\sqrt{1.4 \times 287 \times 260}}$$
$$\boxed{M_2 = 0.773}$$

**At outlet**

From Isentropic flow table for $\gamma = 1.4$ and $M_2 = 0.773 \approx 0.77$

$$\frac{A_2}{A^*} = 1.052$$ *[From Gas tables page no.30]*

$$\Rightarrow \quad A_2 = 1.052 \times A^*$$
$$= 1.052 \times 0.039$$
$$\boxed{\text{Exit area, } A_2 = 0.0410 \text{ m}^2}$$

***Result :***

a. $T_0 = 289.85$ K

b. $c_{max} = 763.08$ m/s

c. $A_2 = 0.0410 \text{ m}^2$

d. $m = 31.36$ kg/s

*8 Steam at a section of a pipe has a pressure of 10 bar, temperature of 550 K, velocity of 125 m/s and datum height of 10m. Calculate the following*

1. *Mach number*
2. *Stagnation pressure*
3. *Stagnation temperature*
4. *Compare the stagnation pressure value with that obtained from Bernoulli equation and comment on the difference.*

*Take $c_p = 2.150$ kJ/kg K ;* $\quad$ *$c_v = 1.615$ kJ/kg K*

***Given***

$p = 10\text{ bar} = 10 \times 10^5\text{ N/m}^2$

$T = 550\text{ K}$

$c = 125\text{ m/s}$

Datum height, $Z = 10\text{ m}$

$c_p = 2.150\text{ kJ/kg K} = 2150\text{ J/kg K}$

$c_v = 1.615\text{ kJ/kg K} = 1615\text{ J/kg k}$

***To find***

1. Mach number, M
2. Stagnation temperature, $T_0$
3. Stagnation pressure, $p_0$
4. Compare the stagnation pressure value with that obtained from the Bernoulli equation and comment on the difference.

***Solution***

We know that

Gas constant, $R = c_p - c_v$

$= 2150 - 1615$

$$\boxed{R = 535\text{ J/kg K}}$$

**This problem can be solved by using gas tables.**

***Solution:***

We know that

$$\text{Gas constant, } R = c_p - c_v$$

$$= 2150 - 1615$$

$$\boxed{R = 535\ \text{J/kgK}}$$

$$\gamma = \frac{c_p}{c_v} = \frac{2150}{1615}$$

$$\boxed{\gamma = 1.33}$$

$$\text{Velocity of sound, } a = \sqrt{\gamma R T}$$

$$= \sqrt{1.33 \times 535 \times 550}$$

$$a = 625.58\text{ m/s}$$

$$\text{Inlet Mach number, } M = \frac{c}{a}$$

$$= \frac{125}{625.58}$$

$$\boxed{M = 0.199}$$

**Refer Isentropic flow table for $\gamma = 1.3$ and $M = 0.199 \approx 0.2$.**

$$\frac{T}{T_0} = 0.994$$

*[From gas tables page no.21]*

$$\frac{p}{p_0} = 0.975$$

$$\Rightarrow \quad \frac{T}{T_0} = 0.994$$

$$T_0 = \frac{T}{0.994} = \frac{550}{0.994} = 553.31\ K$$

$$\boxed{\text{Stagnation temperature, } T_0 = 553.31\ K}$$

$$\Rightarrow \quad \frac{p}{p_0} = 0.975$$

$$p_0 = \frac{p}{0.975} = \frac{10}{0.975} = 10.25\ \text{bar}$$

$$\boxed{\begin{aligned}\text{Stagnation pressure, } p_0 &= 10.25\ \text{bar}\\ &= 10.25 \times 10^5\ N/m^2\end{aligned}}$$

From Bernoulli equation, for incompressible flow, stagnation pressure $p_0 = p + {}^1/_2\, \rho c^2$

$$\Rightarrow \quad p_0 = p + {}^1/_2\ \frac{p}{RT} \times c^2$$

$$= 10 \times 10^5 + {}^1/_2 \times \frac{10 \times 10^5}{535 \times 550} \times (125)^2$$

$$\boxed{p_0 = 10.26 \times 10^5\ N/m^2}$$

***Result***

1. $M = 0.199$
2. $T_0 = 553.31\ K$
3. $p_0 = 10.25 \times 10^5\ N/m^2$

   [for Compressible flow]
4. $p_0 = 10.26 \times 10^5\ N/m^2$

   [for Incompressible flow]

**9** ***An aircraft is flying at an altitude of 11,000 metres, at 800 km/hr. The air is reversibly compressed in an inlet diffuser. The inlet temperature is 216.65 K and pressure is 0.226 bar. If the Mach number at the exit of the diffuser is 0.35, calculate the following***

***1. Entry Mach number***

***2. Velocity, pressure and temperature of air at the diffuser exit.***

***Given***

Altitude, $Z = 11{,}000m$

Air velocity, $c_1 = 800\ km/h = \frac{800 \times 10^3}{3600}\ m/s$

$= 222.2\ m/s$

Inlet temperature, $T_1 = 216.65\ K$

Inlet pressure, $p_1 = 0.226\ bar$

Mach Number at exit, $M_2 = 0.35$

***To find***

1. Entry Mach Number, $M_1$
2. Velocity, pressure and temperature at exit, $c_2, p_2, T_2$.

***Solution***

Velocity of sound, $a_1 = \sqrt{\gamma R T_1}$

$= \sqrt{1.4 \times 287 \times 216.65}$

[For air $\gamma = 1.4$, $R = 287\ J/kg\ K$]

$$\boxed{a_1 = 295.04\ m/s.}$$

Entry Mach number, $M_1 = \frac{c_1}{a_1}$

$$= \frac{222.2}{295.04}$$

$$\boxed{M_1 = 0.753}$$

**At entry,**

Refer Isentropic table for $\gamma = 1.4$ and $M_1 = 0.753 \approx 0.75$

$$\frac{T_1}{T_0} = 0.899$$

*[From Gas tables page no.30]*

$$\frac{p_1}{p_0} = 0.688$$

$$\Rightarrow \quad \frac{T_1}{T_0} = 0.899$$

$$\Rightarrow \quad T_0 = \frac{T_1}{0.899} = \frac{216.65}{0.899} = 240.98 \text{ K}$$

$$\boxed{\text{Stagnation temperature, } T_0 = 240.98 \text{ K}}$$

For Isentropic flow, stagnation temperature remains constant

i.e. $\boxed{T_0 = T_{01} = T_{02} = 240.98 \text{ K}}$

From table

$$\frac{p_1}{p_0} = 0.688$$

$$\frac{0.226}{p_0} = 0.688$$

[Table value– pressure unit is in bar]

$$\Rightarrow \quad p_0 = \frac{0.226}{0.688}$$

$$\boxed{\text{Stagnation pressure, } p_0 = 0.328 \text{ bar}}$$

For Isentropic flow, stagnation pressure remains constant

i.e. $\boxed{p_0 = p_{01} = p_{02} = 0.328 \text{ bar}}$

**At Exit**

Refer Isentropic flow table for $\gamma = 1.4$, $M_2 = 0.35$

$$\frac{T_2}{T_{02}} = 0.976$$

*[From Gas tables page no.29]*

$$\Rightarrow \quad \frac{p_2}{p_{02}} = 0.918$$

$$\Rightarrow \quad \frac{T_2}{T_{02}} = 0.976$$

$$\Rightarrow \quad T_2 = 0.976 \times T_{02}$$

$$= 0.976 \times 240.98$$

$$\boxed{T_2 = 235.19 \text{ K}}$$

$$\Rightarrow \quad \frac{p_2}{p_{02}} = 0.918$$

$$p_2 = 0.918 \times p_{02}$$

$$= 0.918 \times 0..328$$

$$\boxed{p_2 = 0.301 \text{ bar}}$$

Exit sound velocity, $a_2 = \sqrt{\gamma K T_2}$

$$= \sqrt{1.4 \times 287 \times 235.19}$$

$$\boxed{a_2 = 307.41 \text{ m/s.}}$$

Exit Mach number, $M_2 = \dfrac{c_2}{a_2}$

$$\Rightarrow \quad c_2 = M_2 \times a_2$$

$$= 0.35 \times 307.41$$

$$\boxed{c_2 = 107.59 \text{ m/s}}$$

$$\boxed{\text{Velocity of air at exit, } c_2 = 107.59 \text{ m/s}}$$

***Result***

1. $M_1 = 0.753$
2. $T_2 = 235.19$ K

   $p_2 = 0.301$ bar

   $c_2 = 107.59$ m/s.

**Note**

[At $Z = 11{,}000$m, $T_1 = 216.65$ K and $p_1 = 0.226$ bar. So given values are correct (From gas tables page no.19)]

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**1** ***The air moving at a velocity of 150 m/s. The static conditions are 100 KPa and 25° C. Calculate the Mach number and stagnation properties verify the values with table values..***

*[ April'96 Bharathidasan Univ and Apr'97 Bharathiyar Univ]*

***Given :***

Air velocity, $c = 150$ m/s

Static pressure, $p = 100$ kpa $= 1$ bar $= 1 \times 10^5$ N/m$^2$

Static temperature, $T = 25°\text{C} + 273 = 298$ K

***To find :***

1. Mach number, $M$
2. Stagnation temperature, $T_0$
3. Stagnation pressure, $p_0$

***Solution :***

$$\text{Sound velocity, } a = \sqrt{\gamma RT}$$

$$= \sqrt{1.4 \times 287 \times 298} \qquad [\because \text{ For air } \gamma = 1.4, \; R = 287 \text{ J/kg K}]$$

$$\boxed{a = 346.02 \text{ m/s}}$$

$$\text{Mach number, } M = \frac{c}{a}$$

$$= \frac{150}{346.02}$$

$$\boxed{M = 0.434}$$

Stagnation temperature – Mach number relation,

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

$$\Rightarrow \quad \frac{T_0}{298} = 1 + \frac{1.4 - 1}{2} (0.434)^2$$

$$\Rightarrow \quad \frac{T_0}{298} = 1.038$$

$$\Rightarrow \quad T_0 = 298 \times 1.038 = 309.32 \text{ K}$$

$$\boxed{\text{Stagnation temperature, } T_0 = 309.32 \text{ K}}$$

Stagnation temperature – stagnation pressure relation

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\Rightarrow \quad p_0 = p \times \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\Rightarrow \quad = 1 \times 10^5 \times \left(\frac{309.32}{298}\right)^{\frac{1.4}{1.4 - 1}}$$

$$\boxed{\text{Stagnation pressure, } p_0 = 1.13 \times 10^5 \text{ N/m}^2}$$

**To verify $T_0$, $P_0$ value by using gas tables**

Refer Isentropic flow table for $\gamma = 1.4$ and $M = 0.434 \approx 0.43$

$$\frac{T}{T_0} = 0.964$$

$$\frac{p}{p_0} = 0.881$$

*[From gas tables (S.M. Yahya, Fifth edition) page no.29]*

$$\Rightarrow \quad \frac{T}{T_0} = 0.964$$

$$\Rightarrow \quad T_0 = \frac{T}{0.964} = \frac{298}{0.964} = 309.13 \text{ K}$$

$$\boxed{\text{Stagnation temperature, } T_0 = 309.13 \text{ K}}$$

$$\frac{p}{p_0} = 0.881$$

$$\Rightarrow \quad p_0 = \frac{p}{0.881} = \frac{1}{0.881} = 1.13 \text{ bar}$$

$[\because 1 \text{ bar} = 10^5 \text{ N/m}^2]$

$$\boxed{p_0 = 1.13 \times 10^5 \text{ N/m}^2}$$

$$\boxed{\text{Stagnation temperature, } p_0 = 1.13 \times 10^5 \text{ N/m}^2}$$

***Result***

1. $M = 0.434$

2. $T_0 = 309.13 \text{ K}$

3. $p_0 = 1.13 \times 10^5 \text{ N/m}^2$

**8** ***The pressure, temperature and Mach number at the entry of a flow passage are 2.45 bar, 26.5°C and 1.4 respectively. If the exit Mach number is 2.5, determine the following for adiabatic flow of a perfect gas ($\gamma = 1.3$, $R = 0.469$ kJ/kg K).***

1. ***Stagnation temperature***
2. ***Temperature and velocity of gas at exit***
3. ***The flow rate per square metre of the inlet cross section.***

*[Madurai Kamaraj Univ, Apr'96, Manonmanium Sundaranar Univ, Apr'96, Madras Univ, Apr'2000]*

**Given**

$p_1 = 2.45 \text{ bar} = 2.45 \times 10^5 \text{ N/m}^2$

$T_1 = 26.5°C + 273 = 299.5 \text{ K}$

$M_1 = 1.4$

$M_2 = 2.5$

$\gamma = 1.3$

$R = 0.469 \text{ kJ/kgK} = 469 \text{ J/kg K}.$

***To find :***

1. Stagnation temperature, $T_0$
2. Temperature and velocity of gas at exit, $T_2$, $c_2$
3. The flow rate per square metre of the inlet cross section, $\frac{m}{A_1}$

***Solution :***

**At inlet**

Refer Isentropic flow table for $\gamma = 1.3$ and $M_1 = 1.4$.

$$\frac{T_1}{T_0} = 0.773$$ *[ From gas tables page no.23]*

$$\Rightarrow \quad T_{01} = \frac{T_1}{0.773} = \frac{299.5}{0.773} = 387.5 \text{ K}$$

$$\boxed{\text{Stagnation temperature, } T_{01} = 387.5 \text{ K}}$$

For isentropic flow, stagnation temperature remains constant

$$\Rightarrow \quad \boxed{T_0 = T_{01} = T_{02} = 387.5 \text{ K}}$$

**At outlet**

Refer Isentropic flow table for $\gamma = 1.3$ and $M_2 = 2.5$.

$$\frac{T_2}{T_{02}} = 0.516$$ *[From gas tables page no.25]*

$$\Rightarrow \quad T_2 = 0.516 \times T_{02}$$

$$= 0.516 \times 387.5$$

$$\boxed{\text{Exit temperature, } T_2 = 199.95 \text{ K}}$$

Exit Mach number, $M_2 = \frac{c_2}{a_2}$

$$\Rightarrow \qquad M_2 = \frac{c_2}{\sqrt{\gamma R T_2}} \qquad [\because a = \sqrt{\gamma RT}]$$

$$2.5 = \frac{c_2}{\sqrt{1.3 \times 469 \times 199.95}}$$

$$\boxed{\text{Exit velocity} \Rightarrow c_2 = 872.89 \text{ m/s}}$$

From continuity equation, we know that mass flow rate remains constant

$$\Rightarrow \qquad m = \rho A c = \rho_1 A_1 c_1 = \rho_2 A_2 c_2$$

$$\Rightarrow \qquad m = \rho_1 A_1 c_1$$

$$\frac{m}{A_1} = \rho_1 \times c_1$$

$$= \frac{p_1}{RT_1} \times c_1 \qquad [\because \rho = \frac{p}{RT}]$$

$$= \frac{2.45 \times 10^5}{469 \times 299.5} \times c_1$$

$$= 1.74\, c_1$$

$$= 1.74 \times M_1 \times a_1 \qquad [\because M = \frac{c}{a}]$$

$$= 1.74 \times 1.4 \times \sqrt{\gamma RT_1} \qquad [\because a = \sqrt{\gamma RT}]$$

$$= 1.74 \times 1.4 \times \sqrt{1.3 \times 469 \times 299.5}$$

$$\frac{m}{A_1} = 1040.96 \text{ kg/s–m}^2$$

***Result***

1. $T_0 = 387.5$ K
2. $T_2 = 199.95$ K

$c_2 = 872.89$ m/s

3. $\frac{m}{A_1} = 1040.96$ kg/s–m$^2$

$$h_0 = \frac{a^2}{\gamma - 1} + \tfrac{1}{2} c^2 = \tfrac{1}{2} c_{max}^2 \qquad \textit{[From Equation no (1.16)]}$$

$$\Rightarrow \quad \frac{a^2}{\gamma - 1} + \tfrac{1}{2} c^2 = \tfrac{1}{2} c_{max}^2$$

$$\Rightarrow \quad \frac{\gamma RT}{1.4 - 1} + \tfrac{1}{2} c^2 = \tfrac{1}{2} c_{max}^2 \qquad [\because a = \sqrt{\gamma RT}]$$

$$\Rightarrow \quad \frac{1.4 \times 287 \times 573}{1.4 - 1} + \tfrac{1}{2} (120)^2 = \tfrac{1}{2} c_{max}^2$$

$$\Rightarrow \quad c_{max}^2 = 1.165 \times 10^6$$

$$\Rightarrow \quad \boxed{c_{max} = 1079.60 \text{ m/s}}$$

***Define closed and open system.*** *[May 2005 Anna Univ]*

A closed system, does not permit any mass transfer, only energy nsfer takes place.

In open system both the mass and energy transfer takes place.

***What is the difference between intensive and extensive property.*** *[May 2005 Anna Univ]*

**Intensive Properties :**

These properties are independent on the mass of the system.

*Example :* Pressure, Temperature, etc.

**Extensive Properties :**

These properties are dependent upon the mass of the system.

*Example :* Total volume, Total energy, etc.

***Distinguish between Mach wave and normal shock.*** *[Dec 2005 Anna Univ]*

**Mach wave :** The lines at which the pressure difference is concentrated d which generate the cone are called Mach lines or Mach waves.

**Normal Shock :** A shock wave is nothing but a steep finite pressure ave. When the shock wave is right angle to the flow, it is called normal ock.

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# 2

# ISENTROPIC FLOW THROUGH VARIABLE AREA DUCTS

## 2.1 INTRODUCTION

Steady flow is that type of flow, in which the fluid characteristics like velocity, pressure and density do not change with time.

One dimensional flow is that type of flow, in which the flow parameters like velocity, pressure and density are the function of time and one space co-ordinate (x) only.

The one dimensional, steady flow treatment of isentropic flow in variable area passage is discussed in this Chapter.

## 2.2 ADIABATIC PROCESS

In an adiabatic process there is no heat transfer from the fluid to the surroundings or from the surroundings to the fluid.

i.e., $Q = 0$

## 2.3 ISENTROPIC PROCESS

In an isentropic process entropy remains constant and it is reversible. During this process there is no heat transfer from the fluid to the surroundings or from the surroundings to the fluid. Therefore, an isentropic process can be stated as reversible adiabatic process.

Entropy, $S$ = constant

Heat transfer $Q = 0$.

## 2.4 COMPARISON OF ISENTROPIC AND ADIABATIC PROCESSES

An isentropic and adiabatic expansion of a perfect gas between the two states 1 and 2 is shown in fig.2.1

*Isentropic process*

Let, Initial stagnation temperature = $T_{0_1}$

Final stagnation temperature = $T_{0_2} = T_{0_1}$

Initial static temperature = $T_1$

Final static temperature = $T_{2s}$

Initial stagnation pressure = $p_{01s}$

Final stagnation pressure = $p_{02s}$

Initial kinetic energy = $\frac{1}{2} c_1^2$

Final kinetic energy = $\frac{1}{2} c_{2s}^2$

*Adiabatic Process*

Let, Initial stagnation temperature = $T_{0_1}$

Final stagnation temperature = $T_{0_2}$

Initial static temperature = $T_1$

Final static temperature = $T_{2a}$

Initial stagnation pressure = $p_{01a}$

Final stagnation pressure = $p_{02a}$

Initial kinetic energy = $\frac{1}{2} c_1^2$

Final kinetic energy = $\frac{1}{2} c_{2a}^2$

From T– S diagram we came to know that there is an increase in entropy (ds) in adiabatic process but in an isentropic process, entropy remains constant.

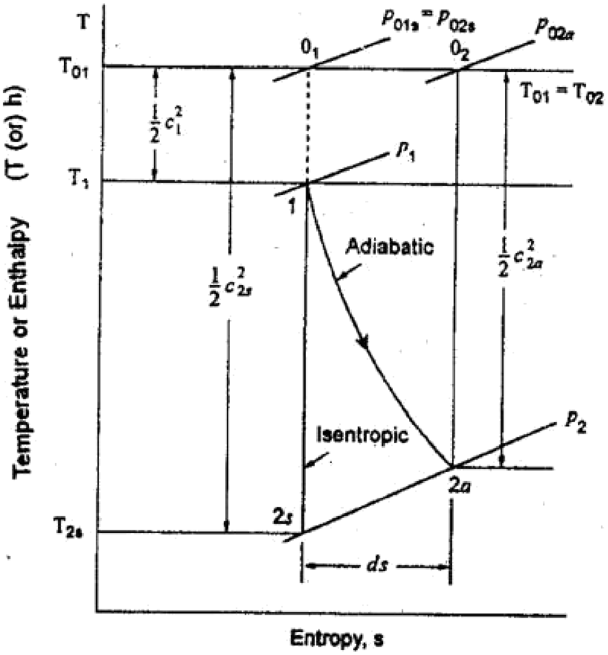


*Fig. 2.1 Temperature-entropy diagram for expansion processes*

In adiabatic process, the final stagnation pressure $p_{02a}$ is lower than its initial value $p_{01s}$. The final kinetic energy $\frac{1}{2} c_{2a}^2$ is also lower than its corresponding isentropic value $\frac{1}{2} c_1^2$.

The stagnation temperature remains constant for both isentropic and adiabatic processes.

From perfect gas equation,

We know that

$$pv = RT$$

For isentropic process at point 2s

$$p_2 v_{2s} = R\,T_{2s}$$

$$v_{2s} = \frac{RT_{2s}}{p_2}$$

For adiabtic process

$$v_{2a} = \frac{RT_{2a}}{p_2}$$

But we know that,

$$T_{2a} > T_{2s}$$

So,

$$v_{2a} > v_{2s}$$

This is shown in p – v diagram.

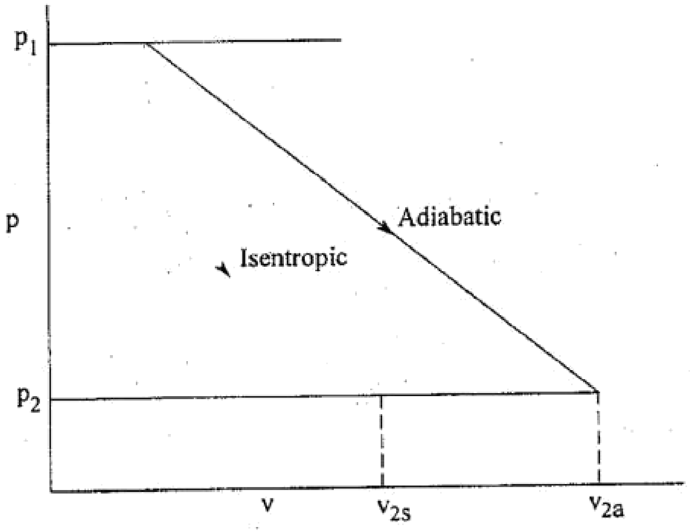


*Fig. 2.2 p-v diagram*

## 2.5 MACH NUMBER VARIATION FOR VARIABLE AREA

From stagnation enthalpy equation, we know that

$$h_0 = h + \frac{c^2}{2}$$

$$\Rightarrow \qquad h + \frac{c^2}{2} = h_0 = \text{constant}$$

Differentiating this equation

$$dh + \frac{2cdc}{2} = 0$$

$$\boxed{dh + cdc = 0} \qquad \text{-----(2.1)}$$

We know that for isentropic flow,

$$dh = \frac{dp}{\rho}$$

$$(2.1) \Rightarrow \quad \frac{dp}{\rho} + cdc = 0$$

$$\frac{dp}{\rho} = -c\,dc$$

$$\boxed{dp = -\rho c\,dc} \qquad \text{-----(2.2)}$$

Mass flow rate, $m = \rho Ac = \text{constant}$

$$\Rightarrow \qquad \rho Ac = \text{constant}$$

Taking log on both sides

$$ln\,\rho + ln\,A + ln\,c = 0$$

Differentiating

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dc}{c} = 0$$

$$\Rightarrow \qquad \frac{dc}{c} = -\left[\frac{d\rho}{\rho} + \frac{dA}{A}\right]$$

$$dc = -c\left[\frac{d\rho}{\rho} + \frac{dA}{A}\right]$$

Substituting dc value in Equation (2.2)

$$(2.2) \Rightarrow \quad dp = -\rho c\left[-c\left[\frac{d\rho}{\rho} + \frac{dA}{A}\right]\right]$$

$$dp = \rho c^2\left[\frac{d\rho}{\rho} + \frac{dA}{A}\right]$$

$$\Rightarrow \quad \frac{d\rho}{\rho} + \frac{dA}{A} = \frac{dp}{\rho c^2}$$

$$\Rightarrow \quad \frac{dA}{A} = \frac{dp}{\rho c^2} - \frac{d\rho}{\rho}$$

$$= \frac{dp}{\rho c^2}\left[1 - \frac{\rho c^2}{dp} \times \frac{d\rho}{\rho}\right]$$

$$\frac{dA}{A} = \frac{dp}{\rho c^2}\left[1 - c^2 \times \frac{d\rho}{dP}\right]$$

For isentropic process

$$\frac{dp}{d\rho} = a^2$$

$$\Rightarrow \quad \frac{dA}{A} = \frac{dp}{\rho c^2}\left[1 - \frac{c^2}{a^2}\right]$$

Mach number, $M = \frac{c}{a}$

$$\Rightarrow \quad \boxed{\frac{dA}{A} = \frac{dp}{\rho c^2}\,[1 - M^2]} \qquad \text{-----}(2.4)$$

This equation is considered for increasing and decreasing area passage for various values of Mach number.

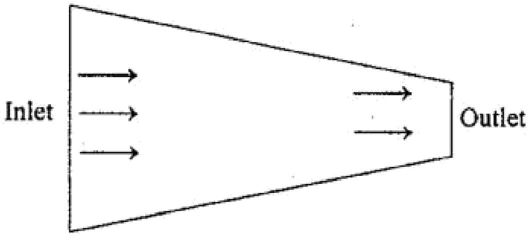
## 2.6 NOZZLE

Nozzle is a duct of varying cross sectional area in which the velocity of fluid increases with the corresponding drop in pressure.

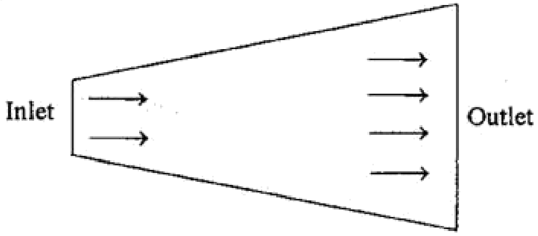
## 2.7 TYPES OF NOZZLES

Following three types of nozzles are important from the subject point of view.

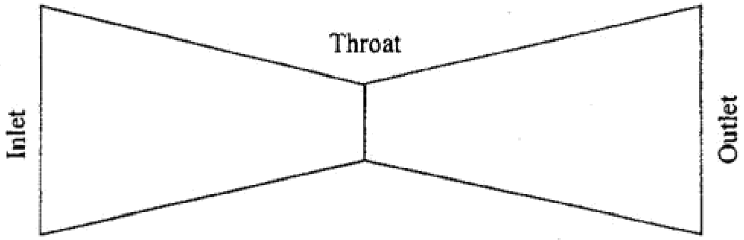
**1. Convergent nozzle :** In convergent nozzle, the cross sectional area decreases from the inlet section to the outlet section.



**2. Divergent nozzle :** In divergent nozzle, the cross sectional area increases from the inlet section to the outlet section.



**3. Convergent – Divergent nozzle (C.D. nozzle) :** In convergent –divergent nozzle the cross sectional area first decreases from the inlet section to throat and then increases from its throat to outlet section.



## 2.8 EXPANSION IN NOZZLES

Gases and vapours are expanded in nozzles. In nozzle velocity of fluid increases with the corresponding drop in pressure. So a pressure drop (dP) in Equation (2.4) is always negative.

$$\Rightarrow \qquad \frac{dA}{A} = \frac{-dp}{\rho c^2}\,[1 - M^2]$$

The following three possible conditions are considered.

*Case (i)*

If $M < 1$, $\frac{dA}{A}$ becomes negative. It means that the area of nozzle decreases from $M = 0$ to $M = 1$. This is convergent type nozzle and flow is subsonic ($\because M < 1$).

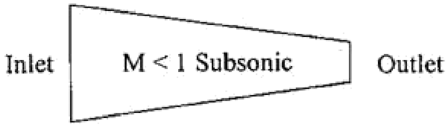


*Fig 2.3 Convergent type nozzle*

*Case (ii)*

If $M = 1, \frac{dA}{A} = 0$. It means that there is no change in area of cross section. This section is referred to as the throat of the passage and the flow is sonic ($\because M = 1$).

*Case (iii)*

If $M > 1$, $\frac{dA}{A}$ becomes positive. It means that the area of nozzle increases. This is divergent type nozzle and the flow is supersonic ($\because M > 1$).

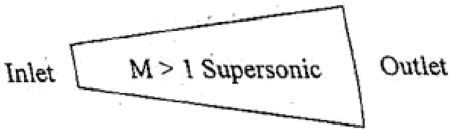


*Fig 2.4 Divergent type nozle*

The above results can be schematically shown in fig. 2.5

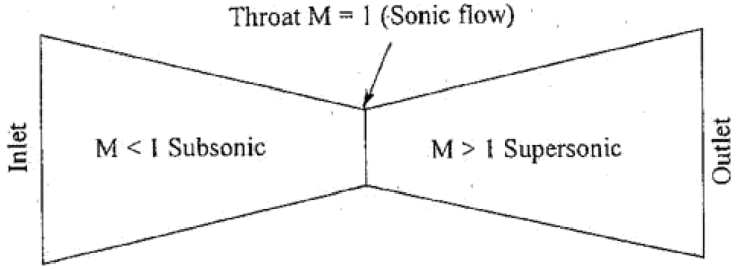


*Fig 2.5 Isentropic flow of a gas in a nozzle.*

## 2.9 DIFFUSER

Diffuser is a device which is used to increase the pressure and decrease the velocity of fluids. So a pressure drop (dp) in equation (2.4) is always positive.

$$\Rightarrow \qquad \frac{dA}{A} = \frac{dp}{\rho c^2}\,[1 - M^2]$$

*Case (i)*

If $M < 1$, $\frac{dA}{A}$ becomes positive. It means that the area of diffuser increases and the flow is subsonic ($\because M < 1$)

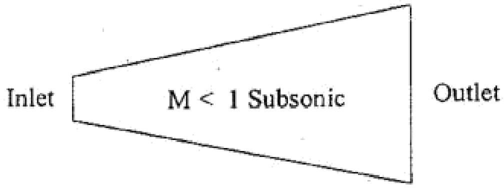


*Fig 2.6 Divergent type diffuser*

*Case (ii)*

If $M = 1$, $\frac{dA}{A} = 0$. It means that there is no change in area of cross section. This section is referred to as the throat of the passage and the flow is sonic ($\because M = 1$).

*Case (iii)*

If $M > 1$, $\frac{dA}{A}$ becomes negative. It means that the area of diffuser decreases and the flow is supersonic ($\because$ $M > 1$).

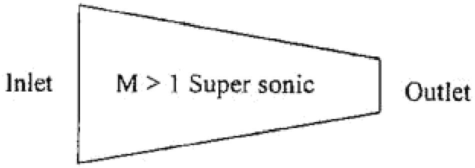


*Fig. 2.7 Convergent type diffuser*

The above results can be schematically shown as in fig 2.8

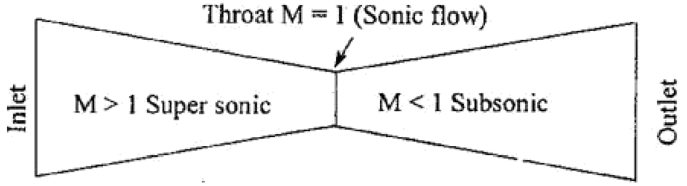


*Fig. 2.8 Isentropic flow of a gas in a diffuser*

## 2.10 CRITICAL TEMPERATURE (T*), CRITICAL PRESSURE (P*) AND CRITICAL DENSITY (ρ*)

Stagnation temperature–Mach number relation

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 \quad \text{[From chapter 1 – Equation no(1.7)]}$$

Stagnation pressure – Mach number relation

$$\frac{p_0}{p} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{\gamma}{\gamma - 1}} \quad \text{[From chapter 1 – Equation no(1.8)]}$$

Stagnation density – Mach number relation

$$\frac{\rho_0}{\rho} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{1}{\gamma - 1}} \quad \text{[From chapter 1 – Equation no(1.9)]}$$

At critical (*) state, M = 1. So the above equations becomes

$$\Rightarrow \frac{T_0}{T^*} = 1 + \frac{\gamma - 1}{2} \qquad \text{[ Put T = T*]}$$

$$= \frac{2 + \gamma - 1}{2}$$

$$\boxed{\frac{T_0}{T^*} = \frac{\gamma + 1}{2}} \qquad ----(2.5)$$

$$\Rightarrow \frac{p_0}{p^*} = \left[1 + \frac{\gamma - 1}{2}\right]^{\frac{\gamma}{\gamma - 1}} \qquad \text{[Put p = p*]}$$

$$\boxed{\frac{p_0}{p^*} = \left[\frac{\gamma + 1}{2}\right]^{\frac{\gamma}{\gamma - 1}}} \qquad ----(2.6)$$

$$\Rightarrow \frac{\rho_0}{\rho^*} = \left[1 + \frac{\gamma - 1}{2}\right]^{\frac{1}{\gamma - 1}}$$

$$\boxed{\frac{\rho_0}{\rho^*} = \left[\frac{\gamma + 1}{2}\right]^{\frac{1}{\gamma - 1}}} \qquad ----(2.7)$$

To find the relationship between static and critical state (T, T*, P, P*, ρ, ρ*).

$$\frac{T^*}{T} = \frac{T^*}{T_0} \times \frac{T_0}{T}$$

$$= \frac{2}{\gamma + 1} \times \left[1 + \frac{\gamma - 1}{2} M^2\right]$$

$$\boxed{\frac{T^*}{T} = \frac{2}{\gamma + 1} + \frac{\gamma - 1}{\gamma + 1} M^2} \qquad ----(2.8)$$

We know that

$$\frac{p^*}{p} = \left(\frac{T^*}{T}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\boxed{\frac{p^*}{p} = \left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{\gamma}{\gamma-1}}} \qquad ----(2.9)$$

We know that

$$\frac{\rho^*}{\rho} = \left(\frac{T^*}{T}\right)^{\frac{1}{\gamma-1}}$$

$$\boxed{\frac{\rho^*}{\rho} = \left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{1}{\gamma-1}}} \qquad ----(2.10)$$

## 2.11 AREA RATIO AS A FUNCTION OF MACH NUMBER

Like temperature, pressure and density ratios, area ratio is also a useful quantity.

We know that,

Mass flow rate, $m = \rho A c = \rho^* A^* c^*$

$$\Rightarrow \boxed{\frac{A}{A^*} = \frac{\rho^*}{\rho} \times \frac{c^*}{c}} \qquad ----(2.11)$$

$$\left.\begin{array}{l}\text{Characteristic}\\ \text{Mach number}\end{array}\right\} M^{*2} = \frac{\left(\frac{\gamma+1}{2}\right)M^2}{1 + \frac{\gamma-1}{2}M^2}$$

[From chapter 1 Equation no.(1.28)]

$$\Rightarrow \qquad M^* = \left[\frac{\left(\frac{\gamma+1}{2}\right)M^2}{1 + \frac{\gamma-1}{2}M^2}\right]^{\frac{1}{2}}$$

$$\frac{c}{a^*} = \left[\frac{\left(\frac{\gamma+1}{2}\right)M^2}{1 + \frac{\gamma-1}{2}M^2}\right]^{\frac{1}{2}} \qquad \left[\because M^* = \frac{c}{a^*}\right]$$

$$\Rightarrow \qquad \frac{c}{c^*} = \left[\frac{\left(\frac{\gamma+1}{2}\right)M^2}{1 + \frac{\gamma-1}{2}M^2}\right]^{\frac{1}{2}} \qquad [\because c^* = a^*]$$

$$\Rightarrow \qquad \frac{c^*}{c} = \frac{1}{\left[\frac{\left(\frac{\gamma+1}{2}\right)M^2}{1 + \frac{\gamma-1}{2}M^2}\right]^{\frac{1}{2}}}$$

$$= \left[\frac{1 + \frac{\gamma-1}{2}M^2}{\left(\frac{\gamma+1}{2}\right)M^2}\right]^{\frac{1}{2}}$$

Multiply by 2 on R.H.S

$$\Rightarrow \quad \frac{c^*}{c} = \left[\frac{2 + (\gamma-1)\,M^2}{(\gamma+1)\,M^2}\right]^{\frac{1}{2}}$$

$$= \left[\frac{2}{(\gamma+1)\,M^2} + \frac{\gamma-1}{\gamma+1}\right]^{\frac{1}{2}}$$

Taking $^1/_M$ out

$$\boxed{\frac{c^*}{c} = \frac{1}{M}\left[\frac{2}{(\gamma+1)} + \left(\frac{\gamma-1}{\gamma+1}\right)M^2\right]^{\frac{1}{2}}} \qquad ----(2.12)$$

Substituting $\frac{c^*}{c}$ and $\frac{\rho^*}{\rho}$ values in equation (2.11)

$$(2.11) \Rightarrow \frac{A}{A^*} = \left[\frac{2}{(\gamma+1)} + \frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{1}{\gamma-1}} \times \frac{1}{M}\left[\frac{2}{(\gamma+1)} + \frac{\gamma-1}{\gamma+1}M^2\right]^{1/2}$$

[From equation (2.10) and (2.12)]

$$\Rightarrow \frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{(\gamma+1)} + \frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{1}{\gamma-1}+1/2}$$

$$= \frac{1}{M}\left[\frac{2}{(\gamma+1)} + \frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{2+\gamma-1}{2(\gamma-1)}}$$

$$\Rightarrow \boxed{\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{(\gamma+1)} + \frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}} \quad ----(2.13)$$

For various values of Mach number and area ratio is shown in fig.2.9

We know that

$$\frac{p_0}{p} = \left[1 + \frac{\gamma-1}{2}M^2\right]^{\frac{\gamma}{\gamma-1}} \text{ [From chapter 1– Equation no.1.8]}$$

$$\Rightarrow \frac{A}{A^*} \times \frac{p}{p_0} = \frac{1}{M}\left[\frac{2}{(\gamma+1)} + \frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}} \times \frac{1}{\left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{\gamma}{\gamma-1}}}$$

$$\Rightarrow \frac{A}{A^*} \times \frac{p}{p_0} = \frac{\frac{1}{M}\left[\frac{2}{(\gamma+1)} + \frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}}{\left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{\gamma}{\gamma-1}}}$$

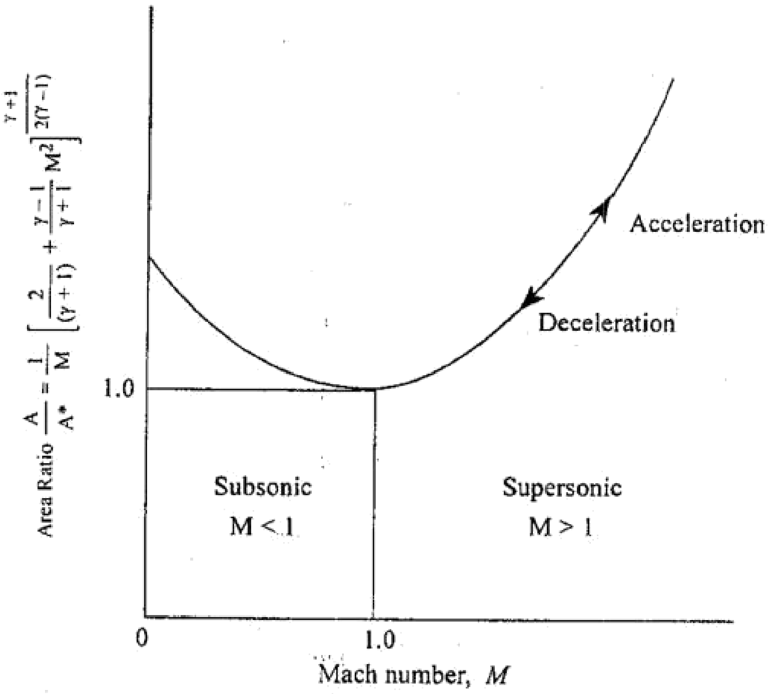


*Fig. 2.9 Variation of area ratio with Mach number*

$$= \frac{\frac{1}{M}\left(\frac{2}{(\gamma+1)}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}}{\left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{\gamma}{\gamma-1}}}$$

$$= \frac{1}{M}\left(\frac{2}{(\gamma+1)}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \times \frac{\left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}}{\left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{\gamma}{\gamma-1}}}$$

$$= \frac{1}{M}\left(\frac{2}{(\gamma+1)}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \times \left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\times \left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{-\gamma}{\gamma-1}}$$

$$= \frac{1}{M}\left(\frac{2}{(\gamma+1)}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \times \left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}-\frac{\gamma}{\gamma-1}}$$

$$= \frac{1}{M}\left(\frac{2}{(\gamma+1)}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \times \left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{1}{\gamma-1}\left[\frac{\gamma+1}{2}-\frac{\gamma}{1}\right]}$$

$$= \frac{1}{M}\left(\frac{2}{(\gamma+1)}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \times \left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{1}{\gamma-1}\left[\frac{\gamma+1-2\gamma}{2}\right]}$$

$$= \frac{1}{M}\left(\frac{2}{(\gamma+1)}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \times \left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{1}{\gamma-1}\left[\frac{1-\gamma}{2}\right]}$$

$$= \frac{1}{M}\left(\frac{2}{(\gamma+1)}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \times \left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{-1/2}$$

$$= \frac{\frac{1}{M}\left(\frac{2}{(\gamma+1)}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{\left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{1/2}}$$

$$\Rightarrow \boxed{\frac{A}{A*}\times\frac{p}{p_0} = \frac{\frac{1}{M}\left(\frac{2}{(\gamma+1)}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{\left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{1/2}}} \qquad \text{----} (2.14)$$

## 2.12 IMPULSE FUNCTION (OR) WALL FORCE FUNCTION

The sum of pressure force (pA) and Impulse force ($\rho Ac^2$) gives Impulse function (F).

Impulse function, $F = pA + \rho Ac^2$ ---- (2.15)

We know that for perfect gas

$$pv = RT$$

$$\Rightarrow \quad \frac{p}{\rho} = RT \qquad [\because \rho = \frac{1}{v}]$$

$$\Rightarrow \quad \rho = \frac{p}{RT}$$

Multiply by $c^2$ on both sides

$$\Rightarrow \quad \rho c^2 = \frac{pc^2}{RT}$$

$$\Rightarrow \quad \rho c^2 = \frac{\gamma pc^2}{\gamma RT}$$

$$= \frac{\gamma pc^2}{a^2} \qquad [\because a = \sqrt{\gamma RT}]$$

$$\boxed{\rho c^2 = \gamma pM^2} \qquad [\because M = \frac{c}{a}]$$

Substitute $\rho c^2$ value in Equation (2.15)

$$(2.15) \Rightarrow \quad F = pA + A \times \gamma pM^2$$

$$F = pA\,[1+\gamma M^2] \qquad \text{----} (2.16)$$

One dimensional flow through a symmetrical straight duct is shown in fig.2.10 The thrust or wall force (F) experienced by the duct also shown in fig.2.10.

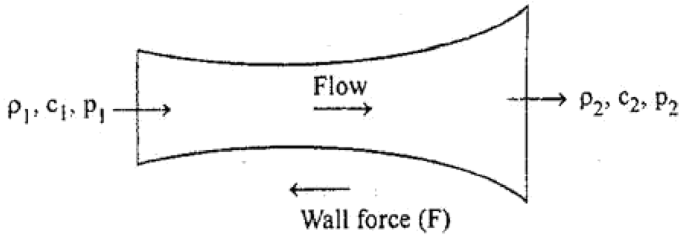


***Fig.2.10 Impulse function***

At sonic condition M = 1

At M = 1, F = F*

$$F^* = p^*A^*\,[1+\gamma] \qquad ----(2.17)$$

$$\Rightarrow \frac{F}{F^*} = \frac{pA\,[1+\gamma M^2]}{p^*A^*\,[1+\gamma]}$$

$$\Rightarrow \frac{F}{F^*} = \frac{p}{p^*} \times \frac{A}{A^*} \times \left[\frac{1+\gamma M^2}{1+\gamma}\right] \qquad ----(2.18)$$

We know that

$$\frac{p}{p^*} = \frac{1}{\left[\frac{2}{\gamma+1}+\frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{\gamma}{\gamma-1}}} \qquad \text{[From equation no.(2.9)]}$$

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}+\frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}} \qquad \text{[From equation no.(2.13)]}$$

Substitute $\frac{p}{p^*}$ and $\frac{A}{A^*}$ values in Equation (2.18)

$$\frac{F}{F^*} = \frac{1}{\left[\frac{2}{\gamma+1}+\frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{\gamma}{\gamma-1}}} \times \frac{1}{M}\left[\frac{2}{\gamma+1}+\frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}} \times \left[\frac{1+\gamma M^2}{1+\gamma}\right]$$

$$= \left[\frac{2}{\gamma+1}+\frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{-\gamma}{\gamma-1}} \times \frac{1}{M}\left[\frac{2}{\gamma+1}+\frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}} \times \left[\frac{1+\gamma M^2}{1+\gamma}\right]$$

$$= \frac{1}{M}\left[\frac{2}{\gamma+1}+\frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)} - \frac{\gamma}{\gamma-1}} \times \left[\frac{1+\gamma M^2}{1+\gamma}\right]$$

$$= \frac{1}{M}\left[\frac{2}{\gamma+1}+\frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{1}{\gamma-1}\left[\frac{\gamma+1}{2}-\frac{\gamma}{1}\right]} \times \left[\frac{1+\gamma M^2}{1+\gamma}\right]$$

$$= \frac{1}{M}\left[\frac{2}{\gamma+1}+\frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{1}{\gamma-1}\left[\frac{\gamma+1-2\gamma}{2}\right]} \times \left[\frac{1+\gamma M^2}{1+\gamma}\right]$$

$$= \frac{1}{M}\left[\frac{2}{\gamma+1}+\frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{1}{\gamma-1}\left[\frac{-(\gamma-1)}{2}\right]} \times \left[\frac{1+\gamma M^2}{1+\gamma}\right]$$

$$= \frac{1}{M}\left[\frac{2}{\gamma+1}+\frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{-1}{2}} \left[\frac{1+\gamma M^2}{1+\gamma}\right]$$

$$= \frac{1+\gamma M^2}{M\,(1+\gamma)\left[\frac{2}{\gamma+1}+\frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{1}{2}}}$$

$$\frac{F}{F^*} = \frac{1+\gamma M^2}{M(1+\gamma)\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{1}{2}}}$$

$$= \frac{1+\gamma M^2}{M\left[\frac{2(1+\gamma)^2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{1}{2}}}$$

$$= \frac{1+\gamma M^2}{M\left[2(1+\gamma)\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{1}{2}}}$$

$$\boxed{\frac{F}{F^*} = \frac{1+\gamma M^2}{M\sqrt{2(1+\gamma)\left(1+\frac{\gamma-1}{2}M^2\right)}}} \qquad ----(2.19)$$

Another non dimensional expressin for the impulse function may e obtianed as follows

$$\Rightarrow \quad \frac{F}{p_0 A^*} = \frac{p A\,[1+\gamma M^2]}{p_0 A^*} \qquad [\because F = pA(1+\gamma M^2)]$$

$$\frac{F}{p_0 A^*} = \frac{p}{p_0} \times \frac{A}{A^*}\,[1+\gamma M^2] \qquad ----(2.20)$$

We know that

$$\frac{p_0}{p} = \left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{\gamma}{\gamma-1}}$$

[From Chapter 1
Equation no.(1.8)]

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}+\frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}} \qquad \text{[From equation no.(2.13)]}$$

Substitute $\frac{p_0}{p^*}$ and $\frac{A}{A^*}$ values in Equation (2.20)

$$\frac{F}{p_0 A^*} = \frac{1}{\left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{\gamma}{\gamma-1}}} \times \frac{1}{M}\left[\frac{2}{\gamma+1}+\frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}} \times [1+\gamma M^2]$$

$$= \frac{1}{M}\left[\frac{2}{\gamma+1}+\frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}} \times \left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{-\gamma}{\gamma-1}} \times [1+\gamma M^2]$$

$$= \frac{1}{M}\left[\frac{2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}\left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}} \times \left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{-\gamma}{\gamma-1}} \times [1+\gamma M^2]$$

$$= \frac{1}{M}\left[\frac{2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}\left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}-\frac{\gamma}{\gamma-1}} \times [1+\gamma M^2]$$

$$= \frac{1}{M}\left[\frac{2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}\left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{1}{\gamma-1}\left[\frac{\gamma+1}{2}-\frac{\gamma}{1}\right]} \times [1+\gamma M^2]$$

$$= \frac{1}{M}\left[\frac{2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}\left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{1}{\gamma-1}\left[\frac{\gamma+1-2\gamma}{2}\right]} \times [1+\gamma M^2]$$

$$= \frac{1}{M}\left[\frac{2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}\left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{-1}{2}} \times [1+\gamma M^2]$$

$$= \frac{1}{M}\left[\frac{2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \times \frac{1+\gamma M^2}{\sqrt{1+\frac{\gamma-1}{2}M^2}}$$

$$\boxed{\frac{F}{P_0 A^*} = \left[\frac{2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \times \frac{1+\gamma M^2}{M\sqrt{1+\frac{\gamma-1}{2}M^2}}} \quad \text{----(2.21)}$$

## 2.13 MASS FLOW RATE IN TEMS OF PRESSURE RATIO

We know that, Mass flow rate, $\dot{m} = \rho A c$

Stagnation pressure – Stagnation density relation

$$\frac{\rho_0}{\rho} = \left(\frac{p_0}{p}\right)^{\frac{1}{\gamma}}$$

$$\Rightarrow \quad \rho = \frac{\rho_0}{\left(\frac{p_0}{p}\right)^{\frac{1}{\gamma}}}$$

$$\boxed{\rho = \rho_0 \times \left(\frac{p_0}{p}\right)^{-\frac{1}{\gamma}} = \rho_0 \times \left(\frac{p}{p_0}\right)^{\frac{1}{\gamma}}} \quad \text{----(2.22)}$$

Stagnation temperature, $T_0 = T + \dfrac{c^2}{2c_p}$

$$\Rightarrow \quad (T_0 - T)\, 2c_p = c^2$$

$$\Rightarrow \quad c^2 = 2c_p\,(T_0 - T)$$

$$\Rightarrow \quad c^2 = 2\,\frac{\gamma R}{\gamma-1}\,(T_0 - T) \qquad \left[\because c_p = \frac{\gamma R}{\gamma-1}\right]$$

$$c^2 = 2\,\frac{\gamma R}{\gamma-1} \times T_0\left[1 - \frac{T}{T_0}\right]$$

$$c^2 = 2\,\frac{\gamma R}{\gamma-1} \times T_0\left[1 - \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right] \qquad \left[\because \frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$\Rightarrow \quad \boxed{c = \sqrt{2\,\frac{\gamma}{\gamma-1}\,RT_0\left[1 - \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}} \quad \text{----(2.23)}$$

Substitute ρ and c values in mass flow rate equation

$$\Rightarrow \quad \dot{m} = \rho\, A\, c$$

$$m = \rho_0 \times \left(\frac{p}{p_0}\right)^{\frac{1}{\gamma}} \times A \times \sqrt{2\,\frac{\gamma}{\gamma-1}\,RT_0\left[1 - \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

We know that

$$\rho_0 = \frac{p_0}{RT_0}$$

$$\Rightarrow \dot{m} = \frac{p_0}{RT_0} \times A \times \left(\frac{p}{p_0}\right)^{\frac{1}{\gamma}} \times \sqrt{2\,\frac{\gamma}{\gamma-1}\,RT_0\left[1 - \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$= \frac{Ap_0}{\sqrt{RT_0} \times \sqrt{RT_0}} \times \left(\frac{p}{p_0}\right)^{\frac{1}{\gamma}} \times \sqrt{RT_0} \times \sqrt{2\,\frac{\gamma}{\gamma-1} \times \left[1 - \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$= \frac{Ap_0}{\sqrt{RT_0}} \times \sqrt{2 \times \frac{\gamma}{\gamma-1} \times \left(\frac{p}{p_0}\right)^{\frac{2}{\gamma}} \left[1 - \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$= \frac{Ap_0}{\sqrt{RT_0}} \times \sqrt{2\,\frac{\gamma}{\gamma-1}\left[\left(\frac{p}{p_0}\right)^{\frac{2}{\gamma}} - \left(\frac{p}{p_0}\right)^{\frac{2}{\gamma}+\frac{\gamma-1}{\gamma}}\right]}$$

$$\dot{m} = \frac{Ap_0}{\sqrt{RT_0}} \times \sqrt{2\frac{\gamma}{\gamma-1}\left[\left(\frac{p}{p_0}\right)^{\frac{2}{\gamma}} - \left(\frac{p}{p_0}\right)^{\frac{\gamma+1}{\gamma}}\right]}$$

$$\boxed{\frac{\dot{m}}{A} = \frac{p_0}{\sqrt{RT_0}} \times \sqrt{2\frac{\gamma}{\gamma-1}\left[\left(\frac{p}{p_0}\right)^{\frac{2}{\gamma}} - \left(\frac{p}{p_0}\right)^{\frac{\gamma+1}{\gamma}}\right]}} \text{ ---- (2.24)}$$

$$\frac{\dot{m}}{A} = \frac{p_0}{\sqrt{RT_0}} \times \sqrt{\gamma} \times \sqrt{\frac{2}{\gamma-1} \times \left[\left(\frac{p}{p_0}\right)^{\frac{2}{\gamma}} - \left(\frac{p}{p_0}\right)^{\frac{\gamma+1}{\gamma}}\right]}$$

$$\Rightarrow \frac{\dot{m} \times \sqrt{RT_0}}{A\,p_0\sqrt{\gamma}} = \sqrt{\frac{2}{\gamma-1} \times \left[\left(\frac{p}{p_0}\right)^{\frac{2}{\gamma}} - \left(\frac{p}{p_0}\right)^{\frac{\gamma+1}{\gamma}}\right]}$$

$$\Rightarrow \boxed{\frac{m \times \sqrt{T_0}}{A\,p_0} \times \sqrt{\frac{R}{\gamma}} = \sqrt{\frac{2}{\gamma-1} \times \left[\left(\frac{p}{p_0}\right)^{\frac{2}{\gamma}} - \left(\frac{p}{p_0}\right)^{\frac{\gamma+1}{\gamma}}\right]}} \text{ --- (2.25)}$$

This equation gives mass flow rate in terms of pressure ratio.

For maximum flow rate condition $m = m_{max}$ and $A = A^*$.

We know that,

$$\left(\frac{p}{p_0}\right)_{max} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

Substitute $\left(\frac{p}{p_0}\right)$ value in Equation (2.25)

$$\frac{m_{max} \times \sqrt{T_0}}{A^* p_0} \times \sqrt{\frac{R}{\gamma}} = \sqrt{\frac{2}{\gamma-1}\left[\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1} \times \frac{2}{\gamma}} - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1} \times \frac{\gamma+1}{\gamma}}\right]}$$

$$= \sqrt{\frac{2}{\gamma-1}\left[\left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma-1}} - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]}$$

Multiply and divide by $\frac{2}{\gamma+1}$

$$= \sqrt{\frac{2}{\gamma-1}\left[\left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma-1}} \times \frac{2}{\gamma+1} \times \frac{\gamma+1}{2} - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]}$$

$$= \sqrt{\frac{2}{\gamma-1}\left[\left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma-1}+1} \times \left(\frac{\gamma+1}{2}\right) - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]}$$

$$= \sqrt{\frac{2}{\gamma-1}\left[\left(\frac{2}{\gamma+1}\right)^{\frac{2+\gamma-1}{\gamma-1}} \times \left(\frac{\gamma+1}{2}\right) - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]}$$

$$= \sqrt{\frac{2}{\gamma-1}\left[\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \times \frac{\gamma+1}{2} - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]}$$

$$= \sqrt{\frac{2}{\gamma-1} \times \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \times \left[\frac{\gamma+1}{2} - 1\right]}$$

$$= \sqrt{\frac{2}{\gamma-1} \times \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \times \left[\frac{\gamma+1-2}{2}\right]}$$

$$= \sqrt{\frac{2}{\gamma-1} \times \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \times \left[\frac{\gamma-1}{2}\right]}$$

$$= \sqrt{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

$$= \left(\frac{2}{\gamma+1}\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}$$

$$\boxed{\frac{m_{max} \times \sqrt{T_0}}{A^* \, p_0} \times \sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}} \quad ----(2.26)$$

This is maximum mass flow rate equation in tems of $T_0$, $p_0$ and $A^*$.

Substituting

$\gamma = 1.4$

$R = 287$ J/kg K

$$\Rightarrow \frac{m_{max} \times \sqrt{T_0}}{A^* \, p_0} \times \sqrt{\frac{287}{1.4}} = \left(\frac{2}{1.4+1}\right)^{\frac{1.4+1}{2\,(1.4-1)}}$$

$$\Rightarrow \boxed{\frac{m_{max} \times \sqrt{T_0}}{A^* \, p_0} = 0.0404} \quad ----(2.27)$$

## 2.14 MASS FLOW RATE IN TERMS OF AREA RATIO

From continuity equation, we know that

$$m = \rho A c = \rho^* A^* c^*$$

Divided by A

$$\Rightarrow \quad \frac{m}{A} = \rho c = \frac{A^*}{A} \rho^* c^* \quad ----(2.28)$$

We know that

$$\rho^* = \frac{p^*}{RT^*}$$

$$c^* = a^* = \sqrt{\gamma RT^*}$$

Substitute those values in Equation (2.28)

$$(2.28) \Rightarrow \frac{m}{A} = \frac{A^*}{A} \times \frac{p^*}{RT^*} \times \sqrt{\gamma RT^*}$$

$$\Rightarrow \frac{m}{A} = \frac{A^*}{A} \times \frac{p^*}{\sqrt{T^*}} \times \sqrt{\frac{\gamma}{R}} \quad ----(2.29)$$

From equation (2.5), we know that

$$\frac{T_0}{T^*} = \frac{\gamma+1}{2}$$

$$\Rightarrow \quad \boxed{T^* = \left(\frac{2}{\gamma+1}\right) T_0}$$

From equation (2.6), we know that

$$\frac{p_0}{p^*} = \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}} \quad \Rightarrow \quad p^* = \frac{p_0}{\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}}}$$

$$\Rightarrow \quad \boxed{p^* = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \times p_0}$$

Substitute $T^*$, $p^*$ values in Equation (2.29)

$$(2.29) \Rightarrow \frac{m}{A} = \frac{A^*}{A} \times \frac{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \times p_0}{\sqrt{\left(\frac{2}{\gamma+1}\right) \times T_0}} \times \sqrt{\frac{\gamma}{R}}$$

$$\Rightarrow \frac{m}{A} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \times \sqrt{\frac{\gamma+1}{2}} \times \frac{p_0}{\sqrt{T_0}} \times \sqrt{\frac{\gamma}{R}} \times \frac{A^*}{A}$$

$$\Rightarrow \frac{m\sqrt{T_0}}{A\ p_0} \sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \times \left(\frac{\gamma+1}{2}\right)^{½} \times \frac{A^*}{A}$$

$$\Rightarrow \frac{m\sqrt{T_0}}{A\ p_0} \sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \times \left(\frac{2}{\gamma+1}\right)^{-½} \times \frac{A^*}{A}$$

$$\Rightarrow \frac{m\sqrt{T_0}}{A\ p_0} \times \sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1} - ½} \times \frac{A^*}{A}$$

$$\Rightarrow \frac{m\sqrt{T_0}}{A\ p_0} \times \sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{2\gamma-\gamma+1}{2(\gamma-1)}} \times \frac{A^*}{A}$$

$$\Rightarrow \boxed{\frac{m\sqrt{T_0}}{A\ p_0} \times \sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \times \frac{A^*}{A}} \qquad ----(2.30)$$

This equation gives the mass flow rate in terms of area ratio.

For maximum mass flow rate condition, $m = m_{max}$ and $A = A^*$.

$$(2.30) \Rightarrow \frac{m_{max}\sqrt{T_0}}{A^*\ p_0} \times \sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

This is similar to Equation no. (2.26)

## 2.15 MASS FLOW RATE IN TERMS OF MACH NUMBER

From Equation (2.13), we know that

$$\frac{A}{A^*} = \frac{1}{M}\left(\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

Substituting $\frac{A}{A^*}$ value in Equation no. (2.30)

$$\Rightarrow \frac{m\sqrt{T_0}}{A\ p_0} \times \sqrt{\frac{R}{\gamma}} = \frac{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{\frac{1}{M}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$= \frac{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \times M}{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$= \frac{M}{\left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$\Rightarrow \boxed{\frac{m\sqrt{T_0}}{A\ p_0} \times \sqrt{\frac{R}{\gamma}} = \frac{M}{\left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}} \qquad ----(2.31)$$

This equation gives the mass flow rate in terms of Mach number.

For maximum mass flow rate condition, $m = m_{max}$, $A = A^*$ and $M = 1$.

$$(2.31) \Rightarrow \frac{m_{max}\sqrt{T_0}}{A^*\, p_0} \times \sqrt{\frac{R}{\gamma}} = \frac{1}{\left(1+\frac{\gamma-1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$= \frac{1}{\left(1+\frac{\gamma-1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$= \frac{1}{\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$(2.31) \Rightarrow \frac{m_{max}\sqrt{T_0}}{A^*\, p_0} \times \sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

This is similar to Equation no. (2.26).

## 2.16 FLOW THROUGH NOZZLES

Flow through nozzles occurs in several engineering applications. Subsonic and sonic flows occurs in convergent nozzles. Supersonic flows occurs in convergent–divergent nozzles. Convergent nozzles are used in flow measuring and flow regulating devices. Convergent–divergent nozzles are used in compressors and turbine blade rows.

### 2.16.1 Convergent Nozzles

The air flow from an infinite reservoir to an exhaust chamber through a convergent nozzle is shown in fig. 2.11. The stagnation pressure ($p_0$) and the stagnation temperature ($T_0$) in the reservoir are maintained constant.

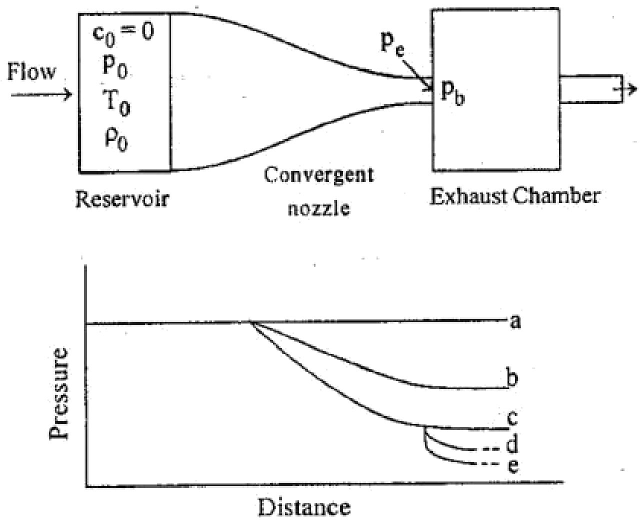


*Fig. 2.11 Isentropic flow through a convergent nozzle*

The exhaust chamber pressure i.e., back pressure ($p_b$) can be varies by means of valve.

Let $p_e$ denotes the pressure in the exit plane of the nozzle. The effects of various values of pressure ratio are shown graphically in curves a, b, c, d and e.

When $\frac{p_b}{p_0} = 1$, the pressure is constant throughout the nozzle and there is no flow which is shown in curve 'a'.

When $p_b$ value is slightly less than $p_0$ value, there will be flow with a constantly decreasing pressure through the nozzle, which is shown in curve 'b'.

When $p_b$ value is further reduced, there will be increasing flow rate nd decreasing pressure through the nozzle which is shown in curve 'c'. ere there is no qualitative change in performance.

When $p_b$ value is further reduced (curve 'd'), $\frac{p_b}{p_0}$ value is equal to he critical pressure ratio and the value of $M_e$ is equal to unity.

Further reduction in $\frac{p_b}{p_0}$ cannot produce further changes within the ozzle [curve ('e')]. The $\frac{p_e}{p_0}$ value cannot be made less than critical pressure atio unless there is a throat upstream of the exit section.

The variation of the nozzle pressure ratio $\left(\frac{p_e}{p_0}\right)$ and the mass flow arameter against the pressure ratio $\left(\frac{p_b}{p_0}\right)$ for $\gamma = 1.4$ is shown in fig 2.12.

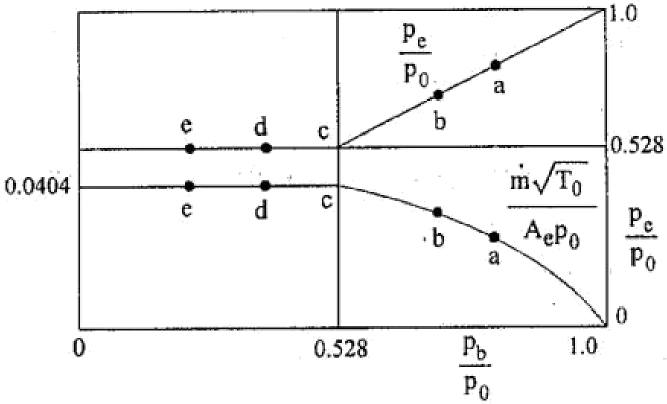


*Fig. 2.12 variation of nozzle pressure and the mass flow parameter for a convergent nozzle*

## 2.16.2 Convergent divergent nozzle

Nozzle is a duct of varying cross sectional area in which the velocity of luid increases with the corresponding drop in pressure.

When the cross section of a nozzle first decreases from the inlet section o throat and then increases from its throat to outlet section, it is called a onvergent – divergent nozzle.

Convergent–divergent nozzle is used to generate supersonic flow. It is sometimes called De Laral Nozzle because it is produce supersonic flow.

Consider the convergent–divergent nozzle shown in fig.2.13. At the throat, the flow is sonic. When $p_e = p_0$, there will be no flow through the nozzle. Let the exit pressure be reduced to a value $p_{e_1}$ which is slightly below $p_0$. This small pressure gradient will cause a flow through the nozzle at low subsonic speeds.

The static pressure will decreases continuously in the convergent portion of the nozzle, reaching a minimum at the throat as shown by the curve 1 in the figure.

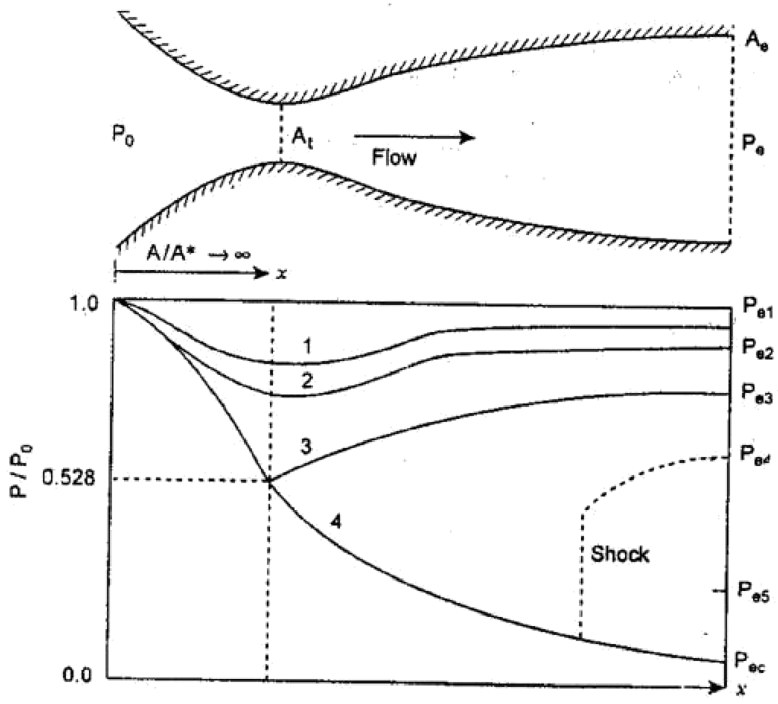


*Fig. 2.13 Flow in a convergent-divergent nozzle*

Now assuming $p_e$ is further reduced to $p_{e_2}$. So, the pressure gradient will be stronger, flow acceleration will be faster, and variation of mach number and static pressure through the duct will be larger, as shown by curve 2. Similarly if $p_e$ is reduced continuously, at some value of $p_e$, the flow will reach sonic velocity at the throat as shown by curve 3.

Now, the sonic flow at the throat will expand further in the divergent portion of the nozzle as supersonic flow if $\frac{p_e}{p_{throat}} < 1$ and will decelerate as a subsonic flow as shown by curve 3 for $\frac{p_{e_3}}{p_{throat}} > 1$.

For the cases discussed above, the mass flow through the duct increases as $p_e$ decreases.

## 2.17 FLOW THROUGH DIFFUSERS

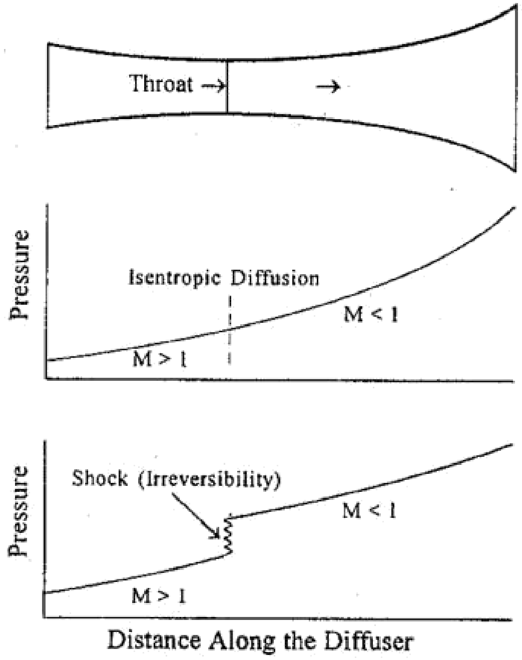


*Fig 2.14 Reversible and irreversible diffusion of supersonic flow*

Diffuser is a device which is used to increase the pressure and decrease the velocity of fluids. The reversible and irreversible diffusion of supersonic flow is shown in fig.2.14. From section 2.9, we know that convergent part of the diffuser is supersonic and divergent part of the diffuser is subsonic. In many applications diffusion occurs through a shock wave i.e., steep finite pressure wave. In an isentropic diffusion, continuous rise in static pressure takes place. This type of diffusion is not possible practically. The pressure rise across the shock wave is sudden and is governed by the Upstream supersonic mach number (M>1) and the mach number after the shock is subsonic (M<1).

## 2.18 FORMULAE USED

1. Mach number, $M = \frac{c}{a}$
2. Velocity of sound, $a = \sqrt{\gamma RT}$
3. Stagnation pressure for incompressible flow, $p_0 = p + \frac{1}{2}\rho c^2$
4. Critical velocity, $c^* = a^* = \sqrt{\gamma RT^*}$
5. Maximum velocity, $\frac{1}{2} c_{max}^2 = \frac{a_0^2}{\gamma - 1}$
6. Characteristic Mach number

$$M^{*2} = \frac{\left(\frac{\gamma+1}{2}\right) M^2}{1 + \left(\frac{\gamma-1}{2}\right) M^2}$$

7. Maximum mass flow rate

$$\frac{m_{max}}{A^*} \frac{\sqrt{T_0}}{p_0} \times \sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \qquad \textit{[From Equation (2.26)]}$$

8. Mas flow rate m, $= \rho_1 A_1 c_1 = \rho_2 A_2 c_2$

$$= \rho^* A^* c^*$$

9. Density, $\rho = \frac{p}{RT}$

10. Power required , $P = mc_p (T_0 - T_t)$

11. Stagnation pressure – stagnation temperature relation

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma - 1}}$$

12. Stagnation temperature

$$T_0 = T + \frac{c^2}{2c_p}$$

13. Maximum mass flow rate (For air)

$$\frac{m_{max}}{A^*} \times \frac{\sqrt{T_0}}{p_0} = 0.0404$$

14. $\frac{T_0}{T^*} = \frac{\gamma + 1}{2}$

15. $\frac{p_0}{p^*} = \left(\frac{\gamma + 1}{2}\right)^{\frac{\gamma}{\gamma - 1}}$

## 2.19 SOLVED PROBLEMS

**1** ***The velocity, pressure and temperature of a duct are 320 m/s, 1 bar and 295 K. Calculate the following***

1. ***Stagnation pressure***
2. ***Stagnation temperature***
3. ***Velocity of sound in dynamic condition***
4. ***Velocity of sound in stagnation condition***
5. ***Stagnation pressure assuming constant density.***

***Take*** $\gamma = 1.4$, $R = 287$ ***J/kg – K.***

***Given***

$c = 320$ m/s

$p = 1$ bar $= 1 \times 10^5$ N/m$^2$

$T = 295$ K

***To find***

1. Stagnation pressure, $p_0$
2. Stagnation temperature, $T_0$
3. Velocity of sound in dynamic condition, a
4. Velocity of sound in stagnation condition, $a_0$
5. Stagnation pressure ($p_0$) assuming constant density.

***Solution***

Velocity of sound in dynamic condition

$$a = \sqrt{\gamma RT}$$

$$= \sqrt{1.4 \times 287 \times 295}$$

$$\boxed{a = 344.28 \text{ m/s}}$$

**This problem can be solved by using gas tables**

***Solution :***

Velocity of sound, $a = \sqrt{\gamma RT}$

$$= \sqrt{1.4 \times 287 \times 295}$$

$$\boxed{a = 344.28 \text{ m/s}}$$

Mach number, $M = \frac{c}{a}$

$$= \frac{320}{344.28}$$

$$\boxed{M = 0.929}$$

Refer Isentropic flow table for $\gamma = 1.4$ and $M = 0.929 \approx 0.93$,

$$\frac{T}{T_0} = 0.853 \; ; \quad \frac{p}{p_0} = 0.572$$

*[From gas tables (S.M.Yahy Fifth edition) page no.31]*

$$\Rightarrow \quad T_0 = \frac{T}{0.853}$$

$$= \frac{295}{0.853}$$

$$\boxed{T_0 = 345.8 \text{ K}}$$

$$\Rightarrow \quad \frac{p}{p_0} = 0.572$$

$$\Rightarrow \quad p_0 = \frac{p}{0.572} = \frac{1 \times 10^5}{0.572}$$

$$\boxed{p_0 = 1.748 \times 10^5 \text{ N/m}^2}$$

Velocity of sound at stagnation condition

$$a_0 = \sqrt{\gamma RT_0}$$

$$= \sqrt{1.4 \times 287 \times 345.8}$$

$$\boxed{a_0 = 372.75 \text{ m/s}}$$

Stagnation pressure at constant density [Incompressible flow]

$$p_0 = p + \frac{1}{2}\rho c^2$$

$$= 1 \times 10^5 + \frac{1}{2} \times 1.18 \times (320)^2$$

$$\boxed{p_0 = 1.6 \times 10^5 \text{ N/m}^2} \qquad \left[\because \rho = \frac{p}{RT}\right]$$

***Result :***

1. $p_0 = 1.748 \times 10^5$ N/m$^2$
2. $T_0 = 345.8$ K
3. $a = 344.28$ m/s
4. $a_0 = 372.75$ m/s.
5. $p_0$ [ constant density] = $1.6 \times 10^5$ N/m$^2$

**8** *Air enters the nozzle from a large reservoir at 7 bar and 320°C. The exit pressure of nozzle is 0.94 bar and mass flow rate is 3500 kg/h. Calculate the following for isentropic flow*

*1. Throat area.*
*2. Throat pressure*
*3. Throat velocity*
*4. Exit area*
*5. Exit Mach number*
*6. Maximum velocity*

***Given***

$p_0 = 7 \text{ bar} = 7 \times 10^5 \text{ N/m}^2$

$T_0 = 320°C + 273 = 593 \text{ K}$

[At reservoir, air maintains stagnation conditions ie., $p_0$, $T_0$]

$p_2 = 0.94 \text{ bar} = 0.94 \times 10^5 \text{ N/m}^2$

$\dot{m} = 3500 \text{ kg/h}$

$$= \frac{3500}{3600} \text{ kg/s}$$

$$\boxed{\dot{m} = 0.972 \text{ kg/s}}$$

***To find :***

1. Throat area, $A^*$
2. Throat pressure, $p^*$
3. Throat velocity, $c^*$
4. Exit area, $A_2$
5. Exit Mach number, $M_2$
6. Maximum velocity, $c_{max}$

***Solution :***

We know that, at throat (*) section, M = 1.

Refer Isentropic flow table for $\gamma = 1.4$ and $M = 1$.

$$\frac{T^*}{T_0} = 0.834$$

$$\frac{p^*}{p_0} = 0.528$$

*[From gas tables page no.31]*

$$\Rightarrow \quad T^* = 0.834 \times T_0$$

$$= 0.834 \times 593$$

$$\boxed{T^* = 494.56 \text{ K}}$$

$$\frac{p^*}{p_0} = 0.528$$

$$\Rightarrow \quad p^* = 0.528 \times p_0$$

$$= 0.528 \times 7 \times 10^5$$

$$\boxed{p^* = 3.69 \times 10^5 \text{ N/m}^2}$$

$$c^* = a^* = \sqrt{\gamma R T^*}$$

$$= \sqrt{1.4 \times 287 \times 494.56}$$

$$\boxed{c^* = 445.77 \text{ m/s}}$$

Mass flow rate, $m = \rho^* A^* c^*$

$$\Rightarrow \quad A^* = \frac{m}{\rho^* \times c^*}$$

$$= \frac{m}{\frac{p^*}{RT^*} \times c^*} \qquad [\because \rho = \frac{p}{RT}]$$

$$= \frac{0.972}{\frac{3.69 \times 10^5}{287 \times 494.56} \times 445.77}$$

$$\boxed{A^* = 8.38 \times 10^{-4} \text{ m}^2}$$

From given data,

$$\frac{p_2}{p_0} = \frac{p_2}{p_{02}} = \frac{0.94 \times 10^5}{7 \times 10^5} = 0.134$$

Refer Isentropic flow table for $\frac{p_2}{p_{02}} = 0.134$ and $\gamma = 1.4$.

$$M_2 = 1.97$$

$$\frac{T_2}{T_{02}} = 0.563$$ *[From gas tables page no.34]*

$$\frac{A_2}{A^*} = 1.646$$

$$\Rightarrow \quad A_2 = 1.646 \times A^*$$

$$= 1.646 \times 8.38 \times 10^{-4}$$

$$\boxed{A_2 = 1.38 \times 10^{-3} \text{ m}^2}$$

Maximum velocity, $\frac{c_{max}}{c^*} = \sqrt{\frac{\gamma + 1}{\gamma - 1}}$ *[From chapter 1. Equation no.(1.24)]*

$$\Rightarrow \quad c_{max} = c^* \sqrt{\frac{\gamma + 1}{\gamma - 1}}$$

$$= 445.77 \sqrt{\frac{1.4 + 1}{1.4 - 1}}$$

$$\boxed{c_{max} = 1091.9 \text{ m/s.}}$$

***Result :***

1. $A^* = 8.38 \times 10^{-4}$ m$^2$
2. $p^* = 3.69 \times 10^5$ N/m$^2$
3. $c^* = 445.77$ m/s
4. $A_2 = 1.38 \times 10^{-3}$ m$^2$
5. $M_2 = 1.97$
6. $c_{max} = 1091.9$ m/s.

**9** ***The pressure, temperature and velocity of air at the entry of a diffuser are 0.7 bar, 345 K and 190 m/s respectively. The entry diameter of a diffuser is 15 cm and exit diameter is 35 cm. Determine the following***

1. ***Exit pressure***
2. ***Exit velocity***
3. ***Force exerted on the diffuser walls. Assuming isentropic flow and take $\gamma = 1.4$, $c_p = 1005$ J/kg K.***

***Given***

$p_1 = 0.7 \text{ bar} = 0.7 \times 10^5 \text{ N/m}^2$

$T_1 = 345 \text{ K}$

$c_1 = 190 \text{ m/s}$

$d_1 = 15 \text{ cm} = 0.15\text{m}$

$d_2 = 35 \text{ cm} = 0.35 \text{ m}$

$\gamma = 1.4$

$c_p = 1005 \text{ J/kg K}$

***To find***

1. Exit pressure, $p_2$
2. Exit velocity, $c_2$
3. Force exerted on the diffuser walls, $(F_2 - F_1)$.

***Solution***

We know that

Velocity of sound at inlet, $a_1 = \sqrt{\gamma R T_1}$

$$= \sqrt{1.4 \times 287 \times 345}$$

$$\boxed{a_1 = 372.32 \text{ m/s}}$$

Mach number at inlet, $M_1 = \dfrac{c_1}{a_1}$

$$= \frac{190}{372.32}$$

$$\boxed{M_1 = 0.510}$$

From Isentropic flow table for $\gamma = 1.4$ and $M_1 = 0.510$,

$$\frac{T_1}{T_{01}} = 0.951 \qquad \frac{p_1}{p_{01}} = 0.837$$

$$\frac{A_1}{A_1{}^*} = 1.321$$ *[From gas tables page no. 29]*

$$\frac{F_1}{F_1{}^*} = 1.190$$

$$\Rightarrow \quad T_{01} = \frac{T_1}{0.951} = \frac{345}{0.951} = 362.78 \text{ K}$$

$$\boxed{T_{01} = 362.78 \text{ K} = T_{02}}$$

[∵ For Isentropic flow, $T_{01} = T_{02}$]

$$\Rightarrow \quad p_{01} = \frac{p_1}{0.837} = \frac{0.7}{0.837} = 0.836 \text{ bar}$$

$$\boxed{p_{01} = 0.836 \text{ bar} = p_{02}}$$

[∵ For Isentropic flow, $p_{01} = p_{02}$]

$$\Rightarrow \quad \frac{A_1}{A_1{}^*} = 1.321$$

$$\Rightarrow \quad A_1{}^* = \frac{A_1}{1.321} = \frac{\frac{\pi}{4} d_1{}^2}{1.321}$$

$$= \frac{\frac{\pi}{4} (0.15)^2}{1.321}$$

$$\boxed{A_1{}^* = 0.01337 = A_2{}^*}$$

Area at exit, $A_2 = \frac{\pi}{4} d_2^2$

$$= \frac{\pi}{4} (0.35)^2$$

$$\boxed{A_2 = 0.096 \text{ m}^2}$$

$$\frac{A_2}{A_2^*} = \frac{0.096}{0.01337}$$

$$\boxed{\frac{A_2}{A_2^*} = 7.180}$$

In this problem, $d_2 > d_1 \Rightarrow A_2 > A_1$. So, this is divergent type diffuser. For divergent type diffuser mach number value is less than unity [Refer section 2.9].

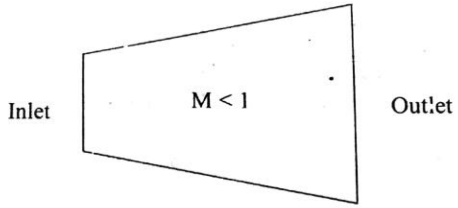


*Fig 2.15*

From Isentropic flow table, for $\frac{A_2}{A^*} = 7.1800 \approx 7.262$ and $\gamma = 1.4$.

$M_2 = 0.08$

$\frac{p_2}{p_{02}} = 0.995$ *[ From gas tables page no.28]*

$\frac{T_2}{T_{02}} = 0.998$

$\frac{F_2}{F_2^*} = 5.753$

[**Note :** For $\frac{A_2}{A^*} = 7.180$ value, we can refer gas tables page no. 28 and page no.38. But we have to take Mach number less than 1 corresponding values because it is divergent type diffuser].

$$\Rightarrow \frac{p_2}{p_{02}} = 0.995$$

$$\Rightarrow p_2 = 0.995 \times 0.836$$

$$p_2 = 0.832 \text{ bar}$$

$$\boxed{\text{Exit pressure, } p_2 = 0.832 \times 10^5 \text{ N/m}^2}$$

$$\frac{T_2}{T_{02}} = 0.998$$

$$\Rightarrow T_2 = 0.998 \times T_{02}$$

$$= 0.998 \times 362.78$$

$$\boxed{T_2 = 362.05 \text{ K}}$$

Mach number, $M_2 = \frac{c_2}{a_2}$

$$\Rightarrow c_2 = M_2 \times a_2$$

$$= M_2 \times \sqrt{\gamma R T_2} \qquad [\because a = \sqrt{\gamma RT}]$$

$$= 0.08 \times \sqrt{1.4 \times 287 \times 362.05}$$

$$\boxed{\text{Exit velocity, } c_2 = 30.51 \text{ m/s}}$$

At throat (*) section, $M = 1,\ \gamma = 1.4$

[From gas tables page no.31]

$$\frac{p^*}{p_0} = 0.528$$

$$\Rightarrow \quad p^* = 0.528 \times p_0$$

$$= 0.528 \times 0.836 \qquad [\because p_0 = p_{01} = p_{02}]$$

$$p^* = 0.441 \text{ bar}$$

$$\Rightarrow \quad \boxed{p^* = p_1^* = p_2^* = 0.441 \times 10^5 \text{ N/m}^2}$$

Force excerted on the diffuser walls is equal to the thrust of the flow

$$\tau = F_2 - F_1$$

$$= 5.753\ F_2^* - 1.190 \times F_1^*$$

$$= [5.753 - 1.190]\ F_1^* \qquad [\because F_1^* = F_2^*]$$

$$= 4.563\ F_1^*$$

$$= 4.563\ [p_1^* \times A_1^*(1+\gamma)] \qquad [\because F^* = p^*A^*(1+\gamma)]$$

$$= 4.563\ [0.441 \times 10^5 \times 0.01337\ (1+1.4)]$$

$$\boxed{\tau = 6457 \text{ N}}$$

**Result**

1. $p_2 = 0.832 \times 10^5 \text{ N/m}^2$
2. $c_2 = 30.51 \text{ m/s}$
3. $\tau = 6457 \text{ N}$

## 2.20 UNIVERSITY SOLVED PROBLEMS

1 ***Air is discharged from a receiver at $p_0 = 6.91$ bar and $T_0 = 325°C$ through a nozzle to an exit pressure of 0.98 bar. If the flow rate is 3600 kg/h. Determine for isentropic flow***

***1. Area, pressure and velocity at throat.***

***2. Area and Mach number at exit***

***3. Maximum possible velocity.***

*[Anna University Nov–2003 and Madras Univ April - 2000]*

***Given :***

$p_0 = 6.91 \text{ bar} = 6.91 \times 10^5 \text{ N/m}^2$

$T_0 = 325°C + 273 = 598 \text{ K}$

$p_2 = 0.98 \text{ bar} = 0.98 \times 10^5 \text{ N/m}^2$

$$\dot{m} = 3600 \text{ kg/h} = \frac{3600 \text{ kg}}{3600} = 1 \text{ kg/s}$$

***To find :***

1. Area, pressure and velocity at throat ($A^*$, $p^*$, $c^*$).
2. Area and Mach number at exit ($A_2$, $M_2$).
3. Maximum possible velocity, $c_{max}$

***Solution :***

We know that at thraot (*) section, $M = 1$.

From Isentropic flow table for $M = 1$ and $\gamma = 1.4$ .

$$\frac{T^*}{T_0} = 0.834$$

$$\frac{p^*}{p_0} = 0.528$$

*[ From gas tables (S.M. Yahya, Fifth edition) page no.31]*

$$\Rightarrow \quad T^* = T_0 \times 0.834$$

$$= 598 \times 0.834$$

$$\boxed{T^* = 498.73 \text{ K}}$$

$$\Rightarrow \quad p^* = 0.528 \times p_0$$

$$= 0.528 \times 6.91 \times 10^5$$

$$\boxed{p^* = 3.65 \times 10^5 \text{ N/m}^2}$$

We know that

$$c^* = a^* = \sqrt{\gamma R T^*}$$

$$= \sqrt{1.4 \times 287 \times 498.73}$$

$$\boxed{c^* = 447.65 \text{ m/s}}$$

Mass flow rate, $m = \rho^* A^* c^*$

$$\Rightarrow \quad A^* = \frac{m}{\rho^* c^*}$$

$$= \frac{m}{\frac{p^*}{RT^*} \times c^*} \qquad \left[\rho = \frac{p}{RT}\right]$$

$$= \frac{1}{\frac{3.66 \times 10^5}{287 \times 498.73} \times 447.65}$$

$$\boxed{A^* = 8.76 \times 10^{-4} \text{ m}^2}$$

From given data,

$$\frac{p_2}{p_0} = \frac{p_2}{p_{02}} = \frac{0.98 \times 10^5}{6.91 \times 10^5} = 0.142$$

$$[\because p_0 = p_{01} = p_{02}]$$

Refer Isentropic flow table for $\frac{p_2}{p_{02}} = 0.142$ and $\gamma = 1.4$ .

$$M_2 = 1.93$$

$$\frac{T_2}{T_{02}} = 0.573 \qquad \textit{[ From gas tables page no.33]}$$

$$\frac{A_2}{A^*} = 1.593$$

$$\Rightarrow \quad A_2 = A^* \times 1.593$$

$$= 8.76 \times 10^{-4} \times 1.593$$

$$\boxed{A_2 = 13.9 \times 10^{-4} \text{ m}^2}$$

Maximum velocity, $\frac{c_{max}}{c^*} = \sqrt{\frac{\gamma + 1}{\gamma - 1}}$ *[From chapter 1 Equation no.(1.24)]*

$$\Rightarrow c_{max} = c^* \sqrt{\frac{\gamma + 1}{\gamma - 1}}$$

$$= 447.65 \sqrt{\frac{1.4 + 1}{1.4 - 1}}$$

$$\boxed{c_{max} = 1096.51 \text{ m/s.}}$$

***Result***

1. $A^* = 8.76 \times 10^{-4} \text{ m}^2$

   $p^* = 3.65 \times 10^5 \text{ N/m}^2$

   $c^* = 447.65 \text{ m/s}$

2. $A_2 = 13.9 \times 10^{-4} \text{ m}^2$

   $M_2 = 1.93$

3. $c_{max} = 1096.51 \text{ m/s.}$

**6** ***Air enters an isentropic diffuser with a mach number of 3.6 and is decelerated to a mach number of 2. The diffuser passes a flow of 15 kg/s. The initial static pressure and temperature of the air are 1.05 bar and 40° C. Assuming $\gamma = 1.4$. Calculate***

1. ***Inlet area, total pressure and total temperature at inlet.***
2. ***Exit area, total pressure, total temperature and static pressure at exit.***

*[MU Oct - 97]*

*Given*

$M_1 = 3.6$

$M_2 = 2$

$m = 15$ kg/s

$p_1 = 1.05 \text{ bar} = 1.05 \times 10^5 \text{ N/m}^2$

$T_1 = 40°\text{C} + 273 = 313 \text{ K}$

$\gamma = 1.4$

*To find*

1. Inlet area ($A_1$), Total pressure and total temperature at inlet. ($p_{01}$, and $T_{01}$)
2. Exit area ($A_2$), Total pressure, total temperature and static pressure at exit. ($p_{02}$, $T_{02}$ and $p_2$ )

***Solution :***

Refer Isentropic flow table for $M_1 = 3.6$ and $\gamma = 1.4$ .

$$\frac{T_1}{T_{01}} = 0.278$$

*[ From gas tables page no.38]*

$$\frac{p_1}{p_{01}} = 11.38 \times 10^{-3}$$

$$\frac{A_1}{A^*} = 7.450$$

$$\Rightarrow \quad T_{01} = \frac{T_1}{0.278} = \frac{313}{0.278} = 1125.89 \text{ K}$$

$$\boxed{T_{01} = 1125.89 \text{ K} = T_{02}}$$

*[ ∵ For Isentropic flow stagnation temperature remains constant]*

$$\Rightarrow \quad p_{01} = \frac{p_1}{11.38 \times 10^{-3}}$$

$$= \frac{1.05 \times 10^5}{11.38 \times 10^{-3}}$$

$$\boxed{p_{01} = 92.26 \times 10^5 \text{ N/m}^2 = p_{02}}$$

*[ ∵ For Isentropic flow stagnation pressure remains constant]*

Mach number at entry, $M_1 = \dfrac{c_1}{a_1}$

$$= \frac{c_1}{\sqrt{\gamma R T_1}}$$

$$\Rightarrow \quad 3.6 = \frac{c_1}{\sqrt{1.4 \times 287 \times 313}}$$

$$\boxed{c_1 = 1276.67 \text{ m/s}}$$

Mass flow rate, $\dot{m} = \rho_1 A_1 c_1$

$$= \frac{p_1}{RT_1} \times A_1 \times c_1$$

$$15 = \frac{1.05 \times 10^5}{287 \times 266} \times A_1 \times 1276.67$$

$$\boxed{A_1 = 0.010 \text{ m}^2}$$

$$\Rightarrow \quad \frac{A_1}{A^*} = 7.450 \quad \text{[From Table]}$$

$$\Rightarrow \quad A^* = \frac{A_1}{7.450} = \frac{0.010}{7.450} = 1.34 \times 10^{-3} \text{ m}^2$$

$$\boxed{A^* = 1.34 \times 10^{-3} \text{ m}^2}$$

Refer Isentropic flow table for $\gamma = 1.4$ and $M_2 = 2$

$$\frac{T_2}{T_{02}} = 0.555 \qquad \textit{[From gas tables page no. 34]}$$

$$\frac{p_2}{p_{02}} = 0.128$$

$$\frac{A_2}{A^*} = 1.687$$

$$\Rightarrow \quad A_2 = 1.687 \times A^*$$

$$= 1.687 \times 1.34 \times 10^{-3}$$

$$\boxed{A_2 = 22.60 \times 10^{-4} \text{ m}^2}$$

$$\Rightarrow \quad p_2 = 0.128 \times p_{02}$$

$$= 0.128 \times 92.26 \times 10^5$$

$$\boxed{p_2 = 11.8 \times 10^5 \text{ N/m}^2}$$

***Result***

1. $A_1 = 0.010 \text{ m}^2$

   $p_{01} = p_{02} = 92.26 \times 10^5 \text{ N/m}^2$

   $T_{01} = T_{02} = 1125.89 \text{ K}$

2. $A_2 = 22.60 \times 10^{-4} \text{ m}^2$

   $p_2 = 11.8 \times 10^5 \text{ N/m}^2$

**7** ***The pressure, velocity and temperature of air ($\gamma = 1.4$, $c_p = 1$ kJ/kg K) at the entry of a nozzle are 2 bar, 145 m/s and 330K. The exit pressure is 1.5 bar. find***

***(a) What is the shape of the nozzle***

***(b) Determine for isentropic flow***

***(i) The Mach number at entry and exit***

***(ii) The flow rate and maximum possible flow rate.***

*[MU Oct - 95]*

***Given***

$\gamma = 1.4$

$c_p = 1$ kJ/kg K $= 1000$ J/kg K

$p_1 = 2$ bar $= 2 \times 10^5$ N/m$^2$

$c_1 = 145$ m/s

$T_1 = 330$ K

$p_2 = 1.5$ bar $= 1.5 \times 10^5$ N/m$^2$

***To find***

a) Shape of the nozzle

b) (i) The Mach number at entry and exit ($M_1$, $M_2$)

(ii) The flow rate (m) and maximum possible flow rate ($m_{max}$)

*Solution :*

Mach number at entry, $M_1 = \dfrac{c_1}{a_1}$

$$= \frac{c_1}{\sqrt{\gamma R T_1}} \qquad [\because a = \sqrt{\gamma R T}\,]$$

$$= \frac{145}{\sqrt{1.4 \times 287 \times 330}}$$

$$\boxed{M_1 = 0.398}$$

Refer Isentropic flow table for $M_1 = 0.398 \approx 0.4$ and $\gamma = 1.4$

$$\frac{T_1}{T_{01}} = 0.969$$

*[From gas tables page no.29]*

$$\frac{p_1}{p_{01}} = 0.895$$

$$\frac{A_1}{A^*} = 1.590$$

$$\Rightarrow \qquad T_{01} = \frac{T_1}{0.969} = \frac{330}{0.969}$$

$$\boxed{T_{01} = 340.56 \text{ K} = T_{02}}$$

$[\because$ For Isentropic flow, $T_{01} = T_{02}]$

$$\Rightarrow \qquad p_{01} = \frac{p_1}{0.895}$$

$$= \frac{2 \times 10^5}{0.895}$$

$$\boxed{p_{01} = 2.23 \times 10^5 \text{ N/m}^2 = p_{02}} \qquad [\because p_{01} = p_{02}]$$

Refer Isentropic flow table for $\dfrac{p_2}{p_{02}} = 0.673 \approx 0.676$ and $\gamma = 1.4$

$$M_2 = 0.77$$

$$\frac{T_2}{T_{02}} = 0.894$$

*[ From gas tables page no.30]*

$$\frac{A_2}{A^*} = 1.052$$

Type of nozzle is convergent because mach number value is less than unity. ($M_2 = 0.77 < 1$, $M_1 = 0.398 < 1$)

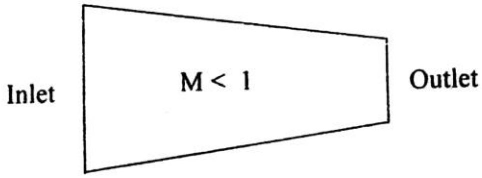


*Fig. 2.17 Convergent nozzle*

Mass flow rate, $\dot{m} = \rho A c = \rho_1 A_1 c_1 = \rho_2 A_2 c_2$

$$\Rightarrow \qquad \dot{m} = \rho_1 A_1 c_1$$

$$\Rightarrow \qquad \frac{\dot{m}}{A_1} = \rho_1 \times c_1$$

$$= \frac{p_1}{RT_1} \times c_1$$

$$= \frac{2 \times 10^5}{287 \times 330} \times 145$$

$$\boxed{\frac{\dot{m}}{A_1} = 306.197 \text{ kg/s–m}^2}$$

Maximum mass flow rate,

$$\frac{\dot{m}_{max}\sqrt{T_0}}{A^* p_0} \times \sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma + 1}{2(\gamma - 1)}}$$ *[From Equation no(2.26)]*

Substitute $\gamma = 1.4$, $R = 287$, $T_0 = 340.56$ K, $p_0 = 2.23 \times 10^5$ N/m$^2$

$$\frac{\dot{m}_{max}}{A^*} \times \frac{\sqrt{340.56}}{2.23 \times 10^5} \times \sqrt{\frac{287}{1.4}} = \left(\frac{2}{1.4 + 1}\right)^{\frac{1.4 + 1}{2(1.4 - 1)}}$$

$$\boxed{\frac{\dot{m}_{max}}{A^*} = 490.43 \text{ kg/s–m}^2}$$

***Result :***

(a) Type of nozzle is convergent

(b) (i) $M_1 = 0.398$

$M_2 = 0.77$

(ii) $\frac{\dot{m}}{A_1} = 306.197$ kg/s–m$^2$

$\frac{\dot{m}_{max}}{A^*} = 490.43$ kg/s–m$^2$

**12** ***A convergent–divergent steam nozzle has an area ratio $A/A^* = 1.44$. Calculate (without using gas tables) the mach number and pressure ratio ($p/p_0$) for isentropic supersonic flow taking $\gamma = 1.3$.***

***Recalculate the above values in the subsonic section of the nozzle for the same area ratio.***

*[MSU April - 95]*

***Given***

$$\frac{A}{A^*} = 1.44$$

$\gamma = 1.3$

***To find***

1. Mass flow rate, m.
2. Pressure ratio, $p/p_0$

} For subsonic flow and supersonic flow

***Solution***

We know that,

Area ratio $\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}M^2\right]^{\frac{\gamma+1}{2[\gamma-1]}}$

*[From equation 2.13]*

Substitute $\frac{A}{A^*}$ and $\gamma$ values

$$\Rightarrow 1.44 = \frac{1}{M}\left[\frac{2}{1.3+1} + \frac{1.3-1}{1.3+1}M^2\right]^{\frac{1.3+1}{2[1.3-1]}}$$

$$\Rightarrow 1.44 = \frac{1}{M}[0.869 + 0.130\ M^2]^{3.833}$$

$$\Rightarrow 1.44\ M = [0.869 + 0.130\ M^2]^{3.833}$$

$$\Rightarrow [1.44\ M]^{1/3.833} = 0.869 + 0.130\ M^2$$

$$\Rightarrow [1.44\ M]^{0.261} = 0.869 + 0.130M^2$$

$$\Rightarrow 0.130\ M^2 - [1.44\ M]^{0.261} + 0.869 = 0$$

$$\Rightarrow 0.130\ M^2 - 1.099 \times M^{0.261} + 0.869 = 0 \qquad ----(1)$$

By trail and error method, we can find M = 0.46, and M = 1.764

For subsonic flow, M = 0.46

We know that,

$$\frac{p_0}{p} = \left[1 + \frac{\gamma-1}{2}M^2\right]^{\frac{\gamma}{\gamma-1}}$$

*[From chapter 1- Equation no.(1.8)]*

$$\Rightarrow \quad \frac{p_0}{p} = \left[1 + \frac{1.3-1}{2}(0.46)^2\right]^{\frac{1.3}{1.3-1}}$$

$$\Rightarrow \quad \frac{p_0}{p} = 1.144$$

$$\Rightarrow \quad \boxed{\frac{p}{p_0} = 0.873}$$

For supersonic flow, M = 1.764

$$\Rightarrow \quad \frac{p_0}{p} = \left[1 + \frac{\gamma-1}{2}M^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\Rightarrow \quad \frac{p_0}{p} = \left[1 + \frac{1.3-1}{2}(1.764)^2\right]^{\frac{1.3}{1.3-1}}$$

$$\Rightarrow \quad \frac{p_0}{p} = 5.258$$

$$\Rightarrow \quad \boxed{\frac{p}{p_0} = 0.190}$$

***Result :***

1. For subsonic flow

$M = 0.46$

$$\frac{p}{p_0} = 0.873$$

2. For supersonic flow

$M = 1.764$

$$\frac{p}{p_0} = 0.190$$

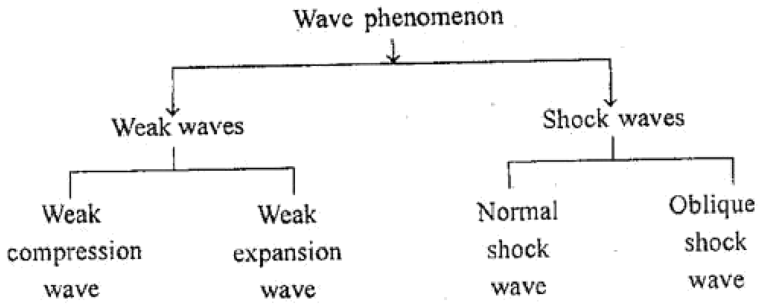
# 4

## 4.1 NORMAL SHOCK WAVES

### 4.1.1 Introduction

When there is a relative motion between a body and fluid, the disturbance is created. If the disturbance is of an infinitely small amplitude, that disturbance is transmitted through the fluid with the speed of sound. If the disturbance is finite amplitude shock waves are created.

Wave phenomenon is classified as follows



### Mach Waves

A mach waves are weak waves. Mach wave has been described in the previous Chapter (Chapter 1).

### Expansion Wave

A wave which is at a lower pressure than the fluid in to which it is moving is called an expansion wave (or) rarefaction wave.

### Compression Wave

A wave which is at a higher pressure than the fluid in to which it is moving is called compression wave.

## Shock Wave

A shock wave is nothing but a steep finite pressure wave. The shock wave may be described as a compression wave front in a supersonic flow field across which there is abrupt change in flow properties. The flow process through the shock wave is highly irreversible and cannot be approximated as being isentropic.

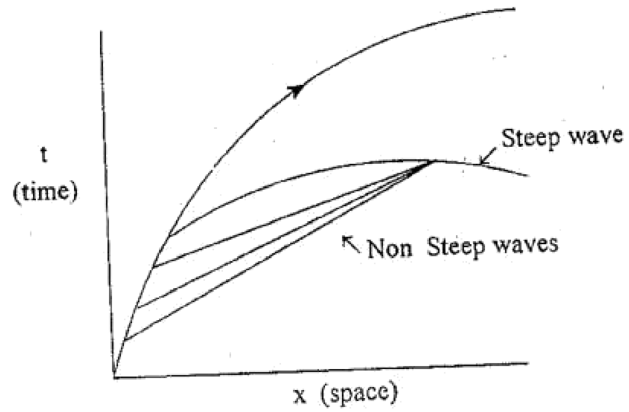


*Fig 4.1 Development shock wave [Time–space diagram]*

### Normal shock

When the shock wave is at right angle to the flow, it is called normal shock.

### Oblique shock

When the shock wave is inclined at an angle to the flow, it is called oblique shock.

### 4.1.2. Prandtl – Meyer Relation

Prandtl–Meyer relation which is the basis of other equation for shock waves. It gives the relationship between the gas velocities before and after the normal shock and the critical velocity of sound.

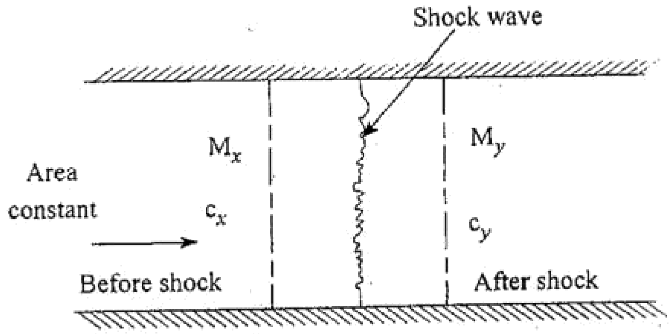


*Fig.4.2*

We know that

Stagnation enthalpy equation

$$h_0 = \frac{a^2}{\gamma - 1} + \frac{c^2}{2} = ½\left(\frac{\gamma+1}{\gamma-1}\right)a*^2$$

[From Chapter 1 – Equation no.1.25]

Applying this equation to the flow before shock wave and after shock wave.

### Before shock wave

$$\frac{a_x^2}{\gamma - 1} + ½\, c_x^2 = ½\left(\frac{\gamma+1}{\gamma-1}\right)a*^2$$

$$\frac{a_x^2}{\gamma - 1} = ½\left(\frac{\gamma+1}{\gamma-1}\right)a*^2 - ½\, c_x^2$$

$$\Rightarrow\ a_x^2 = (\gamma - 1) \times ½\left(\frac{\gamma+1}{\gamma-1}\right)a*^2 - ½\, c_x^2 (\gamma - 1)$$

$$\Rightarrow\ a_x^2 = \left(\frac{\gamma+1}{2}\right)a*^2 - \frac{\gamma - 1}{2}\, c_x^2$$

$$\Rightarrow \frac{a_x^2}{c_x} = \left(\frac{\gamma+1}{2}\right)\frac{a^{*2}}{c_x} - \frac{\gamma-1}{2}c_x$$

$$\Rightarrow \boxed{\frac{a_x^2}{c_x} = \left(\frac{\gamma+1}{2}\right)\frac{a^{*2}}{c_x} - \frac{\gamma-1}{2}c_x} \qquad \text{----(4.1)}$$

**After shock wave**

$$\frac{a_y^2}{\gamma-1} + \frac{1}{2}c_y^2 = \frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)a^{*2}$$

$$\Rightarrow \frac{a_y^2}{\gamma-1} = \frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)a^{*2} - \frac{1}{2}c_y^2$$

$$\Rightarrow a_y^2 = (\gamma-1)\times\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)a^{*2} - \frac{1}{2}c_y^2(\gamma-1)$$

$$\Rightarrow a_y^2 = \left(\frac{\gamma+1}{2}\right)a^{*2} - \frac{\gamma-1}{2}c_y^2$$

$$\Rightarrow \boxed{\frac{a_y^2}{c_y} = \left(\frac{\gamma+1}{2}\right)\frac{a^{*2}}{c_y} - \frac{\gamma-1}{2}c_y} \qquad \text{----(4.2)}$$

We know that from momentum equation

$$(p_x - p_y)\,A = m\,(c_y - c_x)$$

$$\Rightarrow \quad p_x - p_y = \frac{m}{A}(c_y - c_x)$$

$$\Rightarrow \boxed{\frac{p_x - p_y}{\left(\frac{m}{A}\right)} = c_y - c_x}$$

$$\Rightarrow \boxed{\frac{p_x}{\left(\frac{m}{A}\right)} - \frac{p_y}{\left(\frac{m}{A}\right)} = c_y - c_x} \qquad \text{----(4.3)}$$

Mass flow rate, $m = \rho A c = \rho_x A_x c_x = \rho_y A_y c_y$

$$\Rightarrow \boxed{\frac{m}{A} = \rho_x c_x = \rho_y c_y} \qquad \text{----(4.4)}$$

Substitute $\frac{m}{A}$ value in Equation (4.3)

$$(4.3) \Rightarrow \frac{p_x}{\rho_x c_x} - \frac{p_y}{\rho_y c_y} = c_y - c_x$$

Multiplying by $\gamma$

$$\Rightarrow \boxed{\frac{\gamma p_x}{\rho_x c_x} - \frac{\gamma p_y}{\rho_y c_y} = \gamma\,(c_y - c_x)} \qquad \text{----(4.5)}$$

Gas equation, $pv = mRT$

For unit mass

$$pv = RT$$

$$\Rightarrow \quad \frac{p}{\rho} = RT \qquad \left[\because \rho = \frac{1}{v}\right]$$

$$\Rightarrow \quad \frac{p}{\rho} = \frac{\gamma RT}{\gamma}$$

$$\Rightarrow \quad \frac{p}{\rho} = \frac{a^2}{\gamma} \qquad \left[\because a = \sqrt{\gamma RT}\right]$$

$$\Rightarrow \quad \frac{\gamma p}{\rho} = a^2$$

$$\Rightarrow \quad \boxed{\frac{\gamma p_x}{\rho_x} = a_x^2}$$

$$\Rightarrow \quad \boxed{\frac{\gamma p_y}{\rho_y} = a_y^2}$$

Substitue $a_x^2$ and $a_y^2$ values in equation no (4.5)

$$(4.5) \Rightarrow \frac{a_x^2}{c_x} - \frac{a_y^2}{c_y} = \gamma\,[c_y - c_x]$$

$$\Rightarrow \quad \boxed{\frac{a_x^2}{c_x} - \frac{a_y^2}{c_y} = \gamma\,[c_y - c_x]} \qquad \text{----(4.6)}$$

Substitute equation (4.1) and (4.2) in equation no (4.6)

$$(4.6) \Rightarrow \left(\frac{\gamma+1}{2}\right)\frac{a*^2}{c_x} - \left(\frac{\gamma-1}{2}\right)c_x - \left[\left(\frac{\gamma+1}{2}\right)\frac{a*^2}{c_y} - \left(\frac{\gamma-1}{2}\right)c_y\right]$$

$$= \gamma\,[c_y - c_x]$$

$$\Rightarrow \left(\frac{\gamma+1}{2}\right)\frac{a*^2}{c_x} - \left(\frac{\gamma-1}{2}\right)c_x - \left(\frac{\gamma+1}{2}\right)\frac{a*^2}{c_y} + \left(\frac{\gamma-1}{2}\right)c_y$$

$$= \gamma\,[c_y - c_x]$$

$$\Rightarrow \left(\frac{\gamma+1}{2}\right)a*^2\left[\frac{1}{c_x} - \frac{1}{c_y}\right] + \frac{\gamma-1}{2}\,[c_y - c_x] = \gamma\,[c_y - c_x]$$

$$\Rightarrow \left(\frac{\gamma+1}{2}\right)a*^2\left[\frac{c_y - c_x}{c_x c_y}\right] + \left(\frac{\gamma-1}{2}\right)[c_y - c_x] = \gamma\,[c_y - c_x]$$

Multiply by $\dfrac{c_x c_y}{c_y - c_x}$

$$\Rightarrow \left(\frac{\gamma+1}{2}\right)a*^2 + \left(\frac{\gamma-1}{2}\right)c_x c_y = \gamma\,[c_x c_y]$$

Multiplying by 2

$$\Rightarrow \quad (\gamma+1)\,a*^2 + (\gamma-1)\,c_x c_y = 2\gamma\,[c_x c_y]$$

$$\Rightarrow \quad (\gamma+1)\,a*^2 = 2\gamma\,[c_x \times c_y] - (\gamma-1)\,c_x \times c_y$$

$$\Rightarrow \quad (\gamma+1)\,a*^2 = 2\gamma\,c_x c_y - \gamma\,c_x c_y + c_x c_y$$

$$\Rightarrow \quad (\gamma+1)\,a*^2 = \gamma\,c_x c_y + c_x c_y$$

$$\Rightarrow \quad (\gamma+1)\,a*^2 = c_x c_y\,(\gamma+1)$$

$$\Rightarrow \quad a*^2 = \frac{c_x c_y\,(\gamma+1)}{(\gamma+1)}$$

$$\Rightarrow \quad \boxed{a*^2 = c_x c_y} \qquad \text{----(4.7)}$$

This equation is known as Prandtl–Meyer relation.

$$\Rightarrow \quad a* \times a* = c_x c_y$$

$$\Rightarrow \quad 1 = \frac{c_y}{a*} \times \frac{c_x}{a*}$$

$$\Rightarrow \quad 1 = M_x* \times M_y*$$

$$\Rightarrow \quad M_x* \times M_y* = 1 \qquad \text{----(4.8)}$$

This is another useful form of Prandtl–Meyer relation.

### 4.1.3. Down stream mach number (or) Mach number after the shock ($M_y$)

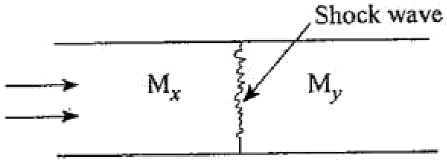


*Fig.4.3*

A fluid is flowing in a constant area duct as shown in fig and the shock wave is generated.

where,

$M_x$ – Upstream Mach number (or) Mach number before the shock.

$M_y$ – Down stream Mach number (or) Mach number after the shock.

We know that

$$\frac{a^{*2}}{a_0^2} = \frac{2}{\gamma+1} \qquad \text{[From Chapter 1– Equation no.1.20]}$$

$$\Rightarrow \quad a^{*2} = \frac{2}{\gamma+1} \times a_0^2$$

$$\Rightarrow \quad a^{*2} = \frac{2\gamma}{\gamma+1} RT_0 \qquad [\because a_0 = \sqrt{\gamma RT_0}]$$

From equation no (4.7), we know that,

$$a^{*2} = c_x c_y$$

$$\Rightarrow \boxed{\frac{2\gamma}{\gamma+1} \times RT_0 = c_x c_y} \qquad ----(4.9)$$

We know that

$$M_x = \frac{c_x}{a_x}$$

$$\Rightarrow \quad c_x = M_x \times a_x$$

$$\Rightarrow \quad \boxed{c_x = M_x \times \sqrt{\gamma RT_x}}$$

Similarly

$$\boxed{c_y = M_y \times \sqrt{\gamma RT_y}}$$

Substituting $c_x c_y$ values in equation no (4.9)

$$(4.9) \Rightarrow \frac{2\gamma}{\gamma+1} \times RT_0 = M_x \sqrt{\gamma RT_x} \times M_y \sqrt{\gamma RT_y}$$

Squaring on both sides

$$\Rightarrow \left(\frac{2}{\gamma+1}\right)^2 \times \gamma^2 R^2 T_0^2 = M_x^2 \gamma RT_x \times M_y^2 \gamma RT_y$$

$$\Rightarrow \left(\frac{2}{\gamma+1}\right)^2 \times T_0^2 = M_x^2 \times M_y^2 \times T_x \times T_y$$

$$\Rightarrow \quad M_x^2 \times M_y^2 = \frac{\left(\frac{2}{\gamma+1}\right)^2 \times T_0^2}{T_x \times T_y}$$

$$\Rightarrow \boxed{M_x^2 \times M_y^2 = \left(\frac{2}{\gamma+1}\right)^2 \times \frac{T_0}{T_x} \times \frac{T_0}{T_y}} \qquad ----(4.10)$$

We know that

$$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2$$

$$\Rightarrow \quad \frac{T_0}{T_x} = 1 + \frac{\gamma-1}{2} M_x^2$$

Similarly

$$\frac{T_0}{T_y} = 1 + \frac{\gamma-1}{2} M_y^2$$

Substituting $\frac{T_0}{T_x}$ and $\frac{T_0}{T_y}$ values in equation no (4.10)

$$\Rightarrow \quad M_x^2 \times M_y^2 = \left(\frac{2}{\gamma+1}\right)^2 \times \left[1+\frac{\gamma-1}{2}M_x^2\right] \times \left[1+\frac{\gamma-1}{2}M_y^2\right]$$

$$\Rightarrow \quad M_x^2 \times M_y^2 = \frac{4}{(\gamma+1)^2} \times \left[1+\frac{\gamma-1}{2}M_x^2\right] \times M_y^2\left[\frac{1}{M_y^2}+\frac{\gamma-1}{2}\right]$$

$$\Rightarrow \quad \frac{M_x^2}{\frac{4}{(\gamma+1)^2}\times\left[1+\frac{\gamma-1}{2}M_x^2\right]} = \frac{M_y^2\left[\frac{1}{M_y^2}+\frac{\gamma-1}{2}\right]}{M_y^2}$$

$$\Rightarrow \quad \frac{M_x^2}{\frac{4}{(\gamma+1)^2}\times\left[1+\frac{\gamma-1}{2}M_x^2\right]} = \frac{1}{M_y^2}+\frac{\gamma-1}{2}$$

$$\Rightarrow \quad \frac{(\gamma+1)^2 M_x^2}{4\left[1+\frac{\gamma-1}{2}M_x^2\right]} = \frac{1}{M_y^2}+\frac{\gamma-1}{2}$$

$$\Rightarrow \quad \frac{1}{M_y^2} = \frac{(\gamma+1)^2 M_x^2}{4\left[1+\frac{\gamma-1}{2}M_x^2\right]} - \frac{\gamma-1}{2}$$

$$\Rightarrow \quad \frac{1}{M_y^2} = \frac{2(\gamma+1)^2 M_x^2 - 4\left[1+\frac{\gamma-1}{2}M_x^2\right](\gamma-1)}{8\left[1+\frac{\gamma-1}{2}M_x^2\right]}$$

$$\Rightarrow \quad M_y^2 = \frac{8\left[1+\frac{\gamma-1}{2}M_x^2\right]}{2(\gamma+1)^2 M_x^2 - 4\left[1+\frac{\gamma-1}{2}M_x^2\right](\gamma-1)}$$

$$= \frac{8\left[1+\frac{\gamma-1}{2}M_x^2\right]}{2(\gamma+1)^2 M_x^2 - [4+2(\gamma-1)\ M_x^2]\times(\gamma-1)}$$

$$= \frac{8\left[1+\frac{\gamma-1}{2}M_x^2\right]}{2(\gamma+1)^2 M_x^2 - [4(\gamma-1)+2(\gamma-1)^2 M_x^2]}$$

$$= \frac{8\left[1+\frac{\gamma-1}{2}M_x^2\right]}{2(\gamma+1)^2 M_x^2 - 4(\gamma-1) - 2(\gamma-1)^2\ M_x^2}$$

$$= \frac{8+4(\gamma-1)M_x^2}{2(\gamma+1)^2 M_x^2 - 4(\gamma-1) - 2(\gamma-1)^2\ M_x^2}$$

$$= \frac{2\,[4+2(\gamma-1)M_x^2]}{2\,[(\gamma+1)^2 M_x^2 - 2(\gamma-1) - (\gamma-1)^2\ M_x^2]}$$

$$= \frac{4+2(\gamma-1)M_x^2}{(\gamma+1)^2 M_x^2 - 2(\gamma-1) - (\gamma-1)^2\ M_x^2}$$

Divided by 2(γ − 1)

$$\Rightarrow \quad M_y^2 = \frac{\frac{2}{\gamma-1}+M_x^2}{\frac{(\gamma+1)^2}{2(\gamma-1)}M_x^2 - 1 - \frac{(\gamma-1)}{2}M_x^2}$$

$$= \frac{\frac{2}{\gamma-1}+M_x^2}{\frac{(\gamma+1)^2}{2(\gamma-1)}M_x^2-\left[1+\frac{\gamma-1}{2}M_x^2\right]}$$

$$= \frac{\frac{2}{\gamma-1}+M_x^2}{\frac{(\gamma^2+2\gamma+1)}{2(\gamma-1)}M_x^2-\left[1+\frac{\gamma-1}{2}M_x^2\right]}$$

$$= \frac{\frac{2}{\gamma-1}+M_x^2}{\frac{M_x^2\gamma^2+2\gamma M_x^2+M_x^2}{2(\gamma-1)}-\left[1+\left(\frac{\gamma-1}{2}\right)M_x^2\right]}$$

$$= \frac{\frac{2}{\gamma-1}+M_x^2}{\frac{M_x^2\gamma^2+2\gamma M_x^2+M_x^2}{2(\gamma-1)}-\left[1+\frac{\gamma M_x^2-M_x^2}{2}\right]}$$

$$= \frac{\frac{2}{\gamma-1}+M_x^2}{\frac{M_x^2\gamma^2+2\gamma M_x^2+M_x^2}{2(\gamma-1)}-\left[\frac{2+\gamma M_x^2-M_x^2}{2}\right]}$$

$$= \frac{\frac{2}{\gamma-1}+M_x^2}{\frac{2[M_x^2\gamma^2+2\gamma M_x^2+M_x^2]-2(\gamma-1)\,[2+\gamma M_x^2-M_x^2]}{2\times 2(\gamma-1)}}$$

$$= \frac{\frac{2}{\gamma-1}+M_x^2}{\frac{[M_x^2\gamma^2+2\gamma M_x^2+M_x^2]-(\gamma-1)\,[2+\gamma M_x^2-M_x^2]}{2(\gamma-1)}}$$

$$= \frac{\left[\frac{2}{\gamma-1}+M_x^2\right]2(\gamma-1)}{M_x^2\gamma^2+2\gamma M_x^2+M_x^2-2(\gamma-1)-\gamma(\gamma-1)M_x^2+(\gamma-1)M_x^2}$$

$$= \frac{\left[\frac{2}{\gamma-1}+M_x^2\right]2(\gamma-1)}{M_x^2\gamma^2+2\gamma M_x^2+M_x^2-2\gamma+2-\gamma^2M_x^2+\gamma M_x^2+\gamma M_x^2-M_x^2}$$

$$= \frac{\left[\frac{2}{\gamma-1}+M_x^2\right]2(\gamma-1)}{4\gamma M_x^2-2\gamma+2}$$

$$= \frac{\left[\frac{2}{\gamma-1}+M_x^2\right]2(\gamma-1)}{4\gamma M_x^2-2(\gamma-1)}$$

$$= \frac{\left[\frac{2}{\gamma-1}+M_x^2\right]}{\frac{4\gamma M_x^2-2(\gamma-1)}{2(\gamma-1)}}$$

$$= \frac{\frac{2}{\gamma-1}+M_x^2}{\frac{4\gamma M_x^2}{2(\gamma-1)}-1}$$

$$M_y^2 = \frac{\frac{2}{\gamma-1} + M_x^2}{\frac{2\gamma M_x^2}{(\gamma-1)} - 1}$$

$$\boxed{M_y^2 = \frac{\frac{2}{\gamma-1} + M_x^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1}} \qquad ---(4.11)$$

### 4.1.4. Static Pressure Ratio Across the Shock

We know that

Force, $F = p + \rho c^2$

Force acting before the normal shock

$$F_x = p_x + \rho_x c_x^2$$

Force acting after the normal shock

$$F_y = p_y + \rho_y c_y^2$$

We know that

$$F_x = F_y$$

$$\Rightarrow \quad p_x + \rho_x c_x^2 = p_y + \rho_y c_y^2$$

$$\Rightarrow \quad p_x + \rho_x \times M_x^2 a_x^2 = p_y + \rho_y \times M_y^2 a_y^2$$

$[\because c = M \times a]$

$$\Rightarrow \quad p_x + \frac{p_x}{RT_x} \times M_x^2 \times \gamma RT_x = p_y + \frac{p_y}{RT_y} \times M_y^2 \times \gamma RT_y$$

$[\because \rho = \frac{p}{RT}$

$a = \sqrt{\gamma RT}]$

$$\Rightarrow \quad p_x + \gamma p_x M_x^2 = p_y + \gamma p_y M_y^2$$

$$\Rightarrow \quad p_x [1 + \gamma M_x^2] = p_y [1 + \gamma M_y^2]$$

$$\Rightarrow \quad \frac{p_y}{p_x} = \frac{1 + \gamma M_x^2}{1 + \gamma M_y^2}$$

Substituting $M_y^2$ value from equation no (4.11)

$$\Rightarrow \quad \frac{p_y}{p_x} = \frac{1 + \gamma M_x^2}{1 + \gamma \times \left[\frac{\frac{2}{\gamma-1} + M_x^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1}\right]}$$

$$= \frac{1 + \gamma M_x^2}{1 + \frac{\frac{2\gamma}{\gamma-1} + \gamma M_x^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1}}$$

$$= \frac{1 + \gamma M_x^2}{\frac{\left(\frac{2\gamma}{\gamma-1} M_x^2 - 1\right) + \left(\frac{2\gamma}{\gamma-1} + \gamma M_x^2\right)}{\left(\frac{2\gamma}{\gamma-1} M_x^2 - 1\right)}}$$

$$= \frac{(1 + \gamma M_x^2)\left(\frac{2\gamma}{\gamma-1} M_x^2 - 1\right)}{\left(\frac{2\gamma}{\gamma-1} M_x^2 - 1\right) + \left(\frac{2\gamma}{\gamma-1} + \gamma M_x^2\right)}$$

$$= \frac{(1+\gamma M_x^2)\left(\frac{2\gamma}{\gamma-1}M_x^2-1\right)}{\frac{2\gamma M_x^2}{\gamma-1}-1+\frac{2\gamma}{\gamma-1}+\gamma M_x^2}$$

$$= \frac{(1+\gamma M_x^2)\left(\frac{2\gamma}{\gamma-1}M_x^2-1\right)}{\gamma M_x^2\left[\frac{2}{\gamma-1}+1\right]+\frac{2\gamma}{\gamma-1}-1}$$

$$= \frac{(1+\gamma M_x^2)\left(\frac{2\gamma}{\gamma-1}M_x^2-1\right)}{\gamma M_x^2\left(\frac{2+\gamma-1}{\gamma-1}\right)+\left(\frac{2\gamma-\gamma+1}{\gamma-1}\right)}$$

$$= \frac{(1+\gamma M_x^2)\left(\frac{2\gamma}{\gamma-1}M_x^2-1\right)}{\gamma M_x^2\left(\frac{\gamma+1}{\gamma-1}\right)+\left(\frac{\gamma+1}{\gamma-1}\right)}$$

$$= \frac{(1+\gamma M_x^2)\left(\frac{2\gamma}{\gamma-1}M_x^2-1\right)}{\frac{\gamma+1}{\gamma-1}\,[\gamma M_x^2+1]}$$

$$= \frac{\left(\frac{2\gamma}{\gamma-1}M_x^2-1\right)}{\frac{\gamma+1}{\gamma-1}}$$

$$= \left(\frac{2\gamma}{\gamma-1}M_x^2-1\right)\times\frac{\gamma-1}{\gamma+1}$$

$$\boxed{\frac{p_y}{p_x} = \frac{2\gamma}{\gamma+1}M_x^2-\left(\frac{\gamma-1}{\gamma+1}\right)} \quad \text{... (4.12)}$$

From equation (4.12), we came to know, if $M_x > 1$, $\frac{p_y}{p_x} > 1$. So there is always pressure rise across it.

$$\text{If} \quad M_x = 1 \Rightarrow \frac{p_y}{p_x} = 1$$

$$\text{If} \quad M_x < 1 \Rightarrow \frac{p_y}{p_x} < 1 \Rightarrow \text{Not possible}$$

### 4.1.5. Static Temperature Ratio Across the Shock

Stagnation temperature – Mach number relation

$$\frac{T_0}{T} = 1+\frac{\gamma-1}{2}M^2$$

For upstream Mach number

$$\frac{T_{0x}}{T_x} = 1+\frac{\gamma-1}{2}M_x^2$$

For downstream Mach number

$$\frac{T_{0y}}{T_y} = 1+\frac{\gamma-1}{2}M_y^2$$

Stagnation enthalpy remains constant

$$\Rightarrow \qquad T_0 = T_{0y} = T_{0x}$$

$$\Rightarrow \frac{T_y}{T_x} = \frac{T_{0x}}{T_x} \times \frac{T_y}{T_{0y}} \qquad [\because T_{0x} = T_{0y}]$$

$$= \left[1 + \frac{\gamma - 1}{2} M_x^2\right] \times \frac{1}{1 + \frac{\gamma - 1}{2} M_y^2}$$

$$= \frac{1 + \frac{\gamma - 1}{2} M_x^2}{1 + \frac{\gamma - 1}{2} M_y^2}$$

Substituting $M_y^2$ value from equation no (4.11)

$$\frac{T_y}{T_x} = \frac{1 + \frac{\gamma - 1}{2} M_x^2}{1 + \frac{\gamma - 1}{2} \times \left[\frac{\frac{2}{\gamma - 1} + M_x^2}{\frac{2\gamma}{\gamma - 1} M_x^2 - 1}\right]}$$

$$= \frac{1 + \frac{\gamma - 1}{2} M_x^2}{1 + \frac{\left[1 + M_x^2 \times \frac{(\gamma - 1)}{2}\right]}{\left[\frac{2\gamma}{\gamma - 1} M_x^2 - 1\right]}}$$

$$= \frac{1 + \frac{\gamma - 1}{2} M_x^2}{\frac{\left[\frac{2\gamma}{\gamma - 1} M_x^2 - 1\right] + \left[1 + M_x^2 \times \frac{(\gamma - 1)}{2}\right]}{\left[\frac{2\gamma}{\gamma - 1} M_x^2 - 1\right]}}$$

$$= \frac{\left[\frac{2\gamma}{\gamma - 1} M_x^2 - 1\right] \left[1 + \frac{\gamma - 1}{2} M_x^2\right]}{\left[\frac{2\gamma}{\gamma - 1} M_x^2 - 1\right] + \left[1 + M_x^2 \times \frac{(\gamma - 1)}{2}\right]}$$

$$= \frac{\left[\frac{2\gamma}{\gamma - 1} M_x^2 - 1\right] \left[1 + \frac{\gamma - 1}{2} M_x^2\right]}{\frac{2\gamma}{\gamma - 1} M_x^2 - 1 + 1 + M_x^2 \times \frac{(\gamma - 1)}{2}}$$

$$= \frac{\left[\frac{2\gamma}{\gamma - 1} M_x^2 - 1\right] \left[1 + \frac{\gamma - 1}{2} M_x^2\right]}{\frac{2\gamma M_x^2}{\gamma - 1} + \frac{M_x^2 (\gamma - 1)}{2}}$$

$$= \frac{\left[\frac{2\gamma}{\gamma - 1} M_x^2 - 1\right] \left[1 + \frac{\gamma - 1}{2} M_x^2\right]}{\frac{M_x^2}{\gamma - 1} \left[2\gamma + \frac{(\gamma - 1)^2}{2}\right]}$$

$$= \frac{\left[\frac{2\gamma}{\gamma - 1} M_x^2 - 1\right] \left[1 + \frac{\gamma - 1}{2} M_x^2\right]}{\frac{M_x^2}{\gamma - 1} \left[\frac{4\gamma + (\gamma - 1)^2}{2}\right]}$$

$$= \frac{\left[\frac{2\gamma}{\gamma - 1} M_x^2 - 1\right] \left[1 + \frac{\gamma - 1}{2} M_x^2\right]}{\frac{M_x^2}{2(\gamma - 1)} \times \left[4\gamma + (\gamma - 1)^2\right]}$$

$$= \frac{\left[\frac{2\gamma}{\gamma-1}M_x^2-1\right]\left[1+\frac{\gamma-1}{2}M_x^2\right]}{\frac{M_x^2}{2(\gamma-1)}\times\left[4\gamma+\gamma^2-2\gamma+1\right]}$$

$$= \frac{\left[\frac{2\gamma}{\gamma-1}M_x^2-1\right]\left[1+\frac{\gamma-1}{2}M_x^2\right]}{\frac{M_x^2}{2(\gamma-1)}\times\left[\gamma^2+2\gamma+1\right]}$$

$$\boxed{\frac{T_y}{T_x} = \frac{\left[\frac{2\gamma}{\gamma-1}M_x^2-1\right]\left[1+\frac{\gamma-1}{2}M_x^2\right]}{\frac{M_x^2}{2(\gamma-1)}\times(\gamma+1)^2}} \quad \text{----(4.13)}$$

### 4.1.6 Velocity of Sound Across the Shock

We know that velocity of sound, $a = \sqrt{\gamma RT}$

For upstream,

$$a_x = \sqrt{\gamma RT_x}$$

For downstream,

$$a_y = \sqrt{\gamma RT_y}$$

$$\Rightarrow \quad \frac{a_y}{a_x} = \frac{\sqrt{\gamma RT_y}}{\sqrt{\gamma RT_x}} = \sqrt{\frac{T_y}{T_x}}$$

Substituting $\frac{T_y}{T_x}$ value from equation no (4.13)

$$\Rightarrow \quad \frac{a_y}{a_x} = \sqrt{\frac{\left[\frac{2\gamma}{\gamma-1}M_x^2-1\right]\left[1+\frac{\gamma-1}{2}M_x^2\right]}{\frac{M_x^2}{2(\gamma-1)}\times(\gamma+1)^2}} \quad \text{----(4.14)}$$

### 4.1.7. Rankine – Hugoniot Equation [Density Ratio Across the Shock]

We know that density, $\rho = \frac{p}{RT}$

For upstream shock wave

$$\rho_x = \frac{p_x}{RT_x}$$

For down stream shock wave

$$\rho_y = \frac{p_y}{RT_y}$$

$$\Rightarrow \quad \frac{\rho_y}{\rho_x} = \frac{\frac{p_y}{RT_y}}{\frac{p_x}{RT_x}}$$

$$\Rightarrow \quad \boxed{\frac{\rho_y}{\rho_x} = \frac{p_y}{p_x} \times \frac{T_x}{T_y}} \quad \text{----(4.15)}$$

From equation no (4.12), we know

$$\frac{p_y}{p_x} = \frac{2\gamma}{\gamma+1} M_x^2 - \left(\frac{\gamma-1}{\gamma+1}\right)$$

$$\Rightarrow \quad \frac{2\gamma}{\gamma+1} M_x^2 = \frac{p_y}{p_x} + \frac{\gamma-1}{2\gamma}$$

$$\Rightarrow \quad \boxed{M_x^2 = \frac{\gamma+1}{2\gamma}\left(\frac{p_y}{p_x}\right) + \frac{\gamma-1}{2\gamma}} \qquad ----(4.16)$$

From equation no (4.13), we know that,

$$\frac{T_y}{T_x} = \frac{\left[\frac{2\gamma}{\gamma-1} M_x^2 - 1\right]\left[1 + \frac{\gamma-1}{2} M_x^2\right]}{\frac{M_x^2}{2(\gamma-1)} \times (\gamma+1)^2}$$

Substituting $M_x^2$ values

$$\frac{T_y}{T_x} = \frac{\frac{2\gamma}{\gamma-1}\left[\frac{\gamma+1}{2\gamma}\left(\frac{p_y}{p_x}\right) + \frac{\gamma-1}{2\gamma}\right] - 1 \times \left[1 + \frac{\gamma-1}{2}\left(\frac{\gamma+1}{2\gamma}\left(\frac{p_y}{p_x}\right) + \frac{\gamma-1}{2\gamma}\right)\right]}{\left[\frac{\gamma+1}{2\gamma}\left(\frac{p_y}{p_x}\right) + \frac{\gamma-1}{2\gamma}\right] \times \frac{(\gamma+1)^2}{2(\gamma-1)}}$$

$$\frac{T_y}{T_x} = \frac{\left[\frac{\gamma+1}{\gamma-1}\frac{p_y}{p_x} + 1 - 1\right] \times \left[1 + \frac{(\gamma-1)(\gamma+1)}{4\gamma}\frac{p_y}{p_x} + \frac{(\gamma-1)^2}{4\gamma}\right]}{\frac{(\gamma+1)^3}{4\gamma(\gamma-1)}\left(\frac{p_y}{p_x}\right) + \frac{(\gamma+1)^2}{4\gamma}}$$

$$= \frac{\left[\frac{\gamma+1}{\gamma-1}\frac{p_y}{p_x}\right]\left[1 + \frac{(\gamma-1)(\gamma+1)}{4\gamma}\frac{p_y}{p_x} + \frac{(\gamma-1)^2}{4\gamma}\right]}{\frac{(\gamma+1)^3}{4\gamma(\gamma-1)}\left(\frac{p_y}{p_x}\right) + \frac{(\gamma+1)^2}{4\gamma}}$$

$$= \frac{\frac{\gamma+1}{\gamma-1}\frac{p_y}{p_x} + \frac{(\gamma+1)^2}{4\gamma}\left[\frac{p_y}{p_x}\right]^2 + \frac{(\gamma-1)(\gamma+1)}{4\gamma} \times \frac{p_y}{p_x}}{\frac{(\gamma+1)^3}{4\gamma(\gamma-1)}\left(\frac{p_y}{p_x}\right) + \frac{(\gamma+1)^2}{4\gamma}}$$

Taking out $\frac{(\gamma+1)^3}{4\gamma(\gamma-1)} \times \frac{p_y}{p_x}$

$$= \frac{\frac{(\gamma+1)^3}{4\gamma(\gamma-1)} \times \frac{p_y}{p_x}\left[\frac{4\gamma}{(\gamma+1)^2} + \left(\frac{\gamma-1}{\gamma+1}\right)\frac{p_y}{p_x} + \frac{(\gamma-1)^2}{(\gamma+1)^2}\right]}{\frac{(\gamma+1)^3}{4\gamma(\gamma-1)}\left(\frac{p_y}{p_x} + \frac{\gamma-1}{\gamma+1}\right)}$$

$$= \frac{\frac{p_y}{p_x}\left[\frac{4\gamma}{(\gamma+1)^2} + \left(\frac{\gamma-1}{\gamma+1}\right)\frac{p_y}{p_x} + \frac{(\gamma-1)^2}{(\gamma+1)^2}\right]}{\left(\frac{p_y}{p_x} + \frac{\gamma-1}{\gamma+1}\right)}$$

$$= \frac{\frac{p_y}{p_x}\left[\left(\frac{\gamma-1}{\gamma+1}\right)\frac{p_y}{p_x} + \frac{4\gamma + (\gamma-1)^2}{(\gamma+1)^2}\right]}{\left(\frac{p_y}{p_x} + \frac{\gamma-1}{\gamma+1}\right)}$$

$$= \frac{\frac{p_y}{p_x}\left[\frac{\gamma-1}{\gamma+1}\times\frac{p_y}{p_x}+\frac{4\gamma+(\gamma^2-2\gamma+1)}{(\gamma+1)^2}\right]}{\left(\frac{p_y}{p_x}+\frac{\gamma-1}{\gamma+1}\right)}$$

$$= \frac{\frac{p_y}{p_x}\left[\frac{\gamma-1}{\gamma+1}\times\frac{p_y}{p_x}+\frac{\gamma^2+2\gamma+1}{(\gamma+1)^2}\right]}{\frac{p_y}{p_x}+\frac{\gamma-1}{\gamma+1}}$$

$$= \frac{\frac{p_y}{p_x}\left[\frac{\gamma-1}{\gamma+1}\times\frac{p_y}{p_x}+\frac{(\gamma+1)^2}{(\gamma+1)^2}\right]}{\frac{p_y}{p_x}+\frac{\gamma-1}{\gamma+1}}$$

$$\frac{T_y}{T_x} = \frac{\frac{p_y}{p_x}\left[\frac{\gamma-1}{\gamma+1}\times\frac{p_y}{p_x}+1\right]}{\frac{p_y}{p_x}+\frac{\gamma-1}{\gamma+1}}$$

$$\Rightarrow \boxed{\frac{T_y}{T_x} = \frac{\frac{p_y}{p_x}\left[\frac{\gamma-1}{\gamma+1}\times\frac{p_y}{p_x}+1\right]}{\frac{p_y}{p_x}+\frac{\gamma-1}{\gamma+1}}} \qquad ----(4.17)$$

$$\Rightarrow \quad \frac{p_y}{p_x} = \frac{\frac{T_y}{T_x}\times\left(\frac{p_y}{p_x}+\frac{\gamma-1}{\gamma+1}\right)}{\left[1+\frac{\gamma-1}{\gamma+1}\times\frac{p_y}{p_x}\right]}$$

$$\Rightarrow \boxed{\frac{p_y}{p_x}\times\frac{T_x}{T_y} = \frac{\frac{p_y}{p_x}+\frac{\gamma-1}{\gamma+1}}{1+\frac{\gamma-1}{\gamma+1}\times\frac{p_y}{p_x}}} \qquad ----(4.18)$$

From equation (4.15), we know that,

$$\frac{\rho_y}{\rho_x} = \frac{p_y}{p_x}\times\frac{T_x}{T_y}$$

$$\Rightarrow \quad \frac{\rho_y}{\rho_x} = \frac{\frac{p_y}{p_x}+\frac{\gamma-1}{\gamma+1}}{1+\frac{\gamma-1}{\gamma+1}\times\frac{p_y}{p_x}} \qquad \text{[From equation (4.18)]}$$

$$\Rightarrow \quad \frac{\rho_y}{\rho_x} = \frac{\frac{\gamma-1}{\gamma+1}\left[1+\frac{\gamma+1}{\gamma-1}\times\frac{p_y}{p_x}\right]}{\frac{\gamma-1}{\gamma+1}\left[\frac{\gamma+1}{\gamma-1}+\frac{p_y}{p_x}\right]}$$

$$\Rightarrow \quad \boxed{\frac{\rho_y}{\rho_x} = \frac{\left[1+\frac{\gamma+1}{\gamma-1}\times\frac{p_y}{p_x}\right]}{\left[\frac{\gamma+1}{\gamma-1}+\frac{p_y}{p_x}\right]}} \qquad ----(4.19)$$

$$\Rightarrow \quad \frac{\rho_y}{\rho_x}\left[\frac{\gamma+1}{\gamma-1}+\frac{p_y}{p_x}\right] = 1+\frac{\gamma+1}{\gamma-1}\times\frac{p_y}{p_x}$$

$$\Rightarrow \quad \frac{\rho_y}{\rho_x}\left(\frac{\gamma+1}{\gamma-1}\right)+\frac{p_y}{p_x}\left(\frac{\rho_y}{\rho_x}\right) = 1+\frac{\gamma+1}{\gamma-1}\times\frac{p_y}{p_x}$$

$$\Rightarrow \quad \frac{\rho_y}{\rho_x}\left(\frac{\gamma+1}{\gamma-1}\right) = 1+\frac{\gamma+1}{\gamma-1}\times\frac{p_y}{p_x}-\frac{p_y}{p_x}\left(\frac{\rho_y}{\rho_x}\right)$$

$$\Rightarrow \quad \frac{\rho_y}{\rho_x}\left(\frac{\gamma+1}{\gamma-1}\right) = 1+\frac{p_y}{p_x}\left[\frac{\gamma+1}{\gamma-1}-\frac{\rho_y}{\rho_x}\right]$$

$$\Rightarrow \quad \frac{\rho_y}{\rho_x}\left(\frac{\gamma+1}{\gamma-1}\right)-1 = \frac{p_y}{p_x}\left[\frac{\gamma+1}{\gamma-1}-\frac{\rho_y}{\rho_x}\right]$$

$$\Rightarrow \quad \boxed{\frac{p_y}{p_x} = \frac{\frac{\rho_y}{\rho_x}\left(\frac{\gamma+1}{\gamma-1}\right)-1}{\left[\frac{\gamma+1}{\gamma-1}-\frac{\rho_y}{\rho_x}\right]}} \qquad ----(4.20)$$

Equation (4.19) and (4.20) are known s Rankine–Hugoniot equation. This gives the pressure–density relation.

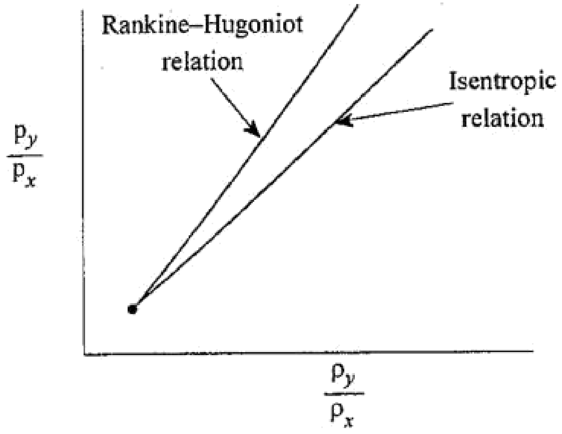


*Fig. 4.4 Comparison of Rankine–Hugoniot and Isentropic relations*

### 4.1.8 Stagnation Pressure Ratio Across the Shock

We know that,

$$\frac{p_{0y}}{p_{0x}} = \frac{p_{0y}}{p_y}\times\frac{p_y}{p_x}\times\frac{p_x}{p_{0x}} \qquad ----(4.21)$$

and

$$\frac{p_0}{p} = \left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{\gamma}{\gamma-1}} \qquad \text{[From Chapter 1 – Equation no.1.8]}$$

Similarly

$$\frac{p_{0y}}{p_y} = \left[1+\frac{\gamma-1}{2}M_y{}^2\right]^{\frac{\gamma}{\gamma-1}}$$

Substituting $M_y{}^2$ value from equation no (4.11)

$$\Rightarrow \quad \frac{p_{0y}}{p_y} = \left[1+\frac{\gamma-1}{2}\times\left(\frac{\frac{2}{\gamma-1}+M_x{}^2}{\frac{2\gamma}{\gamma-1}M_x{}^2-1}\right)\right]^{\frac{\gamma}{\gamma-1}}$$

$$\Rightarrow \quad \frac{p_{0y}}{p_y} = \left[1+\frac{1+\frac{\gamma-1}{2}\times M_x{}^2}{\frac{2\gamma}{\gamma-1}M_x{}^2-1}\right]^{\frac{\gamma}{\gamma-1}}$$

$$\Rightarrow \quad \frac{p_{0y}}{p_y} = \left[\frac{\frac{2\gamma}{\gamma-1}M_x{}^2-1+1+\frac{\gamma-1}{2}M_x{}^2}{\frac{2\gamma}{\gamma-1}M_x{}^2-1}\right]^{\frac{\gamma}{\gamma-1}}$$

$$= \left[ \frac{\frac{2\gamma}{\gamma-1} M_x^2 + \frac{\gamma-1}{2} M_x^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1} \right]^{\frac{\gamma}{\gamma-1}}$$

$$= \left[ \frac{\frac{M_x^2}{2} \left( \frac{4\gamma}{\gamma-1} + \gamma - 1 \right)}{\frac{2\gamma}{\gamma-1} M_x^2 - 1} \right]^{\frac{\gamma}{\gamma-1}}$$

$$= \left[ \frac{\frac{M_x^2}{2(\gamma-1)} \left( 4\gamma + (\gamma-1)^2 \right)}{\frac{2\gamma}{\gamma-1} M_x^2 - 1} \right]^{\frac{\gamma}{\gamma-1}}$$

$$= \left[ \frac{\frac{M_x^2}{2(\gamma-1)} \left( 4\gamma + \gamma^2 - 2\gamma + 1 \right)}{\frac{2\gamma}{\gamma-1} M_x^2 - 1} \right]^{\frac{\gamma}{\gamma-1}}$$

$$= \left[ \frac{\frac{M_x^2}{2(\gamma-1)} \left( \gamma^2 + 2\gamma + 1 \right)}{\frac{2\gamma}{\gamma-1} M_x^2 - 1} \right]^{\frac{\gamma}{\gamma-1}}$$

$$= \left[ \frac{\frac{M_x^2}{2(\gamma-1)} (\gamma+1)^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1} \right]^{\frac{\gamma}{\gamma-1}}$$

$$= \left[ \frac{M_x^2 (\gamma+1)^2}{2(\gamma-1) \left[ \frac{2\gamma}{\gamma-1} M_x^2 - 1 \right]} \right]^{\frac{\gamma}{\gamma-1}}$$

$$= \left[ \frac{M_x^2 (\gamma+1)^2}{4\gamma M_x^2 - 2(\gamma-1)} \right]^{\frac{\gamma}{\gamma-1}}$$

$$= \left[ \frac{M_x^2 (\gamma+1)^2}{2(2\gamma M_x^2 - (\gamma-1)} \right]^{\frac{\gamma}{\gamma-1}}$$

Divided by $(\gamma + 1)$

$$= \left[ \frac{\frac{M_x^2}{(\gamma+1)} (\gamma+1)^2}{2 \left[ \frac{2\gamma M_x^2}{\gamma+1} - \frac{\gamma-1}{\gamma+1} \right]} \right]^{\frac{\gamma}{\gamma-1}}$$

$$= \left[ \frac{M_x^2 (\gamma+1)}{2 \left[ \frac{2\gamma M_x^2}{\gamma+1} - \frac{\gamma-1}{\gamma+1} \right]} \right]^{\frac{\gamma}{\gamma-1}}$$

$$\boxed{\frac{p_{0y}}{p_y} = \left[ \frac{\frac{M_x^2}{2} (\gamma+1)}{\frac{2\gamma}{\gamma+1} M_x^2 - \frac{\gamma-1}{\gamma+1}} \right]^{\frac{\gamma}{\gamma-1}}} \quad \text{----} (4.22)$$

Similarly,

$$\frac{p_{0x}}{p_x} = \left[1+\frac{\gamma-1}{2}M_x^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\Rightarrow \boxed{\frac{p_x}{p_{0x}} = \left[1+\frac{\gamma-1}{2}M_x^2\right]^{\frac{-\gamma}{\gamma-1}}} \qquad ----(4.23)$$

From equation no (4.12), we know that

$$\frac{p_y}{p_x} = \frac{2\gamma}{\gamma+1}M_x^2 - \left(\frac{\gamma-1}{\gamma+1}\right)$$

Substituting $\frac{p_{0y}}{p_y}$, $\frac{p_y}{p_x}$ and $\frac{p_x}{p_{0x}}$ values in equation no (4.21)

$$\frac{p_{0y}}{p_{0x}} = \left[\frac{\frac{M_x^2}{2}(\gamma+1)}{\frac{2\gamma}{\gamma+1}M_x^2-\frac{\gamma-1}{\gamma+1}}\right]^{\frac{\gamma}{\gamma-1}} \times \frac{2\gamma}{\gamma+1}M_x^2-\left(\frac{\gamma-1}{\gamma+1}\right)$$

$$\times\left[1+\frac{\gamma-1}{2}M_x^2\right]^{\frac{-\gamma}{\gamma-1}}$$

$$\Rightarrow \frac{p_{0y}}{p_{0x}} = \left[\frac{\frac{M_x^2}{2}(\gamma+1)}{\frac{2\gamma}{\gamma+1}M_x^2-\frac{\gamma-1}{\gamma+1}}\right]^{\frac{\gamma}{\gamma-1}} \times \frac{2\gamma}{\gamma+1}M_x^2-\left(\frac{\gamma-1}{\gamma+1}\right)$$

$$\times \frac{1}{\left[1+\frac{\gamma-1}{2}M_x^2\right]^{\frac{\gamma}{\gamma-1}}}$$

$$= \left[\frac{M_x^2}{2}(\gamma+1)\right]^{\frac{\gamma}{\gamma-1}} \times \left[\frac{2\gamma}{\gamma+1}M_x^2-\frac{\gamma-1}{\gamma+1}\right]^{\frac{-\gamma}{\gamma-1}+1}$$

$$\times \frac{1}{\left[1+\frac{\gamma-1}{2}M_x^2\right]^{\frac{\gamma}{\gamma-1}}}$$

$$= \left[\frac{\frac{M_x^2}{2}(\gamma+1)}{1+\frac{\gamma-1}{2}M_x^2}\right]^{\frac{\gamma}{\gamma-1}} \times \left[\frac{2\gamma}{\gamma+1}M_x^2-\frac{\gamma-1}{\gamma+1}\right]^{\frac{-\gamma+\gamma-1}{\gamma-1}}$$

$$= \left[\frac{\frac{M_x^2}{2}(\gamma+1)}{1+\frac{\gamma-1}{2}M_x^2}\right]^{\frac{\gamma}{\gamma-1}} \times \left[\frac{2\gamma}{\gamma+1}M_x^2-\frac{\gamma-1}{\gamma+1}\right]^{\frac{-1}{\gamma-1}}$$

$$\boxed{\frac{p_{0y}}{p_{0x}} = \left[\frac{\frac{M_x^2}{2}(\gamma+1)}{1+\frac{\gamma-1}{2}M_x^2}\right]^{\frac{\gamma}{\gamma-1}} \times \left[\frac{2\gamma}{\gamma+1}M_x^2-\frac{\gamma-1}{\gamma+1}\right]^{\frac{-1}{\gamma-1}}}$$

----(4.24)

This equation gives the stagnation pressure ratio across the shock.

## 4.1.9. Change in Entropy Across the Shock

We know that,

Change in entropy, $\Delta s = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$

$$= c_p \ln \frac{T_2}{T_1} - c_p \left(\frac{\gamma - 1}{\gamma}\right) \ln \frac{p_2}{p_1}$$

$$\left[\because R = c_p \frac{(\gamma - 1)}{\gamma}\right]$$

$$= c_p \ln \frac{T_2}{T_1} - c_p \ln \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\Delta s = c_p \ln \left[\frac{\frac{T_2}{T_1}}{\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}}\right]$$

For shock waves

$$\Delta s = c_p \ln \left[\frac{\frac{T_y}{T_x}}{\left(\frac{p_y}{p_x}\right)^{\frac{\gamma-1}{\gamma}}}\right] \qquad \text{----}(4.25)$$

We know that,

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

Similarly

$$\frac{T_{0y}}{T_y} = 1 + \frac{\gamma - 1}{2} M_y^2$$

$$\frac{T_{0x}}{T_x} = 1 + \frac{\gamma - 1}{2} M_x^2$$

$$\Rightarrow \frac{\frac{T_{0x}}{T_x}}{\frac{T_{0y}}{T_y}} = \frac{1 + \frac{\gamma - 1}{2} M_x^2}{1 + \frac{\gamma - 1}{2} M_y^2}$$

$$\Rightarrow \frac{T_y}{T_x} = \frac{1 + \frac{\gamma - 1}{2} M_x^2}{1 + \frac{\gamma - 1}{2} M_y^2} \qquad \text{-----}(4.26)$$

$$[\because T_{0x} = T_{0y}]$$

We know that,

$$\frac{p_0}{p} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{\gamma}{\gamma-1}}$$

Similarly

$$\frac{p_{0y}}{p_y} = \left[1 + \frac{\gamma - 1}{2} M_y^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p_{0x}}{p_x} = \left[1 + \frac{\gamma - 1}{2} M_x^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\frac{p_{0x}}{p_x}}{\frac{p_{0y}}{p_y}} = \frac{\left[1 + \frac{\gamma - 1}{2} M_x^2\right]^{\frac{\gamma}{\gamma-1}}}{\left[1 + \frac{\gamma - 1}{2} M_y^2\right]^{\frac{\gamma}{\gamma-1}}}$$

$$\frac{p_y}{p_x} \times \frac{p_{0x}}{p_{0y}} = \frac{\left[1 + \frac{\gamma - 1}{2} M_x^2\right]^{\frac{\gamma}{\gamma-1}}}{\left[1 + \frac{\gamma - 1}{2} M_y^2\right]^{\frac{\gamma}{\gamma-1}}}$$

$$\Rightarrow \quad \frac{p_y}{p_x} = \frac{\left[1 + \frac{\gamma - 1}{2} M_x^2\right]^{\frac{\gamma}{\gamma-1}}}{\left[1 + \frac{\gamma - 1}{2} M_y^2\right]^{\frac{\gamma}{\gamma-1}}} \times \frac{p_{0y}}{p_{0x}}$$

$$\Rightarrow \quad \left[\frac{p_y}{p_x}\right]^{\frac{\gamma-1}{\gamma}} = \frac{1 + \frac{\gamma - 1}{2} M_x^2}{1 + \frac{\gamma - 1}{2} M_y^2} \times \left[\frac{p_{0y}}{p_{0x}}\right]^{\frac{\gamma-1}{\gamma}} \quad \text{----(4.27)}$$

Substituting equation (4.26) and (4.27) in equation (4.25)

$$(4.25) \Rightarrow \qquad \Delta s = c_p \, ln \left[ \frac{\dfrac{1 + \frac{\gamma - 1}{2} M_x^2}{1 + \frac{\gamma - 1}{2} M_y^2}}{\dfrac{1 + \frac{\gamma - 1}{2} M_x^2}{1 + \frac{\gamma - 1}{2} M_y^2} \times \left(\frac{p_{0y}}{p_{0x}}\right)^{\frac{\gamma-1}{\gamma}}} \right]$$

$$= c_p \, ln \left(\frac{p_{0y}}{p_{0x}}\right)^{-\left(\frac{\gamma-1}{\gamma}\right)}$$

$$\Delta s = - c_p \left(\frac{\gamma-1}{\gamma}\right) ln \left(\frac{p_{0y}}{p_{0x}}\right)$$

$$= - \frac{\gamma R}{\gamma-1} \times \frac{\gamma-1}{\gamma} \times ln \left(\frac{p_{0y}}{p_{0x}}\right)$$

$$\left[\because c_p = \frac{\gamma R}{\gamma-1}\right]$$

$$= - R \, ln \left(\frac{p_{0y}}{p_{0x}}\right)$$

$$\frac{\Delta s}{R} = - ln \left(\frac{p_{0y}}{p_{0x}}\right)$$

Substitute $\frac{p_{0y}}{p_{0x}}$ value [from equation no. (4.24)]

$$\Rightarrow - \frac{\Delta s}{R} = ln \left[ \frac{\frac{(\gamma + 1)}{2} M_x^2}{1 + \frac{\gamma - 1}{2} M_x^2} \right]^{\frac{\gamma}{\gamma-1}} \times \left[\frac{2\gamma}{\gamma + 1} M_x^2 - \frac{\gamma - 1}{\gamma + 1}\right]^{\frac{-1}{\gamma-1}}$$

$$\Rightarrow - \frac{\Delta s}{R} = ln \left[ \frac{\frac{(\gamma + 1)}{2} M_x^2}{1 + \frac{\gamma - 1}{2} M_x^2} \right]^{\frac{\gamma}{\gamma-1}} + ln \left[\frac{2\gamma}{\gamma + 1} M_x^2 - \frac{\gamma - 1}{\gamma + 1}\right]^{\frac{-1}{\gamma-1}}$$

$$= \frac{\gamma}{\gamma-1} \, ln \left[ \frac{\frac{(\gamma + 1)}{2} M_x^2}{1 + \frac{\gamma - 1}{2} M_x^2} \right] - \frac{1}{\gamma-1} \, ln \left[\frac{2\gamma}{\gamma + 1} M_x^2 - \frac{\gamma - 1}{\gamma + 1}\right]$$

$$\Rightarrow \boxed{\frac{\Delta s}{R} = - \frac{\gamma}{\gamma-1} \, ln \left[ \frac{\frac{(\gamma + 1)}{2} M_x^2}{1 + \frac{\gamma - 1}{2} M_x^2} \right] + \frac{1}{\gamma-1} \, ln \left[\frac{2\gamma}{\gamma + 1} M_x^2 - \frac{\gamma - 1}{\gamma + 1}\right]} \quad \text{----(4.28)}$$

This equation gives change in entropy equation in terms of Mach number.

### 4.1.10 Strength of a Shock Wave

It is defined as the ratio of difference in down stream and upstream shock pressures $(p_y - p_x)$ to upstream shock pressures $(p_x)$. It is denoted by $\xi$.

$$\xi = \frac{p_y - p_x}{p_x}$$

$$\boxed{\xi = \frac{p_y}{p_x} - 1} \qquad ----(4.29)$$

Substituting $\frac{p_y}{p_x}$ value from equation no (4.12)

$$\xi = \left[\frac{2\gamma}{\gamma+1} M_x^2 - \left(\frac{\gamma-1}{\gamma+1}\right)\right] - 1$$

$$= \frac{1}{\gamma+1}\left[2\gamma M_x^2 - (\gamma-1) - (\gamma+1)\right]$$

$$= \frac{1}{\gamma+1}\left[2\gamma M_x^2 - \gamma + 1 - \gamma - 1\right]$$

$$= \frac{1}{\gamma+1}\left[2\gamma M_x^2 - 2\gamma\right]$$

$$\boxed{\xi = \frac{2\gamma}{\gamma+1}[M_x^2 - 1]} \qquad ----(4.30)$$

From equation (4.30), we came to know, strength of shock wave is directly proportional to $(M_x^2 - 1)$.

$$\boxed{\xi \propto M_x^2 - 1} \qquad ---- (4.31)$$

The strength of shock wave may be expressed in another form using Rankine–Hugoniot equation.

From equation (4.20), [Rankine –Hugoniot equation]

$$\frac{p_y}{p_x} = \frac{\frac{\rho_y}{\rho_x}\left(\frac{\gamma+1}{\gamma-1}\right) - 1}{\frac{\gamma+1}{\gamma-1} - \frac{\rho_y}{\rho_x}}$$

We know that

$$\xi = \frac{p_y}{p_x} - 1$$

$$\Rightarrow \quad \xi = \frac{\frac{\rho_y}{\rho_x}\left(\frac{\gamma+1}{\gamma-1}\right) - 1}{\frac{\gamma+1}{\gamma-1} - \frac{\rho_y}{\rho_x}} - 1$$

$$= \frac{\frac{\rho_y}{\rho_x}\left(\frac{\gamma+1}{\gamma-1}\right) - 1 - \frac{\gamma+1}{\gamma-1} + \frac{\rho_y}{\rho_x}}{\frac{\gamma+1}{\gamma-1} - \frac{\rho_y}{\rho_x}}$$

$$= \frac{\frac{\gamma+1}{\gamma-1}\left[\frac{\rho_y}{\rho_x} - 1\right] + \left[\frac{\rho_y}{\rho_x} - 1\right]}{\frac{\gamma+1+1-1}{\gamma-1} - \frac{\rho_y}{\rho_x}}$$

[Adding and subtracting 1]

$$= \frac{\frac{\gamma+1}{\gamma-1}\left[\frac{\rho_y}{\rho_x} - 1\right] + \left[\frac{\rho_y}{\rho_x} - 1\right]}{\frac{\gamma-1}{\gamma-1} + \frac{2}{\gamma-1} - \frac{\rho_y}{\rho_x}}$$

$$= \frac{\left[\frac{\rho_y}{\rho_x} - 1\right]\left[\frac{\gamma+1}{\gamma-1} + 1\right]}{\frac{2}{\gamma-1} + 1 - \frac{\rho_y}{\rho_x}}$$

$$= \frac{\left[\frac{\rho_y}{\rho_x}-1\right]\left[\frac{\gamma+1+\gamma-1}{\gamma-1}\right]}{\frac{2}{\gamma-1}-\left[\frac{\rho_y}{\rho_x}-1\right]}$$

$$= \frac{\left[\frac{\rho_y}{\rho_x}-1\right]\left[\frac{2\gamma}{\gamma-1}\right]}{\frac{2}{\gamma-1}-\left[\frac{\rho_y}{\rho_x}-1\right]}$$

$$\Rightarrow \boxed{\xi = \frac{\left[\frac{2\gamma}{\gamma-1}\right]\left[\frac{\rho_y}{\rho_x}-1\right]}{\frac{2}{\gamma-1}-\left[\frac{\rho_y}{\rho_x}-1\right]}} \qquad ----(4.32)$$

From equation (4.32) we came to know strength of shock wave is directly proportional to $\left[\frac{\rho_y}{\rho_x}-1\right]$

$$\Rightarrow \boxed{\xi \propto \left[\frac{\rho_y}{\rho_x}-1\right]} \qquad ----(4.33)$$

### 4.1.11 Supersonic Wind Tunnels

In supersonic wind tunnel, the diffusion of the supersonic flow after the test section takes place through a shock wave. It is shown in fig.4.5. It consists of nozzle, test section and diffuser.

At designed running condition there is no shock wave anywhere in the tunnel. It happens during ideal flow. The ideal flow is reversibly accelerated in the nozzle to supersonic mach number and the model can be

tested at supersonic velocity in the test section. In the supersonic diffuser, the supersonic flow leaving the test section is decelerated and pressure of the gas is raised.

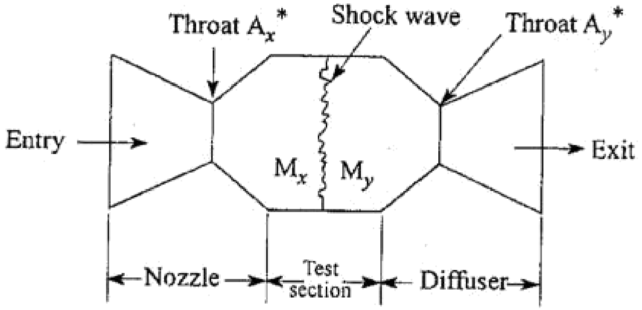


*Fig.4.5 Supersonic wind tunnel with shock in the test section*

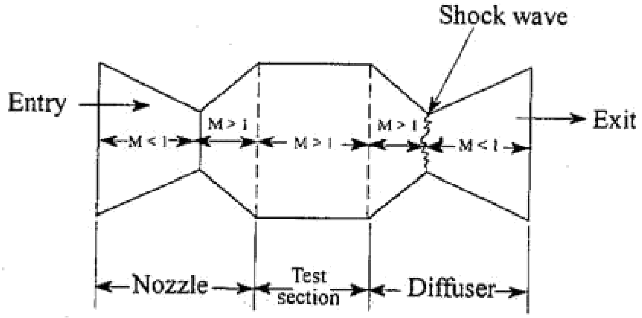


*Fig.4.6 Supersonic wind tunnel with shock at the diffuser throat*

### 4.1.12 FORMULAE USED

1. Down stream Mach number (or) Mach number after the shock

$$M_y^2 = \frac{\frac{2}{\gamma-1} + M_x^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1}$$

2. Static pressure ratio across the shock

$$\frac{p_y}{p_x} = \frac{2\gamma}{\gamma+1} M_x^2 - \frac{\gamma-1}{\gamma+1}$$

3. Static temperature ratio across the shock

$$\frac{T_y}{T_x} = \frac{\left[1 + \frac{\gamma-1}{2} M_x^2\right]\left[\frac{2\gamma}{\gamma-1} M_x^2 - 1\right]}{\frac{1}{2}\frac{(\gamma+1)^2}{\gamma-1} M_x^2}$$

4. Density ratio across the shock

$$\frac{\rho_y}{\rho_x} = \frac{1 + \frac{\gamma+1}{\gamma-1} \times \frac{p_y}{p_x}}{\frac{\gamma+1}{\gamma-1} + \frac{p_y}{p_x}}$$

5. Stagnation pressure loss, $\Delta p_0 = p_{0x} - p_{0y}$

6. Percentage of stagnation pressure loss, $\Delta p_0 = \frac{p_{0x} - p_{0y}}{p_{0x}} \times 100$

7. Increase in entropy, $\Delta s = R\, ln \left(\frac{p_{0x}}{p_{0y}}\right)$

8. Diffuser efficiency, $\eta_D = \dfrac{\frac{T_{01}}{T_1}\left[\frac{p_{0y}}{p_{0x}}\right]^{\frac{\gamma-1}{\gamma}} - 1}{\frac{\gamma-1}{2} M_1^2}$

### 4.1.13 SOLVED PROBLEMS

1 ***The upstream mach number, pressure and temperature of normal shock wave are 2.4, 2 bar and 270 K respectively. Calculate the Mach number, pressure, temperature and velocity of the gas for the down stream of the shock. Check the calculated values with those given in the gas tables. Take $\gamma$ = 1.3 R = 460 J/kg K.***

***Given***

$M_x = 2.4$

$p_x = 2 \text{ bar} = 2 \times 10^5 \text{ N/m}^2$

$T_x = 270 \text{ K}$

$\gamma = 1.3$

$R = 460 \text{ J/kg K}$

***To find***

1. Down stream Mach number, $M_y$
2. Down stream pressure, $p_y$
3. Down stream temperature, $T_y$
4. Down stream velocity, $c_y$

***Solution***

We know that,

1. Down stream Mach number, $M_y^2 = \frac{\frac{2}{\gamma-1} + M_x^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1}$

$$\Rightarrow \quad M_y^2 = \frac{\frac{2}{1.3-1} + (2.4)^2}{\frac{2 \times 1.3}{1.3-1}(2.4)^2 - 1}$$

$$= 0.254$$

Downstream Mach number (or) Mach number after the shock wave $\Rightarrow$ $M_y = 0.504$

2. Static pressure ratio across the shock

$$\frac{p_y}{p_x} = \frac{2\gamma}{\gamma+1} M_x^2 - \frac{\gamma-1}{\gamma+1}$$

$$= \frac{2 \times 1.3}{1.3+1} (2.4)^2 - \frac{1.3-1}{1.3+1}$$

$$\boxed{\frac{p_y}{p_x} = 6.380}$$

$$\Rightarrow \quad p_y = 6.380 \times p_x$$

$$= 6.380 \times 2 \times 10^5$$

$$p_y = 12.76 \times 10^5 \text{ N/m}^2$$

Downstream pressure (or)
Pressure after the shock $\Rightarrow p_y = 12.76 \times 10^5 \text{ N/m}^2$

3. Static temperature ratio across the shock

$$\frac{T_y}{T_x} = \frac{\left[1 + \frac{\gamma-1}{2} M_x^2\right]\left[\frac{2\gamma}{\gamma-1} M_x^2 - 1\right]}{\frac{1}{2} \frac{(\gamma+1)^2}{\gamma-1} M_x^2}$$

$$= \frac{\left[1 + \frac{1.3-1}{2} + (2.4)^2\right]\left[\frac{2 \times 1.3}{1.3-1} (2.4)^2 - 1\right]}{\frac{1}{2} \times \frac{(1.3+1)^2}{1.3-1} \times (2.4)^2}$$

$$\boxed{\frac{T_y}{T_x} = 1.795}$$

$$\Rightarrow \quad T_y = 1.795 \times T_x$$

$$= 1.795 \times 270$$

$$T_y = 484.65 \text{ K}$$

Downstream temperature (or)
Temperature after the shock $T_y = 484.65 \text{ K}$

4. Downstream velocity (or) velocity after the shock

$$c_y = M_y \times a_y$$

$$= M_y \times \sqrt{\gamma R T_y}$$

$$= 0.504 \times \sqrt{1.3 \times 460 \times 484.65}$$

$$\boxed{c_y = 271.32 \text{ m/s}}$$

Refer Normsl shocks tables for $M_x = 2.4$ and $\gamma = 1.3$ .

$$M_y = 0.505$$

$$\frac{p_y}{p_x} = 6.380$$

*[From gas tables (S.M. Yahya, Fifth edition) page no.49]*

$$\frac{T_y}{T_x} = 1.797$$

From these values we came to know calculated values and gas tables values are same.

***Result :***

1. $M_y = 0.505$
2. $p_y = 12.76 \times 10^5 \text{ N/m}^2$
3. $T_y = 484.65 \text{ K}$
4. $c_y = 271.32 \text{ m/s}$

**2** *A jet of air at 270 K and 0.7 bar has an initial Mach number of 1.9. If it passes through a normal shock wave, determine the following for downstream of the shock.*

1. *Mach number*
2. *Pressure*
3. *Temperature*
4. *Speed of sound*
5. *Jet velocity*
6. *Density*

***Given :***

$T_x = 270$ K

$p_x = 0.7$ bar $= 0.7 \times 10^5$ N/m$^2$

$M_x = 1.9$

***To find : [ At downstream]***

1. $M_y$
2. $p_y$
3. $T_y$
4. $a_y$
5. $c_y$
6. $\rho_y$

***Solution :***

Refer Normal shocks tables for $M_x = 1.9$ and $\gamma = 1.4$.

$$M_y = 0.596$$

$$\frac{p_y}{p_x} = 4.045 \qquad \textit{[From gas tables page no.53]}$$

$$\frac{T_y}{T_x} = 1.608$$

$$\Rightarrow \quad p_y = 4.045 \times p_x$$

$$= 4.045 \times 0.7 \times 10^5$$

$$\boxed{p_y = 2.831 \times 10^5 \text{ N/m}^2}$$

$$\Rightarrow \quad T_y = 1.608 \times T_x$$

$$= 1.608 \times 270$$

$$\boxed{T_y = 434.16 \text{ K}}$$

Speed of sound at downstream of the shock

$$a_y = \sqrt{\gamma R T_y}$$

$$= \sqrt{1.4 \times 287 \times 434.16}$$

$$\boxed{a_y = 417.66 \text{ m/s}}$$

Jet velocity at downstream of the shock

$$c_y = M_y \times a_y$$

$$= 0.596 \times 417.66$$

$$\boxed{c_y = 248.93 \text{ m/s}}$$

$$\text{Density} = \rho_y = \frac{p_y}{RT_y} = \frac{2.83 \times 10^5}{287 \times 434.16} = 2.27 \text{ kg/m}^3$$

$$\boxed{\rho_y = 2.27 \text{ kg/m}^3}$$

***Result***

1. $M_y = 0.596$
2. $p_y = 2.831 \times 10^5$ N/m$^2$

3. $T_y = 434.16$ K

4. $a_y = 417.66$ m/s

5. $c_y = 248.93$ m/s

6. $\rho_y = 2.27$ kg/m$^3$.

$\boxed{3}$ ***Air at a Mach number of 1.6, pressure of 0.9 bar, and temperature of 370 K passes through a normal shock. Calculate the density after the shock. Compare this value with isentropic compression through the same pressure ratio.***

***Given :***

$M_x = 1.6$

$p_x = 0.9 \text{ bar} = 0.9 \times 10^5 \text{ N/m}^2$

$T_x = 370$ K

***To find :***

1. Density after the shock, $\rho_y$

2. Compare this value with isentropic compression.

***Solution :***

Refer Normal shocks tables for $M_x = 1.6$ and $\gamma = 1.4$.

$$M_y = 0.668$$

$$\frac{p_y}{p_x} = 2.820 \qquad \textit{[From gas tables page no.52]}$$

$$\frac{T_y}{T_x} = 1.388$$

$$\Rightarrow \quad p_y = 2.820 \times p_x$$

$$= 2.820 \times 0.9 \times 10^5$$

$$\boxed{p_y = 2.538 \times 10^5 \text{ N/m}^2}$$

$$\Rightarrow \quad T_y = 1.388 \times T_x$$

$$= 1.388 \times 370$$

$$\boxed{T_y = 513.56 \text{ K}}$$

We know that

$$\rho_y = \frac{p_y}{RT_y}$$

$$= \frac{2.538 \times 10^5}{287 \times 513.56}$$

$$\boxed{\rho_y = 1.722 \text{ kg/m}^3}$$

$$\boxed{\text{Density after the shock, } \rho_y = 1.722 \text{ kg/m}^3}$$

For Isentropic flow,

$$\frac{\rho_y}{\rho_x} = \left(\frac{p_y}{p_x}\right)^{\frac{1}{\gamma}}$$

$$\Rightarrow \quad \rho_y = \rho_x \left(\frac{p_y}{p_x}\right)^{\frac{1}{\gamma}}$$

$$= \frac{p_x}{RT_x}\left(\frac{p_y}{p_x}\right)^{\frac{1}{\gamma}}$$

$$= \frac{0.9 \times 10^5}{287 \times 370} [2.820]^{\frac{1}{1.4}}$$

$$\boxed{\rho_y = 1.77 \text{ kg/m}^3}$$

From that we came to know that the final density in the isentropic process is greater than in shock process.

***Result***

1. $\rho_y$ [ For shock wave process] = 1.722 kg/m$^3$.
2. $\rho_y$ [ For Isentropic process] = 1.77 kg/m$^3$

**4** ***An aircraft flies at a Mach number of 1.1 at an altitude of 15,000 metres. The compression in its engine is partly achieved by a normal shock wave standing at the entry of its diffuser. Determine the following for downstream of the shock.***

*1. Mach number*

*2. Temperature of the air*

*3. Pressure of the air*

*4. Stagnation pressure loss across the shock.*

***Given***

$M_x = 1.1$

Altitude, Z = 15,000 m

***To find [At downstream]***

1. $M_y$
2. $T_y$
3. $p_y$
4. $\Delta p_0 = p_{0x} - p_{0y}$

***Solution :***

Refer gas tables for Z = 15,000 m

*[From gas tables page no.20]*

$T_x = 216.6$ K

$p_x = 0.120$ bar

$p_x = 0.120 \times 10^5$ N/m$^2$

Refer Normal shocks tables for $M_x = 1.1$ and $\gamma = 1.4$.

$M_y = 0.911$

$\frac{p_y}{p_x} = 1.245$ *[From gas tables page no.52]*

$\frac{T_y}{T_x} = 1.065$

$\frac{p_{0y}}{p_{0x}} = 0.998$

$\frac{p_{0y}}{p_x} = 2.133$

$\Rightarrow \quad p_y = 1.245 \times p_x$

$= 1.245 \times 0.120 \times 10^5$

$$\boxed{p_y = 0.149 \times 10^5 \text{ N/m}^2}$$

$\Rightarrow \quad T_y = 1.065 \times T_x$

$= 1.065 \times 216.6$

$$\boxed{T_y = 230.67 \text{ K}}$$

$$\Rightarrow \quad p_{0y} = 2.133 \times p_x$$

$$= 2.133 \times 0.120 \times 10^5$$

$$\boxed{p_{0y} = 0.2559 \times 10^5 \text{ N/m}^2}$$

$$\frac{p_{0y}}{p_{0x}} = 0.998$$

$$\Rightarrow \quad p_{0x} = \frac{p_{0y}}{0.998}$$

$$= \frac{0.2559 \times 10^5}{0.998}$$

$$\boxed{p_{0x} = 0.2564 \times 10^5 \text{ N/m}^2}$$

**Stagnation pressure loss**

$$\Delta p_0 = p_{0x} - p_{0y}$$

$$= 0.2564 \times 10^5 - 0.2559 \times 10^5$$

$$\boxed{\Delta p_0 = 50 \text{ N/m}^2}$$

***Result*** :

1. $M_y = 0.911$
2. $T_y = 230.67$ K
3. $p_y = 0.149 \times 10^5$ N/m$^2$
4. $\Delta p_0 = 50$ N/m$^2$

**5** ***Air enters into a diffuser through a normal shock wave at an initial Mach number of 1.4. Determine the following***

***1. Efficiency***

***2. Percentage of stagnation pressure loss***

***3. Increase in entropy***

***Given :***

$M_x = 1.4$

$\gamma = 1.4$

$R = 287$ J/kg K

***To find :***

1. Diffuser efficiency, $\eta_D$
2. Percentage of stagnation pressure loss, $\Delta p_0$
3. Increase in entropy, $\Delta s$.

***Solution :***

Refer Isentropic flow table for $M_x = M_1 = 1.4$ and $\gamma = 1.4$.

$$\frac{T_1}{T_{01}} = 0.718 \qquad \textit{[From gas tables page no. 32]}$$

Refer Normal shocks tables for $M_x = 1.4$ and $\gamma = 1.4$.

$$\frac{p_{0y}}{p_{0x}} = 0.958 \qquad \textit{[From gas tables page no. 52]}$$

$$\text{Diffuser efficiency, } \eta_D = \frac{\dfrac{T_{01}}{T_1}\left[\dfrac{p_{0y}}{p_{0x}}\right]^{\frac{\gamma-1}{\gamma}} - 1}{\dfrac{\gamma-1}{2} M_1^2}$$

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$$= \frac{\frac{1}{0.718} \times [0.958]^{\frac{1.4-1}{1.4}} - 1}{\frac{1.4-1}{2} \times (1.4)^2}$$

$$= 0.958$$

$$\boxed{\eta_D = 95.8\,\%}$$

Percentage of stagnation pressure loss, $\Delta p_0 = \frac{p_{0x} - p_{0y}}{p_{0x}} \times 100$

$$= \left(1 - \frac{p_{0y}}{p_{0x}}\right) \times 100$$

$$= [1 - 0.958] \times 100$$

$$\boxed{\Delta p_0 = 4.2\,\%}$$

Increase in entropy, $\Delta s = R\, ln \left(\frac{p_{0x}}{p_{0y}}\right)$

$$= 287\ ln \left(\frac{1}{0.958}\right)$$

$$\boxed{\Delta s = 12.3\ \text{J/kg K}}$$

***Result***

1. $\eta_D = 95.8\,\%$
2. $\Delta p_0 = 4.2\,\%$
3. $\Delta s = 12.3$ J/kg K.

---

**1** ***The state of the gas [$\gamma$ = 1.3 and R = 0.469 kJ/kg K] upstream of a normal shock wave is given by the following data : $M_x$ = 2.5, $p_x$ = 2 bar, $T_x$ = 275 K. Calculate the mach number, pressure, temperature of the gas downstream of the shock.***

*[Anna Univ-May 2005]*

***Given :***

$\gamma = 1.3$

$R = 0.469$ kJ/kg K = 469 J/kg K

$M_x = 2.5$

$p_x = 2$ bar $= 2 \times 10^5$ N/m$^2$

$T_x = 275$ K

***To find***

1. Down stream mach number, $M_y$.
2. Down stream pressure, $p_y$.
3. Down stream temperature, $T_y$

***Solution :***

Refer Normal shocks table for $M_x = 2.5$ and $\gamma = 1.3$

$M_y = 0.493$

$\frac{p_y}{p_x} = 6.935$

*[From gas tables (S.M. Yahya, Fifth edition) page no.49]*

$\frac{T_y}{T_x} = 1.869$

$\Rightarrow \quad p_y = 6.935 \times p_x$

$= 6.935 \times 2 \times 10^5$

$$\boxed{p_y = 13.87 \times 10^5\ \text{N/m}^2}$$

$$\Rightarrow \quad T_y = 1.869 \times T_x$$

$$= 1.869 \times 275$$

$$\boxed{T_y = 513.97 \text{ K}}$$

***Result :***

1. $M_y = 0.493$

2. $p_y = 13.87 \times 10^5$ N/m$^2$

3. $T_y = 513.97$ K

**2** ***Air flows adiabatically in a pipe. A normal shock wave is formed. The pressure and temperature of air before the shock are 150 kN/m$^2$ and 25°C respectively. The pressure just after the normal shock is 350 kN/m$^2$. Calculate***

- ***(i) Mach number before the shock***
- ***(ii) Mach number, static temperature and velocity of air after the shock wave.***
- ***(iii) Increase in density of air***
- ***(iv) Loss of stagnation pressure of air***
- ***(v) Change in entropy***

*[Anna university, May 2004]*

***Given***

$p_x = 150$ kN/m$^2$ $= 150 \times 10^3$ N/m$^2$

$T_x = 25°\text{ C} + 273 = 298$ K

$p_y = 350$ kN/m$^2$ $= 350 \times 10^3$ N/m$^2$

***To find :***

1. Mach number before the shock, $M_x$
2. Mach number, static temperature and velocity after the shock, $[M_y, T_y, c_y]$
3. Increase in density of air, $[\rho_y - \rho_x]$
4. Loss of stagnation pressure, $(\Delta p_0)$
5. Change in entropy, $(\Delta s)$

***Solution***

$$\frac{p_y}{p_x} = \frac{350 \times 10^3}{150 \times 10^3}$$

$$\boxed{\frac{p_y}{p_x} = 2.333}$$

Refer Normal shocks tables for $\frac{p_y}{p_x} = 2.333 \approx 2.320$ and $\gamma = 1.4$.

$M_x = 1.46$

$M_y = 0.716$

$\frac{T_y}{T_x} = 1.294$ *[From gas tables page no.52]*

$\frac{p_{0y}}{p_{0x}} = 0.942$

$\frac{p_{0y}}{p_x} = 3.265$

$$\Rightarrow \quad T_y = 1.294 \times T_x$$

$$= 1.294 \times 298$$

$$\boxed{T_y = 385.61 \text{ K}}$$

$$\Rightarrow \quad p_{0y} = 3.265 \times p_x$$

$$= 3.265 \times 150 \times 10^3$$

$$\boxed{p_{0y} = 489.75 \times 10^3 \text{ N/m}^2}$$

$$\Rightarrow \quad p_{0x} = \frac{p_{0y}}{0.942} = \frac{489.75 \times 10^5}{0.942} = 519.9 \times 10^3 \text{ N/m}^2$$

$$\boxed{p_{0x} = 519.9 \times 10^3 \text{ N/m}^2}$$

Stagnation pressure loss

$$\Delta p_0 = p_{0x} - p_{0y}$$

$$= 519.9 \times 10^3 - 489.75 \times 10^3$$

$$\boxed{\Delta p_0 = 30.15 \times 10^3 \text{ N/m}^2}$$

Mach number after the shock

$$M_y = \frac{c_y}{a_y}$$

$$\Rightarrow \quad c_y = M_y \times a_y$$

$$= M_y \times \sqrt{\gamma R T_y}$$

$$= 0.716 \times \sqrt{1.4 \times 287 \times 385.61}$$

$$\boxed{c_y = 281.83 \text{ m/s}}$$

Density before the shock

$$\rho_x = \frac{p_x}{R T_x}$$

$$= \frac{150 \times 10^3}{287 \times 298}$$

$$\boxed{\rho_x = 1.753 \text{ kg/m}^3}$$

Density after the shock

$$\rho_y = \frac{p_y}{R T_y}$$

$$= \frac{350 \times 10^3}{287 \times 385.61}$$

$$\boxed{\rho_y = 3.162 \text{ kg/m}^3}$$

Increase in density $= \rho_y - \rho_x$

$$= 3.162 - 1.753$$

$$\boxed{\text{Increase in density} = 1.409 \text{ kg/m}^3}$$

Change in entropy, $\Delta s = R \, ln \left( \frac{p_{0x}}{p_{0y}} \right)$

$$= 287 \, ln \left( \frac{1}{0.942} \right)$$

$$\boxed{\Delta s = 17.148 \text{ J/kg K}}$$

***Result :***

1. $M_x = 1.46$
2. $M_y = 0.716$, $T_y = 385.61$ K, $c_y = 281.83$ m/s
3. $\rho_y - \rho_x = 1.409$ kg/m$^3$
4. $\Delta p_0 = 30.15 \times 10^3$ N/m$^2$
5. $\Delta s = 17.148$ J/kg K

**4** *A convergent divergent nozzle is designed to expand air from a reservoir in which the pressure is 700 kPa and temperature is 5°C and the nozzle inlet mach number is 0.2. The nozzle throat area is 46 cm² and the exit area is 230 cm². A normal shock appears at a section where the area is 175 cm². Find the exit pressure and temperature. Also find the increase in entropy across the shock.*

*[Anna Univ-Dec'04]*

***Given :***

$p_0 = 700 \text{ kPa} = 700 \times 10^3 \text{ Pa} = 700 \times 10^3 \text{ N/m}^2$

$T_0 = 5°C + 273 = 278K$

$M_1 = 0.2$

$A^* = 46 \text{ cm}^2 = 46 \times 10^{-4} \text{ m}^2 = A_x^*$ $[\because A^* = A_x^*]$

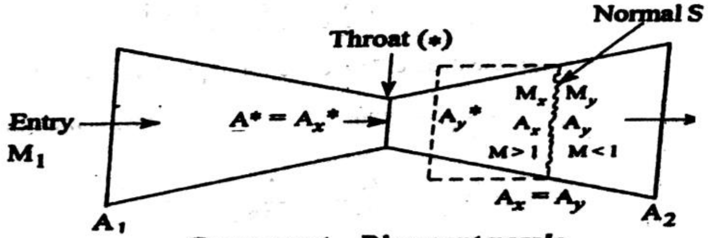
$A_2 = 230 \text{ cm}^2 = 230 \times 10^{-4} \text{ m}^2$

$A_x = 175 \text{ cm}^2 = 175 \times 10^{-4} \text{ m}^2$

***To find :***

1. Exit pressure, $p_2$
2. Exit temperature, $T_2$
3. Increase in entropy, $\Delta s$

***Solution :***



*Convergent – Divergent nozzle*

$$\frac{A_x}{A_x^*} = \frac{175 \times 10^{-4}}{46 \times 10^{-4}} = 3.80$$

Refer Isentropic flow tables for $\frac{A_x}{A_x^*} = 3.80 \approx 3.813$ and $\gamma = 1.4$.

$M_x = 2.89$ *[From gas tables page no.37]*

[**Note :**Upstream Mach number, $M_x$ value is always greater than one]

Refer Normal shock table for $M_x = 2.89$ and $\gamma = 1.4$.

$$M_y = 0.482$$

$$\frac{p_{0y}}{p_{0x}} = 0.361 \qquad \textit{[From gas tables page no.55]}$$

$$\Rightarrow \quad p_{0y} = 0.361 \times p_{0x}$$

$$= 0.361 \times 700 \times 10^3 \qquad [\because p_0 = p_{0x}]$$

$$\boxed{p_{0y} = 2.52 \times 10^5 \text{ N/m}^2}$$

Increase in entropy $\Delta s = R\, ln\left(\frac{p_{0x}}{p_{0y}}\right)$

$$= 287\ ln\left(\frac{1}{0.361}\right)$$

$$\boxed{\Delta s = 292.42 \text{ J/kg K}}$$

Refer Isentropic flow table for $M_y = 0.482 \approx 0.48$ and $\gamma = 1.4$.

$$\frac{A_y}{A_y{}^*} = 1.380 \qquad \textit{[From gas tables page no.29]}$$

$$\Rightarrow \quad A_y{}^* = \frac{A_y}{1.380} = \frac{A_x}{1.380} \qquad [\because A_x = A_y]$$

$$= \frac{175 \times 10^{-4}}{1.380}$$

$$\boxed{A_y{}^* = 0.01268 \text{m}^2}$$

$$\Rightarrow \quad \frac{A_2}{A_y{}^*} = \frac{230 \times 10^{-4}}{0.01268} = 1.81$$

$$\boxed{\frac{A_2}{A_y{}^*} = 1.81}$$

Refer Isentropic flow table for $\frac{A_2}{A_y{}^*} = 1.81 \approx 1.823$ and $\gamma = 1.4$.

$$M_2 = 0.34$$

$$\frac{p_2}{p_{0y}} = 0.923 \qquad \textit{[From gas tables page no.29]}$$

$$\frac{T_2}{T_{0y}} = 0.977$$

$$\Rightarrow \quad p_2 = 0.923 \times p_{0y}$$

$$= 0.923 \times 2.52 \times 10^5$$

$$\boxed{p_2 = 2.32 \times 10^5 \text{ N/m}^2}$$

$$\Rightarrow \quad T_2 = 0.977 \times T_{0y}$$

$$= 0.977 \times 278 \qquad [\because T_0 = T_{0x} = T_{0y}]$$

$$\boxed{T_2 = 271.60 \text{ K}}$$

***Result***

1. $p_2 = 2.32 \times 10^5$ N/m$^2$
2. $T_2 = 271.60$ K
3. $\Delta s = 292.42$ J/kg–K

**5** ***When a converging divergine nozzle is operated at off-design condition a normal shock occurs at a section where the cross sectional area is 18.75 cm² in the diverging portion. At inlet to the nozzle the stagnation state is given as 0.21 MPa and 36°C. The throat area is 12.5 cm² and exit area is 25 cm². Estimate the exit mach number, exit pressure and loss in stagnation pressure for flow through nozzle.***

*[Anna Univ-Dec'05]*

***Given :***

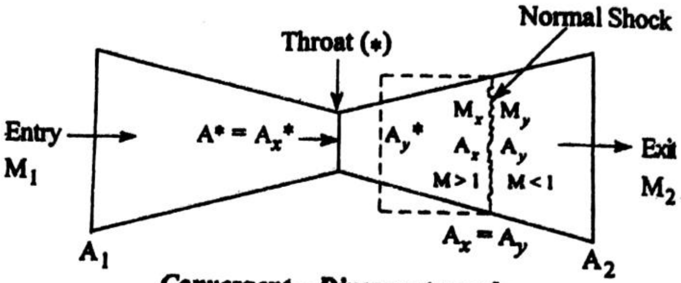
$A_x = 18.75 \text{ cm}^2 = 18.75 \times 10^{-4} \text{ m}^2 = A_y$

$p_0 = 0.21 \text{ MPa} = 0.21 \times 10^6 \text{ Pa} = 0.21 \times 10^6 \text{ N/m}^2$

$T_0 = 36°\text{C} + 273 = 309 \text{ K}$

$A^* = A_x^* = 12.5 \text{ cm}^2 = 12.5 \times 10^{-4} \text{ m}^2$ $[\because A^* = A_x^*]$

$A_2 = 25 \text{ cm}^2 = 25 \times 10^{-4} \text{ m}^2$



***Convergent – Divergent nozzle***

***To find :***

1. Exit Mach number, $M_2$
2. Exit pressure, $p_2$
3. Loss in stagnation pressure, $(\Delta p_0)$

***Solution :***

$$\frac{A_x}{A_x^*} = \frac{18.75 \times 10^{-4}}{12.5 \times 10^{-4}} = 1.5$$

Refer Isentropic flow tables for $\frac{A_x}{A_x^*} = 1.5$ and $\gamma = 1.4$.

$M_x = 1.86$ *[From gas tables page no.34]*

**[Note :** Upstream Mach number, $M_x$ value is always greater than one]

Refer Normal shock table for $M_x = 1.86$ and $\gamma = 1.4$.

$M_y = 0.604$

$\frac{p_{0y}}{p_{0x}} = 0.786$ *[From gas tables page no.53]*

$\Rightarrow p_{0y} = 0.786 \times p_{0x}$

$= 0.786 \times 0.21 \times 10^6$

$\boxed{p_{0y} = 1.65 \times 10^5 \text{ N/m}^2}$ $[\because p_0 = p_{0x}]$

Refer Isentropic flow table for $M_y = 0.604 \approx 0.60$ and $\gamma = 1.4$.

$\frac{A_y}{A_y^*} = 1.188$ *[From gas tables page no.29]*

$\Rightarrow A_y^* = \frac{A_y}{1.188}$ $[\because A_x = A_y]$

$= \frac{18.75 \times 10^{-4}}{1.188}$

$\boxed{A_y^* = 1.578 \times 10^{-3} \text{ m}^2}$

$\Rightarrow \frac{A_2}{A_y^*} = \frac{25 \times 10^{-4}}{1.578 \times 10^{-3}} = 1.584$

Refer Isentropic flow table for $\frac{A_2}{A_y^*} = 1.584 \approx 1.590$ and $\gamma = 1.4$.

$M_2 = 0.40$

$\frac{p_2}{p_{0y}} = 0.895$ *[From gas tables page no.29]*

$\Rightarrow p_2 = 0.895 \times p_{0y}$

$= 0.895 \times 1.65 \times 10^5$

$\boxed{p_2 = 1.476 \times 10^5 \text{ N/m}^2}$

We know that,

Stagnation pressure loss, $\Delta p = p_{0x} - p_{0y}$

$$= (0.21 \times 10^6) - (1.65 \times 10^5) \qquad [\because p_0 = p_{0x}]$$

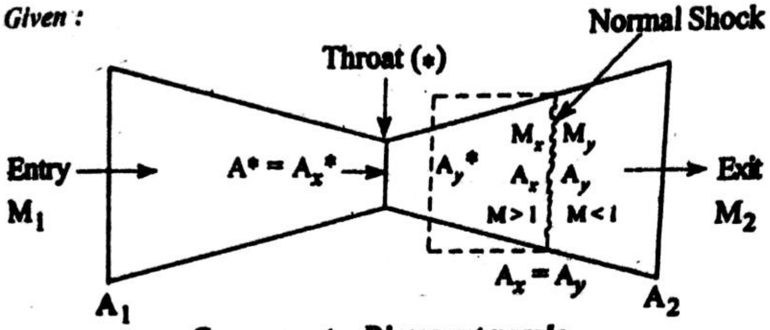
$$\boxed{\Delta p = 0.45 \times 10^5 \text{ N/m}^2}$$

***Result :***

1. $M_2 = 0.40$
2. $p_2 = 1.476 \times 10^5$ N/m$^2$
3. $\Delta p = 0.45 \times 10^5$ N/m$^2$

**7** ***A convergent–divergent air nozzle has exit to throat area ratio of 3. A normal shock appears at the divergent section where the existing area ratio is 2.2. Find the Mach number, before and after the shock. If the inlet stagnation properties are 500 kPa and 450K, find the properties of air at exit and entropy increase across the shock.*** *[Madras Univ - Apr'99]*

***Given :***



***Convergent – Divergent nozzle***

$$\frac{A_2}{A^*} = \frac{A_2}{A_x^*} = 3$$

$$\frac{A_x}{A_x^*} = 2.2$$

$p_0 = 500$ kpa $= 500 \times 10^3$ Pa $= 500 \times 10^3$ N/m$^2$

$$\boxed{p_0 = 5 \times 10^5 \text{ N/m}^2}$$

$T_0 = 450$ K

***To find :***

1. Mach number before and after the shock ($M_x$, $M_y$)
2. Properties of air at exit ($p_2$, $T_2$, $c_2$)
3. Increase in entropy ($\Delta s$).

***Solution :***

Refer Isentropic flow table for $\frac{A_x}{A_x^*} = 2.2 \approx 2.213$ and $\gamma = 1.4$.

$M_x = 2.31$ *[From gas tables page no.35]*

Refer Normal shock table for $M_x = 2.31$ and $\gamma = 1.4$.

$M_y = 0.533$

$\frac{p_{0y}}{p_{0x}} = 0.5785$ *[From gas tables page no.54]*

$$\Rightarrow \quad p_{0y} = 0.5785 \times p_{0x}$$

$$= 0.5785 \times 5 \times 10^5 \qquad [\because p_0 = p_{0x}]$$

$$\boxed{p_{0y} = 2.89 \times 10^5 \text{ N/m}^2}$$

Increase in entropy, $\Delta s = R \ ln \left(\frac{p_{0x}}{p_{0y}}\right)$

$$= 287 \ ln \left(\frac{1}{0.5785}\right)$$

$$\boxed{\Delta s = 157.07 \text{ J/kg K}}$$

Refer Isentropic flow table for $M_y = 0.533 \approx 0.53$ and $\gamma = 1.4$.

$\frac{A_y}{A_y{*}} = 1.287$ *[From gas tables page no.29]*

We know that,

$$\frac{A_2}{A_y{*}} = \frac{A_2}{A_x{*}} \times \frac{A_x{*}}{A_x} \times \frac{A_y}{A_y{*}} \qquad [\because A_x = A_y]$$

$$= 3 \times \frac{1}{2.2} \times 1.287$$

$$\boxed{\frac{A_2}{A_y{*}} = 1.755}$$

Refer Isentropic flow table for $\frac{A_2}{A_y{*}} = 1.755 \approx 1.735$ and $\gamma = 1.4$.

$M_2 = 0.36$ *[From gas tables page no.29]*

$$\frac{T_2}{T_{0y}} = 0.975$$

$$\frac{p_2}{p_{0y}} = 0.914$$

$$\Rightarrow \quad T_2 = 0.975 \times T_{0y}$$

$$= 0.975 \times 450 \qquad [\because T_0 = T_{0x} = T_{0y} = 450]$$

$$\boxed{T_2 = 438.75 \text{ K}}$$

$$\Rightarrow \quad p_2 = 0.914 \times p_{0y}$$

$$= 0.914 \times 2.89 \times 10^5$$

$$\boxed{p_2 = 2.65 \times 10^5 \text{ N/m}^2}$$

Exit velocity of air, $c_2 = M_2 \times a_2$

$$= M_2 \times \sqrt{\gamma R T_2}$$

$$= 0.36 \times \sqrt{1.4 \times 287 \times 438.75}$$

$$\boxed{c_2 = 151.15 \text{ m/s}}$$

***Result :***

1. $M_x = 2.31$

   $M_y = 0.533$

2. $p_2 = 2.65 \times 10^5 \text{ N/m}^2$

   $T_2 = 438.75 \text{ K}$

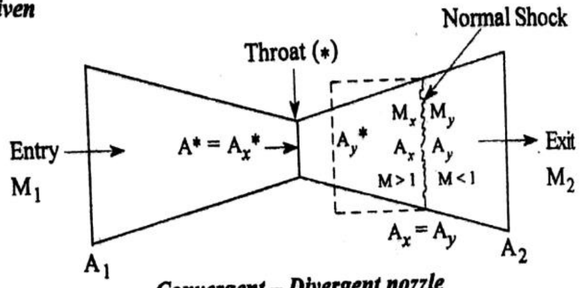
   $c_2 = 151.15 \text{ m/s}$

3. $\Delta s = 157.07 \text{ J/kg K}$

**8** ***A Convergent– divergent nozzle has an exit area to throat area ratio of 2.5. The total properties of air at inlet are 7 bar and 87°C. The throat area is 6.5 cm$^2$. Determine Mach number, static pressure, static temperature and stagnation pressure at exit, when a plane normal shock stands at a point where the Mach number is 2. Assume Isentropic flow before and after the shock.***

*[Manonmanium Sundaranar Univ - Apr'97]*

**Given**



*Convergent – Divergent nozzle*

$$\frac{A_2}{A^*} = \frac{A_2}{A_x^*} = 2.5 \qquad [\because A^* = A_x^*]$$

$p_0 = 7 \text{ bar} = 7 \times 10^5 \text{ N/m}^2$

$T_0 = 87°C + 273 = 360 \text{ K}$

$A^* = A_x^* = 6.5 \text{ cm}^2 = 6.5 \times 10^{-4} \text{ m}^2$

$M_x = 2$

**To find**

1. Exit Mach number, $M_2$
2. Exit State pressure, $p_2$
3. Exit static temperature, $T_2$
4. Exit stagnation pressure, $p_{02}$

**Solution**

Refer Isentropic flow table for $M_x = 2$ and $\gamma = 1.4$.

$$\frac{A_x}{A_x^*} = 1.687 \qquad \textit{[From gas tables page no. 34]}$$

Refer Normal shocks table for $M_x = 2$ and $\gamma = 1.4$.

$M_y = 0.577$

$$\frac{p_{0y}}{p_{0x}} = 0.721 \qquad \textit{[From gas tables page no. 53]}$$

$$\Rightarrow \quad p_{0y} = 0.721 \times p_{0x}$$

$$= 0.721 \times 7 \times 10^5 \qquad [\because p_0 = p_{0x}]$$

$$\boxed{p_{0y} = 5.047 \times 10^5 \text{ N/m}^2}$$

Refer Isentropic flow table for $M_y = 0.577 \approx 0.58$ and $\gamma = 1.4$.

$$\frac{A_y}{A_y^*} = 1.213 \qquad \textit{[From gas tables page no. 29]}$$

We know that,

$$\boxed{\frac{A_2}{A_y^*} = \frac{A_2}{A_x^*} \times \frac{A_x^*}{A_x} \times \frac{A_y}{A_y^*}} \qquad [\because A_x = A_y]$$

$$= 2.5 \times \frac{1}{1.687} \times 1.213$$

$$\boxed{\frac{A_2}{A_y^*} = 1.7975}$$

Refer Isentropic flow table for $\frac{A_2}{A_y^*} = 1.797 \approx 1.778$ and $\gamma = 1.4$.

$$M_2 = 0.35 \qquad \textit{[From gas tables page no. 29]}$$

$$\frac{T_2}{T_{0y}} = 0.976$$

$$\frac{p_2}{p_{0y}} = 0.918$$

$$\Rightarrow \quad T_2 = 0.976 \times T_{0y} \qquad [\because T_0 = T_{0x} = T_{0y}]$$

$$= 0.976 \times 360$$

$$\boxed{\text{Exit temperature, } T_2 = 351.36 \text{ K}}$$

$$\Rightarrow \quad p_2 = 0.912 \times p_{0y}$$

$$= 0.912 \times 5.047 \times 10^5$$

$$\boxed{\text{Exit pressure, } p_2 = 4.60 \times 10^5 \text{ N/m}^2}$$

We know,

Stagnation pressure at exit, $p_{02} = p_{0y}$

$$\Rightarrow \quad \boxed{p_{02} = 5.047 \times 10^5 \text{ N/m}^2}$$

***Result***

1. $M_2 = 0.35$
2. $p_2 = 4.60 \times 10^5 \text{ N/m}^2$
3. $T_2 = 351.36 \text{ K}$
4. $p_{02} = 5.047 \times 10^5 \text{ N/m}^2$

## 4.2 OBLIQUE SHOCK WAVES

### 4.2.1 Introduction

When the shock wave is inclined at an angle to flow, it is called oblique shock. It is also referred to as a two dimensional plane shock wave.

### 4.2.2 FlowThrough Oblique Shock Waves

Consider a stationary wedge in a flow system as shown in fig.4.7. The gas undergoes a change of direction due to this concave corner. A shock will appear at the wedge as shown and this will make an angle $\sigma$ with the original flow direction. This angle is called the wave angle ($\sigma$) and it will be greater than the deflection angle ($\delta$).

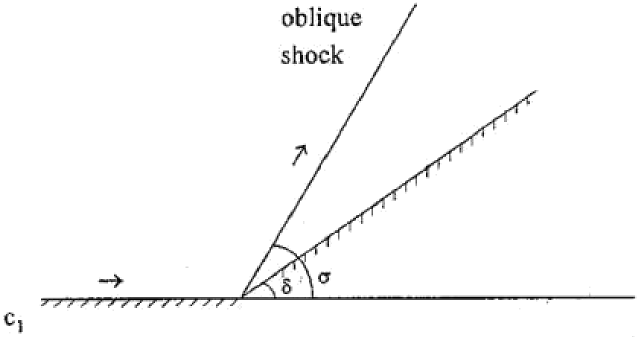


*Fig. 4.7*

A stronger oblique shock wave is nearer to a normal shock wave and has a larger wave angle (i.e., nearer to 90°).

A weak oblique shock wave has smaller wave angle and closer to the Mach waves.

Oblique shocks usually occur when a supersonic flow is turned in to itself. The opposite of this, i.e., when a supersonic flow is turned away from itself, expansion fan is formed.

The following assumptions are used for oblique shock flow

1. Flow is steady, adiabatic and frictionless.
2. The gas is perfect with constant specific heats.
3. Absence of work transfer across the boundaries.
4. Absence of body forces.

### 4.2.3 Formulae Used

1. Mach number at entry (or) Upstream Mach number of Oblique shock

$$M_1 = \frac{M_x}{\sin \sigma}$$

where

$M_x$ – Upstream Mach number of normal shock

$\sigma$ – Wave angle.

2. Mach number at exit (or) Downstream Mach number of Oblique shock

$$M_2 = \frac{M_y}{\sin (\sigma - \delta)}$$

where

$M_y$ – Downstream Mach number of normal shock

$\delta$ – Deflection angle (or) Wedge angle.

3. Deflection angle

$$\tan \delta = 2 \cot \sigma \frac{M_1^2 \sin^2 \sigma - 1}{2 + \gamma M_1^2 + M_1^2 (1 - 2\sin^2 \sigma)}$$

## 4.2.4 Solved Problems

$\boxed{1}$ ***An oblique shock wave occurs at the leading edge of a symmetrical wedge. Air has a Mach number of 2.1 and deflection angle (δ) of 15°. Determine the following for strong and weak waves.***

*1) Wave angle*

*2) Pressure ratio*

*3) Density ratio*

*4) Temperature ratio*

*5) Down stream Mach number*

***Given***

Upstream Mach number of oblique shock, $M_1 = 2.1$

Deflection angle, $\delta = 15°$

***To find***

For strong and weak waves

1) Wave angle, $\sigma$

2) Pressure ratio, $\frac{p_2}{p_1}$

3) Density ratio, $\frac{\rho_2}{\rho_1}$

4) Temperature ratio, $\frac{T_2}{T_1}$

5) Downstream Mach number of oblique shock, $M_2$.

***Solution***

We know that,

Deflection angle, $\tan\delta = 2\cot\sigma \; \frac{M_1^2 \sin^2\sigma - 1}{2 + \gamma M_1^2 + M_1^2(1 - 2\sin^2\sigma)}$

Substitute $\delta = 15°$ and $M_1 = 2.1$

$$\Rightarrow \tan 15 = 2\cot\sigma \times \frac{(2.1)^2 \sin^2\sigma - 1}{2 + 1.4 \times (2.1)^2 + (2.1)^2(1 - 2\sin^2\sigma)}$$

$$0.267 = 2\cot\sigma \times \frac{4.41 \times \sin^2\sigma - 1}{8.174 \; + 4.41\,(1 - 2\sin^2\sigma)}$$

$$0.267 = 2\cot\sigma \times \frac{4.41 \times \sin^2\sigma - 1}{8.174 \; + 4.41 - 8.82\sin^2\sigma}$$

$$\Rightarrow 0.1335 = \cot\sigma \times \frac{4.41 \times \sin^2\sigma - 1}{12.584 - 8.82\sin^2\sigma}$$

By iteration, we can find

$$\sigma = 80.8°$$

and

$$\sigma = 43°$$

The angle being closer to 90° is the angle of strong shock wave. Similarly the angle much lesser than 90° is the angle of weak wave.

So,

$\sigma_{strong} = 80.8°$

$\sigma_{weak} = 43°$

**For strong shock wave**

Upstream Mach number of Normal shock $\quad M_x = M_1 \sin\sigma$

$= 2.1 \times \sin 80.8$

$$\boxed{M_x = 2.07}$$

Refer normal shocks table for $M_x = 2.07$ and $\gamma = 1.4$.

$$M_y = 0.565$$

$$\frac{p_y}{p_x} = 4.832 = \frac{p_2}{p_1}$$

$$\frac{T_y}{T_x} = 1.745 = \frac{T_2}{T_1}$$

*[ From gas tables (S.M. Yahya, Fifth edition) page no.53]*

We know that,

Down stream Mach number of oblique shock, $M_2 = \frac{M_y}{\sin(\sigma - \delta)}$

$$= \frac{0.565}{\sin(80.8° - 15°)}$$

$$\boxed{M_2 = 0.619}$$

Density at exit, $\rho_2 = \frac{p_2}{RT_2}$

Density at entry, $\rho_1 = \frac{p_1}{RT_1}$

$$\Rightarrow \quad \frac{\rho_2}{\rho_1} = \frac{\frac{p_2}{RT_2}}{\frac{p_1}{RT_1}}$$

$$= \frac{p_2}{p_1} \times \frac{T_1}{T_2}$$

$$= 4.832 \times \frac{1}{1.745}$$

$$\boxed{\frac{\rho_2}{\rho_1} = 2.769}$$

**For Weak shock wave**

Upstream Mach number of normal shock, $M_x = M_1 \sin\sigma$

$$= 2.1 \times \sin 43°$$

$$\boxed{M_x = 1.432}$$

Refer normal shocks table for $M_x = 1.432$ and $\gamma = 1.4$.

$$M_y = 0.727$$

$$\frac{p_y}{p_x} = 2.219 = \frac{p_2}{p_1}$$

$$\frac{T_y}{T_x} = 1.274 = \frac{T_2}{T_1}$$

*[From gas tables page no.52]*

We know that,

Down stream Mach number of oblique shock, $M_2 = \frac{M_y}{\sin(\sigma - \delta)}$

$$= \frac{0.727}{\sin(43 - 15)}$$

$$\boxed{M_2 = 1.548}$$

Density at exit, $\rho_2 = \frac{p_2}{RT_2}$

Density at entry $\rho_1 = \frac{p_1}{RT_1}$

$$\Rightarrow \quad \frac{\rho_2}{\rho_1} = \frac{\frac{p_2}{RT_2}}{\frac{p_1}{RT_1}}$$

$$= \frac{p_2}{p_1} \times \frac{T_1}{T_2}$$

$$= 2.219 \times \frac{1}{1.274}$$

$$\boxed{\frac{\rho_2}{\rho_1} = 1.741}$$

***Result***

**For Strong shock wave**

1. Wave angle, $\sigma_{strong} = 80.8°$
2. Pressure ratio, $\frac{p_2}{p_1} = 4.832$
3. Density ratio, $\frac{\rho_2}{\rho_1} = 2.769$
4. Temperature ratio, $\frac{T_2}{T_1} = 1.745$
5. Downstream Mach number, $M_2 = 0.619$

**For Weak shock wave**

1. Wave angle, $\sigma_{weak} = 43°$
2. Pressure ratio, $\frac{p_2}{p_1} = 2.219$
3. Density ratio, $\frac{\rho_2}{\rho_1} = 1.741$
4. Temperature ratio, $\frac{T_2}{T_1} = 1.274$
5. Downstream Mach number, $M_2 = 1.548$

**2** *A gas at a pressure of 340 m bar, temperature of 355 K and entry Mach number of 1.4 is expanded isentropically to 140 m bar. Calculate the following*

*1) Deflection angle*

*2) Final Mach number*

*3) Final temperature of the gas.*

*Take $\gamma = 1.3$*

***Given :***

$p_1 = 340$ m bar $= 340 \times 10^{-3}$ bar $= 0.340$ bar $= 0.340 \times 10^5$ N/m$^2$

$T_1 = 355$ K

$M_1 = 1.4$

$p_2 = 140$ m bar $= 140 \times 10^{-3}$ bar $= 0.140$ bar $= 0.140 \times 10^5$ N/m$^2$

***To find :***

1) Deflection angle, $\delta$

2) Final Mach number, $M_2$

3) Final temperature, $T_2$

***Solution :***

Refer Isentropic flow table for $M_1 = 1.4$ and $\gamma = 1.3$.

$$\frac{T_1}{T_{01}} = 0.773$$ *[From gas tables page no.23]*

$$\frac{p_1}{p_{01}} = 0.327$$

$$\Rightarrow \quad T_{01} = \frac{T_1}{0.773} = \frac{355}{0.773} = 459.24 \text{ K}$$

$$\boxed{T_{01} = 459.24 \text{ K} = T_{02}}$$ $[\because T_{01} = T_{02}]$

$$\Rightarrow \quad p_{01} = \frac{p_1}{0.327} = \frac{0.340 \times 10^5}{0.327} = 1.039 \times 10^5 \text{ N/m}^2$$

$$\boxed{p_{01} = 1.039 \times 10^5 \text{ N/m}^2 = p_{02}}$$

$$\frac{p_2}{p_{02}} = \frac{0.140 \times 10^5}{1.039 \times 10^5} = 0.134$$

Refer Isentropic flow table for $\frac{p_2}{p_{02}} = 0.134 \approx 0.135$ and $\gamma = 1.3$.

$$M_2 = 1.98$$

$$\frac{T_2}{T_{02}} = 0.629$$

*[From gas tables page no.24]*

$$\Rightarrow \quad T_2 = 0.629 \times T_{02}$$

$$= 0.620 \times 459.24$$

$$\boxed{T_2 = 288.86 \text{ K}}$$

Refer Prandtl–Meyer functions table for $M_1 = 1.4$ and $\gamma = 1.3$.

$$w(M_1) = 9.542$$

*[From gas tables page no.125]*

Refer Prandtl–Meyer functions table for $M_2 = 1.98$ and $\gamma = 1.3$.

$$w(M_2) = 28.060$$

*[From gas tables page no.126]*

Deflection angle

$$\delta = w(M_1) - w(M_2)$$

$$= 9.542 - 28.060$$

$$\boxed{\delta = -18.51^\circ}$$

**Result**

1. $\delta = -18.51^\circ$
2. $M_2 = 1.98$
3. $T_2 = 288.86 \text{ K}$

**3** ***A jet of air approaches a symmetrical wedge of a Mach number of 2.4 and wave angle of 60°. Determine the following***

***1) Deflection angle***

***2) Pressure ratio***

***3) Temperature ratio***

***4) Final Mach number***

**Given**

Entry Mach number $M_1 = 2.4$

Wave angle $\sigma = 60^\circ$

**To find**

1) Deflection angle, $\delta$

2) Pressure ratio, $\frac{p_2}{p_1}$

3) Temperature ratio, $\frac{T_2}{T_1}$

5) Final Mach number, $M_2$.

**Solution**

We know that,

Deflection angle, $\tan \delta = 2 \cot \sigma \dfrac{M_1^2 \sin^2 \sigma - 1}{2 + \gamma M_1^2 + M_1^2 (1 - 2\sin^2 \sigma)}$

$$\Rightarrow \tan\delta = 2\cot 60^\circ \times \frac{(2.4)^2 \sin^2 60 - 1}{2 + 1.4 \times (2.4)^2 + (2.4)^2 (1 - 2\sin^2\sigma)}$$

$$\Rightarrow \tan\delta = 0.555$$

$$\boxed{\delta = 28.08^\circ}$$

We know that,

$$\text{Entry Mach number, } M_1 = \frac{M_x}{\sin\sigma}$$

$$\Rightarrow \quad M_x = M_1 \times \sin\sigma$$

$$= 2.4 \times \sin 60$$

$$\boxed{M_x = 2.07}$$

Refer Normal shocks table for $M_x = 2.07$ and $\gamma = 1.4$.

$$M_y = 0.565$$

$$\frac{p_y}{p_x} = 4.832 = \frac{p_2}{p_1}$$

*[From gas tables page no.53]*

$$\frac{T_y}{T_x} = 1.745 = \frac{T_2}{T_1}$$

$$\text{Exit Mach number, } M_2 = \frac{M_y}{\sin(\sigma - \delta)}$$

$$= \frac{0.565}{\sin(60 - 28.08)}$$

$$\boxed{M_2 = 1.068}$$

**Result :**

1. $\delta = 28.08^\circ$
2. $\frac{p_2}{p_1} = 4.832$
3. $\frac{T_2}{T_1} = 1.745$
4. $M_2 = 1.068$

**4** ***An air jet at a Mach number of 2.1 is isentropically deflected by 10° in the clockwise direction. The initial pressure is 100 kN/m² and initial temperature is 98°C. Determine the final state of air after expansion.***

***Given :***

$M_1 = 2.1$

Deflection angle, $\delta = 10^\circ \Rightarrow$ clockwise direction, $\delta = -10^\circ$

$p_1 = 100 \text{ kN/m}^2 = 100 \times 10^3 \text{ N/m}^2 = 1 \times 10^5 \text{ N/m}^2$

$T_1 = 98^\circ \text{C} + 273 = 371 \text{ K}$

***To find :***

Final state of air ($T_2, p_2, \rho_2$)

***Solution :***

Refer Isentropic flow table for $M_1 = 2.1$ and $\gamma = 1.4$.

$$\frac{T_1}{T_{01}} = 0.531$$

*[From gas tables page no.34]*

$$\frac{p_1}{p_{01}} = 0.109$$

$$\Rightarrow \quad T_{01} = \frac{T_1}{0.531} = \frac{371}{0.531} = 698.68 \text{ K}$$

$$\boxed{T_{01} = 698.68\text{ K} = T_{02}} \qquad [\because T_{01} = T_{02}]$$

$$\Rightarrow \quad p_{01} = \frac{p_1}{0.109} = \frac{1 \times 10^5}{0.109} = 9.17 \times 10^5 \text{ N/m}^2$$

$$\boxed{p_{01} = 9.17 \times 10^5 \text{ N/m}^2 = p_{02}} \qquad [\because p_{01} = p_{02}]$$

We know that,

Deflection angle, $\delta = \omega(M_1) - \omega(M_2)$ ----(A)

Refer Prandtl–Meyer functions table for $M_1 = 2.1$ and $\gamma = 1.4$.

$$\omega(M_1) = 29.097$$

*[From gas tables page no.126]*

$$(A) \Rightarrow \quad \delta = \omega(M_1) - \omega(M_2)$$

$$-10° = 29.097 - \omega(M_2)$$

$$\Rightarrow \quad \boxed{\omega(M_2) = 39.097}$$

Refer Prandtl–Meyer functions table for $\omega(M_2) = 39.097 \approx 39$ and $\gamma = 1.4$.

$M_2 = 2.495$ *[From gas tables page no131]*

Refer Isentropic flow table for $M_2 = 2.495 \approx 2.5$ and $\gamma = 1.4$

$$\frac{T_2}{T_{02}} = 0.444$$ *[ From gas tables page no.36]*

$$\frac{p_2}{p_{02}} = 0.0585$$

$$\Rightarrow \quad T_2 = 0.444 \times T_{02}$$

$$= 0.444 \times 698.68 \text{ K}$$

$$\boxed{T_2 = 310.21 \text{ K}}$$

$$\Rightarrow \quad p_2 = 0.0585 \times p_{02}$$

$$= 0.0585 \times 9.17 \times 10^5$$

$$\boxed{p_2 = 0.536 \times 10^5 \text{ N/m}^2}$$

Density at exit, $\rho_2 = \dfrac{p_2}{RT_2}$

$$= \frac{0.536 \times 10^5}{287 \times 310.21}$$

$$\boxed{\rho_2 = 0.602 \text{ kg/m}^3}$$

***Result***

1. $T_2 = 310.21$ K
2. $p_2 = 0.536 \times 10^5$ N/m$^2$
3. $\rho_2 = 0.602$ kg/m$^3$

**5** ***An oblique shock wave at an angle of 33° occurs at the leading edge of a symmetrical wedge. Air has a Mach number of 2.1 upstream temperature of 300 K and upstream pressure of 11 bar. Determine the following***

***a) Downstream pressure***

***b) Down stream temperature***

***c) Wedge angle***

***d) Downstream Mach number***

***Given :***

Wave angle, $\sigma = 33°$

Entry Mach number, $M_1 = 2.1$

Upstream pressure, $p_x = 11 \text{ bar} = 11 \times 10^5 \text{ N/m}^2$

Upstream temperature, $T_x = 300 \text{ K}$

***To find :***

1) Downstream pressure, $p_y$

2) Downstream temperature, $T_y$

3) Wedge angle (or) Deflection angle, $(\delta)$

4) Downstream Mach number, $M_2$.

***Solution :***

Mach number at entry (or)

Upstream Mach number of oblique shock, $M_1 = \dfrac{M_x}{\sin\sigma}$

$$\Rightarrow \quad M_x = M_1 \sin\sigma$$

$$= 2.1 \times \sin 33°$$

$$\boxed{M_x = 1.143}$$

Refer Normal shocks table for $M_x = 1.143 \approx 1.14$ and $\gamma = 1.4$.

$$M_y = 0.882$$

$$\frac{T_y}{T_x} = 1.090 \qquad \textit{[From gas tables page no. 52]}$$

$$\frac{p_y}{p_x} = 1.349$$

$$\Rightarrow \quad T_y = 1.090 \times T_x$$

$$= 1.090 \times 300$$

$$\boxed{T_y = 327 \text{ K}}$$

$$\Rightarrow \quad p_y = 1.349 \times p_x$$

$$= 1.349 \times 11 \times 10^5$$

$$\boxed{p_y = 14.83 \times 10^5 \text{ N/m}^2}$$

Deflection angle (or) Wedge angle $\tan\delta = 2\cot\sigma \dfrac{M_1^2 \sin^2\sigma - 1}{2 + \gamma M_1^2 + M_1^2 (1 - 2\sin^2\sigma)}$

$$\Rightarrow \quad \tan\delta = 2\cot 33 \times \frac{(2.1)^2 \sin^2(33) - 1}{2 + 1.4 \times (2.1)^2 + (2.1)^2 [(1 - 2\sin^2(33))]}$$

$$\Rightarrow \quad \tan\delta = 0.0952$$

$$\Rightarrow \quad \boxed{\delta = 5.44°}$$

Downstream Mach number of oblique shock, $M_2 = \dfrac{M_y}{\sin(\sigma - \delta)}$

$$= \frac{0.882}{\sin(33 - 5.44)}$$

$$\boxed{M_2 = 1.91}$$

***Result***

1. $p_y = 14.83 \times 10^5 \text{ N/m}^2$
2. $T_y = 327 \text{ K}$
3. $\delta = 5.44°$
4. $M_2 = 1.91$

# 3

# FLOW THROUGH CONSTANT AREA DUCTS

## 3.1 FLOW IN CONSTANT AREA DUCTS WITH HEAT TRANSFER [RAYLEIGH FLOW]

### 3.1.1 Introduction

Flow in a constant area duct with heat transfer and without friction is known as Rayleigh flow.

In the case of combustion chambers, regenerators, heat exchangers and intercoolers the fluid flow takes place with heat transfer. In such a flow, the following assumptions are made

1. One dimensional steady flow
2. Flow takes place in constant area section.
3. There is no friction
4. The gas is perfect.
5. Absence of work transfer across the boundaries.

### 3.1.2 Rayleigh Line (or) Curve

Flow in a constant area duct with heat transfer and without friction is described by a curve known as Rayleigh line (or) curve.

### 3.1.16 SOLVED PROBLEMS

**NOTE**

Procedure for solving Rayleigh flow problems

- Find entry Mach number ($M_1$) (If not given)
- Find stagnation temperature ($T_0$), stagnation pressure ($p_0$) [or] Static temperature ($T_1$), Static pressure ($p_1$) by referring Isentropic flow table at $M_1$.
- Find $T^*$, $p^*$, $c^*$, $p_0{}^*$, $T_0{}^*$ values by referring Rayleigh flow table at $M_1$.
- Find $T_2$, $p_2$, $c_2$, $p_{0_2}$ values by referring Rayleigh flow table at $M_2$

  (or) $\dfrac{T_{02}}{T_{02}{}^*}$

- $$T_1{}^* = T_2{}^*$$
  $$p_1{}^* = p_2{}^*$$
  $$T_{0_1}{}^* = T_{0_2}{}^*$$
  $$p_{0_1}{}^* = p_{0_2}{}^*$$
  $$c_1{}^* = c_2{}^*$$

1 *The pressure, temperature and velocity of a gas in a combustion chamber at entry are 0.35 bar, 300 K and 55 m/s. The increase in stagnation enthalpy of the gas between entry and exit is 1170 kJ/kg. Calculate the following*

1. *Exit Mach number, $M_2$*
2. *Exit Pressure, $p_2$*
3. *Exit temperature, $T_2$*
4. *Exit velocity, $c_2$*

*Take $c_p = 1.005$ kJ/kg K, $\gamma = 1.4$,*

***Given :***

$p_1 = 0.35 \text{ bar} = 0.35 \times 10^5 \text{ N/m}^2$

$T_1 = 300$ K

$c_1 = 55$ m/s.

Change in enthalpy, $\Delta h_0 = h_{0_2} - h_{0_1} = 1170 \text{ kJ/kg} = 1170 \times 10^3 \text{ J/kg}$.

$C_p = 1.005 \text{ kJ/kg K} = 1005 \text{ J/kg K}$

$\gamma = 1.4$

***To find :***

1. Exit Mach number, $M_2$
2. Exit Pressure, $p_2$
3. Exit temperature, $T_2$
4. Exit velocity, $c_2$

***Solution :***

Mach number at entry,
$$M_1 = \frac{c_1}{a_1} = \frac{c_1}{\sqrt{\gamma R T_1}} = \frac{55}{\sqrt{1.4 \times 287 \times 300}}$$

$$\boxed{M_1 = 0.158}$$

Refer Isentropic flow table for $\gamma = 1.4$ and $M_1 = 0.158 \approx 0.16$

$$\frac{T_1}{T_{0_1}} = 0.9949$$

*[From gas tables (S.M. Yahya, Fifth edition) - page no.28]*

$$\frac{p_1}{p_{0_1}} = 0.982$$

$$T_{01} = \frac{T_1}{0.9949} = \frac{300}{0.9949} = 301.54 \text{ K}$$

$$\boxed{T_{01} = 301.54 \text{ K}}$$

Refer Rayleigh flow table for $\gamma = 1.4$ and $M_1 = 0.158 \approx 0.16$

$$\frac{p_1}{p_1^*} = 2.317$$

$$\frac{p_{01}}{p_{01}^*} = 1.246$$

*[From gas tables page no.111]*

$$\frac{T_1}{T_1^*} = 0.137$$

$$\frac{T_{01}}{T_{01}^*} = 0.115$$

$$\frac{c_1}{c_1^*} = 0.059$$

$$\Rightarrow \quad p_1^* = \frac{p_1}{2.317}$$

$$= \frac{0.35 \times 10^5}{2.317}$$

$$\boxed{p_1^* = 0.151 \times 10^5 \text{ N/m}^2 = p_2^*} \qquad [\because p_1^* = p_2^*]$$

$$\Rightarrow \quad T_{01}^* = \frac{T_{01}}{0.115}$$

$$= \frac{301.54}{0.115}$$

$$\boxed{T_{01}^* = 2622.08 \text{ K} = T_{02}^*} \qquad [\because T_{01}^* = T_{02}^*]$$

$$\Rightarrow \quad T_1^* = \frac{T_1}{0.137}$$

$$= \frac{300}{0.137}$$

$$\boxed{T_1^* = 2189.78 \text{ K} = T_2^*} \qquad [\because T_1^* = T_2^*]$$

$$\Rightarrow \quad c_1^* = \frac{c_1}{0.059}$$

$$= \frac{55}{0.059}$$

$$\boxed{c_1^* = 932.20 \text{ m/s} = c_2^*} \qquad [\because c_1^* = c_2^*]$$

We know that

$$h_{0_2} = c_p T_{02}$$

$$h_{0_1} = c_p T_{01}$$

$$\Rightarrow \quad h_{0_2} - h_{0_1} = c_p T_{02} - c_p T_{01}$$

$$\Rightarrow \quad 1170 \times 10^3 = 1005 [T_{02} - 301.54]$$

$$\Rightarrow \quad 1164.18 = T_{02} - 301.54$$

$$\Rightarrow \quad \boxed{T_{02} = 1465.72 \text{ K}}$$

$$\Rightarrow \quad \frac{T_{02}}{T_{02}^*} = \frac{1465.72}{2622.08} = 0.56$$

Refer Rayleigh flow table for $\gamma = 1.4$ and $\frac{T_{02}}{T_{02}^*} = 0.56 \approx 0.564$

$$M_2 = 0.42$$

$$\frac{p_2}{p_2^*} = 1.925$$

$$\frac{p_{02}}{p_{02}^*} = 1.148$$

*[ From gas tables page no.111]*

$$\frac{T_2}{T_2^*} = 0.653$$

$$\frac{c_2}{c_2^*} = 0.339$$

[Note : For $\frac{T_{02}}{T_{02}^*} = 0.564$ , we can refer gas tables page no.111 and page no 116. But we have to take $M_2 < 1$ corresponding values, since the inlet mach number is subsonic i.e., $M_1 < 1$ Refer fig.3.2]

$$\Rightarrow \qquad p_2 = p_2^* \times 1.925$$

$$= 0.151 \times 10^5 \times 1.925$$

$$\boxed{p_2 = 0.2906 \times 10^5 \text{ N/m}^2}$$

$$\Rightarrow \qquad T_2 = T_2^* \times 0.653$$

$$= 2189.78 \times 0.653$$

$$\boxed{T_2 = 1429.93 \text{ K}}$$

$$\Rightarrow \qquad c_2 = c_2^* \times 0.339$$

$$= 932.20 \times 0.339$$

$$\boxed{c_2 = 316.09 \text{ m/s}}$$

***Result :***

1. $M_2 = 0.42$
2. $p_2 = 0.2906 \times 10^5$ N/m$^2$
3. $T_2 = 1429.93$ K
4. $c_2 = 316.09$ m/s.

**2** ***The pressure, temperature and Mach number of air in a combustion chamber are 4 bar, 100°C and 0.2 respectively. The stagnation temperature of air in a combustion chamber is increased 3 times its initial value. Calculate***

***1. The Mach number, pressure and temperature at the exit***

***2. Stagnation pressure loss***

***3. Heat supplied per kg of air.***

***Given***

$p_1 = 4 \text{ bar} = 4 \times 10^5 \text{ N/m}^2$

$T_1 = 100°\text{C} + 273 = 373 \text{ K}$

$M_1 = 0.2$

$T_{02} = 3\, T_{01}$

***To find***

1. The Mach number, pressure and temperature at exit, $(M_2, p_2, T_2)$
2. Stagnation pressure loss $(\Delta p_0)$
3. Heat supplied (Q).

***Solution***

Refer Isentropic flow table for $\gamma = 1.4$ and $M_1 = 0.2$

$$\frac{T_1}{T_{0_1}} = 0.992 \qquad \textit{[From gas tables page no.28]}$$

$$\frac{p_1}{p_{0_1}} = 0.973$$

$$T_{01} = \frac{T_1}{0.992} = \frac{373}{0.992} = 376 \text{ K}$$

$$\boxed{T_{01} = 376 \text{ K}}$$

$$\Rightarrow \qquad p_{01} = \frac{p_1}{0.973} = \frac{4 \times 10^5}{0.973}$$

$$\boxed{p_{01} = 4.11 \times 10^5 \text{ N/m}^2}$$

Refer Rayleigh flow table for $\gamma = 1.4$ and $M_1 = 0.2$

$$\frac{p_1}{p_1{}^*} = 2.273 \qquad \textit{[From gas tables page no. 111]}$$

$$\frac{p_{01}}{p_{01}{}^*} = 1.235$$

$$\frac{T_1}{T_1{}^*} = 0.207$$

$$\frac{T_{01}}{T_{01}{}^*} = 0.174$$

$$\Rightarrow \quad p_1{}^* = \frac{p_1}{2.273} = \frac{4 \times 10^5}{2.273} = 1.759 \times 10^5 \text{ N/m}^2$$

$$\boxed{p_1{}^* = 1.759 \times 10^5 \text{ N/m}^2 = p_2{}^*}$$

$$\Rightarrow \quad p_{01}{}^* = \frac{p_{01}}{1.235}$$

$$= \frac{4.11 \times 10^5}{1.235}$$

$$\boxed{p_{01}{}^* = 3.327 \times 10^5 \text{ N/m}^2 = p_{02}{}^*} \qquad [\because p_{01}{}^* = p_{02}{}^*]$$

$$\Rightarrow \quad T_1{}^* = \frac{T_1}{0.207}$$

$$= \frac{373}{0.207}$$

$$\boxed{T_1{}^* = 1801.93 \text{ K} = T_2{}^*} \qquad [\because T_1{}^* = T_2{}^*]$$

$$\Rightarrow \quad T_{01}{}^* = \frac{T_{01}}{0.174}$$

$$= \frac{376}{0.174}$$

$$\boxed{T_{01}{}^* = 2160.91\text{K} = T_{02}{}^*} \qquad [\because T_{01}{}^* = T_{02}{}^*]$$

From given, we know that

$$T_{02} = 3 \times T_{01}$$

$$= 3 \times 376$$

$$\boxed{T_{02} = 1128 \text{ K}}$$

$$\Rightarrow \quad \frac{T_{02}}{T_{02}{}^*} = \frac{1128}{2160.91} = 0.522$$

Refer Rayleigh flow table for $\gamma = 1.4$ and $\frac{T_{02}}{T_{02}{}^*} = 0.522 \approx 0.529$

$$M_2 = 0.40 \qquad \textit{[From gas tables page no. 111]}$$

$$\frac{p_2}{p_2{}^*} = 1.961$$

$$\frac{p_{02}}{p_{02}{}^*} = 1.157$$

$$\frac{T_2}{T_2{}^*} = 0.615$$

[**Note :** Inlet Mach number is subsonic, i.e., $M_1 < 1$. So, $M_2 < 1$]

$$\Rightarrow \quad p_2 = p_2{}^* \times 1.961$$

$$= 1.759 \times 10^5 \times 1.961$$

$$\boxed{p_2 = 3.449 \times 10^5 \text{ N/m}^2}$$

$\Rightarrow \quad T_2 = T_2^* \times 0.615$

$= 1801.93 \times 0.615$

$\boxed{T_2 = 1108.18 \text{ K}}$

$\Rightarrow \quad p_{02} = 1.157 \times p_{02}^*$

$= 1.157 \times 3.327 \times 10^5$

$\boxed{p_{02} = 3.849 \times 10^5 \text{ N/m}^2}$

Stagnation pressure loss

$\Delta p_0 = p_{01} - p_{02}$

$= 4.11 \times 10^5 - 3.849 \times 10^5$

$\boxed{\Delta p_0 = 0.261 \times 10^5 \text{ N/m}^2}$

Heat supplied, $Q = mc_p (T_{02} - T_{01})$

For unit mass

$Q = c_p (T_{02} - T_{01})$

$= 1005 [1128 - 376]$

$\boxed{Q = 755.7 \times 10^3 \text{ J/kg}}$

***Result***

1. $M_2 = 0.40$, $p_2 = 3.449 \times 10^5 \text{ N/m}^2$, $T_2 = 1108.18$ K
2. $\Delta p_0 = 0.261 \times 10^5 \text{ N/m}^2$
3. $Q = 755.7 \times 10^3$ J/kg

**[3]** ***The conditions of a gas in a combustion chamber at entry are $M_1 = 0.28$, $T_{01} = 380$ K, $P_{01} = 4.9$ bar. The heat supplied in the combustion chamber is 620 kJ/kg. Determine Mach number, pressure and temperature of the gas at exit and also determine the stagnation pressure loss during heating. Take $\gamma = 1.3$, $c_p = 1.22$ kJ/kg K.***

***Given***

$M_1 = 0.28$,

$T_{01} = 380$ K,

$p_{01} = 4.9 \text{ bar} = 4.9 \times 10^5 \text{ N/m}^2$

$Q = 620 \text{ kJ/kg} = 620 \times 10^3 \text{ J/kg}$

Take $\gamma = 1.3$, $c_p = 1.22 \text{ kJ/kg K} = 1.22 \times 10^3 \text{ J/kg K}$

***To find***

1. Mach number, pressure and temperature of the gas at exit, ($M_2$, $p_2$ and $T_2$)
2. Stagnation pressure loss ($\Delta p_0$)

***Solution***

Refer Isentropic flow table for $\gamma = 1.3$ and $M_1 = 0.28$

$\frac{T_1}{T_{0_1}} = 0.988$ *[From gas tables page no. 21]*

$\frac{p_1}{p_{0_1}} = 0.951$

$\Rightarrow \quad p_1 = p_{01} \times 0.951$

$= 4.9 \times 10^5 \times 0.951$

$\boxed{p_1 = 4.659 \times 10^5 \text{ N/m}^2}$

$$\Rightarrow \qquad T_1 = T_{01} \times 0.988$$

$$= 380 \times 0.988$$

$$\boxed{T_1 = 375.44 \text{ K}}$$

Refer Rayleigh flow table for $\gamma = 1.3$ and $M_1 = 0.28$

$$\frac{p_1}{p_1^*} = 2.087$$

$$\frac{p_{01}}{p_{01}^*} = 1.198 \qquad \textit{[ From gas tables page no.105]}$$

$$\frac{T_1}{T_1^*} = 0.342$$

$$\frac{T_{01}}{T_{01}^*} = 0.300$$

$$\Rightarrow \qquad p_1^* = \frac{p_1}{2.087} = \frac{4.659 \times 10^5}{2.087} = 2.23 \times 10^5 \text{ N/m}^2$$

$$\boxed{p_1^* = 2.23 \times 10^5 \text{ N/m}^2 = p_2^*} \qquad [\because p_1^* = p_2^*]$$

$$\Rightarrow \qquad p_{01}^* = \frac{p_{01}}{1.198}$$

$$= \frac{4.9 \times 10^5}{1.198} = 4.09 \times 10^5 \text{ N/m}^2$$

$$\boxed{p_{01}^* = 4.09 \times 10^5 \text{ N/m}^2 = p_{02}^*} \qquad [\because p_{01}^* = p_{02}^*]$$

$$\Rightarrow \qquad T_1^* = \frac{T_1}{0.342}$$

$$= \frac{375.44}{0.342}$$

$$\boxed{T_1^* = 1097.77 \text{ K} = T_2^*} \qquad [\because T_1^* = T_2^*]$$

$$\Rightarrow \qquad T_{01}^* = \frac{T_{01}}{0.300}$$

$$= \frac{380}{0.300}$$

$$\boxed{T_{01}^* = 1266.6 \text{ K} = T_{02}^*} \qquad [\because T_{01}^* = T_{02}^*]$$

We know

$$Q = mc_p (T_{02} - T_{01})$$

For unit mass

$$Q = c_p (T_{02} - T_{01})$$

$$620 \times 10^3 = 1.22 \times 10^3 [T_{02} - 380]$$

$$\Rightarrow \qquad [T_{02} - 380] = 508.19$$

$$\Rightarrow \qquad \boxed{T_{02} = 888.19 \text{ K}}$$

$$\frac{T_{02}}{T_{02}^*} = \frac{888.19}{1266.6} = 0.701$$

Refer Rayleigh flow table for $\gamma = 1.3$ and $\frac{T_{02}}{T_{02}^*} = 0.701 \approx 0.708$

$$M_2 = 0.52$$

$$\frac{p_2}{p_2^*} = 1.702 \qquad \textit{[From gas tables page no.105]}$$

$$\frac{p_{02}}{p_{02}^*} = 1.103$$

$$\frac{T_2}{T_2^*} = 0.783$$

[ **Note :** Inlet Mach number $M_1 < 1$, So $M_2 < 1$]

$$\Rightarrow \quad p_2 = p_2^* \times 1.702$$

$$= 2.23 \times 10^5 \times 1.702$$

$$\boxed{p_2 = 3.79 \times 10^5 \text{ N/m}^2}$$

$$\Rightarrow \quad T_2 = T_2^* \times 0.783$$

$$= 1097.77 \times 0.783$$

$$\boxed{T_2 = 859.55 \text{ K}}$$

$$\Rightarrow \quad p_{02} = 1.103 \times p_{02}^*$$

$$= 1.103 \times 4.09 \times 10^5$$

$$\boxed{p_{02} = 4.511 \times 10^5 \text{ N/m}^2}$$

Stagnation pressure loss

$$\Delta p_0 = p_{01} - p_{02}$$

$$= 4.9 \times 10^5 - 4.511 \times 10^5$$

$$\boxed{\Delta p_0 = 0.389 \times 10^5 \text{ N/m}^2}$$

***Result***

1. $M_2 = 0.52$

   $p_2 = 3.79 \times 10^5 \text{ N/m}^2$

   $T_2 = 859.55 \text{ K}$

2. Stagnation pressure loss

   $\Delta p_0 = 0.389 \times 10^5 \text{ N/m}^2$

4 ***The pressure, temperature and Mach number of the gas at exit are 2 bar, 1200°C and 0.7 respectively. The ratio of stagnation temperature at exit to entry is 3.85. Calculate the following.***

1. ***Mach number, pressure and temperature of the gas at entry.***
2. ***The heat supplied per kg of gas***
3. ***The maximum heat supplied***
4. ***Is it a cooling or heating process***

***Take $\gamma = 1.3$, $C_p = 1.22$ kJ/kg K***

***Given :***

$p_2 = 2 \text{ bar} = 2 \times 10^5 \text{ N/m}^2$

$T_2 = 1200°\text{C} = 1200 + 273 = 1473 \text{ K}$

$M_2 = 0.7$

$$\frac{T_{02}}{T_{01}} = 3.85$$

$c_p = 1.22 \text{ kJ/kg K} = 1.22 \times 10^3 \text{ J/kg K}; \gamma = 1.3$

***To find :***

1. The Mach number, pressure and temperature of the gas at entry, ($M_1$, $p_1$ and $T_1$)
2. Heat supplied, Q
3. Maximum heat supplied ($Q_{max}$)
4. Whether cooling or heating process

***Solution :***

Refer Isentropic flow table for $\gamma = 1.3$ and $M_2 = 0.7$

$$\frac{T_2}{T_{0_2}} = 0.932 \qquad \textit{[From gas tables page no.22]}$$

$$\frac{p_2}{p_{0_2}} = 0.735$$

$$\Rightarrow \quad T_{02} = \frac{T_2}{0.932} = \frac{1473}{0.932} = 1580.47 \text{ K}$$

$$\boxed{\text{Stagnation temperature at exit, } T_{02} = 1580.47 \text{ K}}$$

From given,

$$\frac{T_{02}}{T_{01}} = 3.85$$

$$\Rightarrow \quad T_{01} = \frac{T_{02}}{3.85}$$

$$= \frac{1580.47}{3.85} = 410.51 \text{ K}$$

$$\boxed{\text{Stagnation temperature at entry, } T_{01} = 410.51 \text{ K}}$$

Refer Rayleigh flow table for $\gamma = 1.3$ and $M_2 = 0.7$

$$\frac{p_2}{p_2^*} = 1.405$$

$$\frac{p_{02}}{p_{02}^*} = 1.042 \qquad \textit{[From gas tables page no.106]}$$

$$\frac{T_2}{T_2^*} = 0.967$$

$$\frac{T_{02}}{T_{02}^*} = 0.903$$

$$\Rightarrow \quad p_2^* = \frac{p_2}{1.405}$$

$$= \frac{2 \times 10^5}{1.405}$$

$$\boxed{p_2^* = 1.423 \times 10^5 \text{ N/m}^2 = p_1^*} \qquad [\because p_2^* = p_1^*]$$

$$\Rightarrow \quad T_2^* = \frac{T_2}{0.967}$$

$$= \frac{1473}{0.967}$$

$$\boxed{T_2^* = 1523.27 \text{ K} = T_1^*} \qquad [\because T_2^* = T_1^*]$$

$$\Rightarrow \quad T_{02}^* = \frac{T_{02}}{0.903}$$

$$= \frac{1580.47}{0.903}$$

$$\boxed{T_{02}^* = 1750.24 \text{ K} = T_{01}^*} \qquad [\because T_{02}^* = T_{01}^*]$$

$$\Rightarrow \quad \frac{T_{01}}{T_{01}^*} = \frac{410.51}{1750.24}$$

$$\frac{T_{01}}{T_{01}^*} = 0.234$$

Refer Rayleigh flow table for $\gamma = 1.3$ and $\frac{T_{01}}{T_{01}^*} = 0.234 \approx 0.231$

$$M_1 = 0.24$$

$$\frac{p_1}{p_1^*} = 2.140 \qquad \textit{[From gas tables page no.105]}$$

$$\frac{p_{01}}{p_{01}^*} = 1.212$$

$$\frac{T_1}{T_1^*} = 0.264$$

[ **Note :** Exit Mach number $M_2 < 1$. So, $M_1 < 1$]

$$\Rightarrow \quad p_1 = 2.140 \times p_1^*$$

$$= 2.140 \times 1.423 \times 10^5$$

$$\boxed{p_1 = 3.046 \times 10^5 \text{ N/m}^2} \qquad [\because p_1^* = p_2^*]$$

$$\Rightarrow \quad T_1 = 0.264 \times T_1^*$$

$$= 0.264 \times 1523.27$$

$$\boxed{T_1 = 402.14 \text{ K}} \qquad [\because T_1^* = T_2^*]$$

We know that

$$Q = mc_p (T_{02} - T_{01})$$

For unit mass

$$Q = c_p (T_{02} - T_{01})$$

$$= 1220 \ [1580.47 - 410.51]$$

$$Q = 1427.35 \times 10^3 \text{ J/kg}$$

$$\boxed{Q = 1427.35 \text{ kJ/kg}}$$

$$\text{Maximum heat supplied, } Q_{max} = \frac{(1-M_1^2)^2}{2(1+\gamma)M_1^2} \times c_pT_1$$

[From equation (3.21)]

$$\Rightarrow \quad Q_{max} = \frac{[1-(0.24)^2]^2}{2(1+1.3)\,(0.24)^2} \times 1220 \times 402.14$$

$$Q_{max} = 1644.47 \times 10^3 \text{ J/kg}$$

$$\boxed{Q_{max} = 1644.47 \text{ kJ/kg}}$$

Since the heat transfer is positive, it is a heating process.

***Result***

1. $M_1 = 0.24$

   $p_1 = 3.046 \times 10^5 \text{ N/m}^2$

   $T_1 = 402.14 \text{ K}$

2. $Q = 1427.35$ kJ/kg

3. $Q_{max} = 1644.47$ kJ/kg

4. Heating process

$\boxed{5}$ ***The condition of a gas ( $\gamma = 1.3$, $R = 465$ J/kg K ) at the entry of a constant area duct are $p_1 = 0.35$ bar, $T_1 = 310$ K, $c_1 = 60$ m/s. The heat supplied in the combustion chamber is 4500 kJ/kg. Calculate at exit***

***1. Pressure***

***2. Temperature***

***3. Mach number***

***4. Velocity of the gas***

***5. What are the pressure and temperature of the gas at sonic condition***

***Given***

$\gamma = 1.3$

$R = 465$ J/kg K

$p_1 = 0.35 \text{ bar} = 0.35 \times 10^5 \text{ N/m}^2$

$T_1 = 310$ K

$c_1 = 60$ m/s.

$Q = 4500 \text{ kJ/kg} = 4500 \times 10^3 \text{ J/kg}.$

***To find***

At exit

1. Pressure, $p_2$

2. Temperature, $T_2$
3. Mach number, $M_2$
4. Velocity of gas $c_2$
5. Pressure, temperature at sonic condition ($p^*$, $T^*$).

***Solution :***

We know,

$$\text{Mach number at entry, } M_1 = \frac{c_1}{a_1}$$

$$= \frac{c_1}{\sqrt{\gamma R T_1}}$$

$$= \frac{60}{\sqrt{1.4 \times 465 \times 310}}$$

$$\boxed{M_1 = 0.138}$$

Refer Isentropic flow table for $\gamma = 1.3$ and $M_1 = 0.138 \approx 0.14$

$$\frac{T_1}{T_{0_1}} = 0.997 \qquad \textit{[From gas tables page no.21]}$$

$$\frac{p_1}{p_{0_1}} = 0.987$$

$$T_{01} = \frac{T_1}{0.997} = \frac{310}{0.997} = 310.93 \text{ K}$$

$$\boxed{T_{01} = 310.93 \text{ K}}$$

$$\Rightarrow \quad p_{01} = \frac{p_1}{0.987}$$

$$= \frac{0.35 \times 10^5}{0.987} = 0.355 \times 10^5 \text{ N/m}^2$$

$$\boxed{p_{01} = 0.355 \times 10^5 \text{ N/m}^2}$$

Refer Rayleigh flow table for $\gamma = 1.3$ and $M_1 = 0.138 \approx 0.14$

$$\frac{p_1}{p_1^*} = 2.243 \qquad \textit{[ From gas tables page no.105}$$

$$\frac{p_{01}}{p_{01}^*} = 1.239$$

$$\frac{T_1}{T_1^*} = 0.098$$

$$\frac{T_{01}}{T_{01}^*} = 0.086$$

$$\frac{c_1}{c_1^*} = 0.044$$

$$\Rightarrow \quad p_1^* = \frac{p_1}{2.243} = \frac{0.35 \times 10^5}{2.243} = 0.156 \times 10^5 \text{ N/m}^2$$

$$\boxed{p_1^* = 0.156 \times 10^5 \text{ N/m}^2 = p_2^*} \qquad [\because p_1^* = p_2^*]$$

$$\Rightarrow \quad p_{01}^* = \frac{p_{01}}{1.239} = \frac{0.355 \times 10^5}{1.239} = 0.286 \times 10^5 \text{ N/m}^2$$

$$\boxed{p_{01}^* = 0.286 \times 10^5 \text{ N/m}^2 = p_{02}^*}$$

$$\Rightarrow \quad T_1^* = \frac{T_1}{0.098}$$

$$= \frac{310}{0.098}$$

$$\boxed{T_1^* = 3163.26 \text{ K} = T_2^*} \qquad [\because T_1^* = T_2^*]$$

$$\Rightarrow \qquad T_{01}^* = \frac{T_{01}}{0.086}$$

$$= \frac{310.93}{0.086}$$

$$\boxed{T_{01}^* = 3615.46\ \text{K} = T_{02}^*}$$

$$\Rightarrow \qquad c_1^* = \frac{c_1}{0.044}$$

$$= \frac{60}{0.044}$$

$$\boxed{c_1^* = 1363.64\ \text{m/s} = c_2^*}$$

We know that,

$$Q = c_p (T_{02} - T_{01})$$

$$4500 \times 10^3 = \frac{\gamma R}{\gamma - 1} [T_{02} - T_{01}] \qquad \left[\because c_p = \frac{\gamma R}{\gamma - 1}\right]$$

$$\Rightarrow \qquad 4500 \times 10^3 = \frac{1.3 \times 465}{1.3 - 1} [T_{02} - 310.93]$$

$$\Rightarrow \qquad [T_{02} - 310.93] = 2233.25$$

$$\Rightarrow \qquad \boxed{T_{02} = 2544.18\ \text{K}}$$

$$\frac{T_{02}}{T_{02}^*} = \frac{2544.18}{3615.46} = 0.703$$

Refer Rayleigh flow table for $\gamma = 1.3$ and $\frac{T_{02}}{T_{02}^*} = 0.703 \approx 0.708$

$$M_2 = 0.52$$

$$\frac{p_2}{p_2^*} = 1.702$$

$$\frac{p_{02}}{p_{02}^*} = 1.103 \qquad \textit{[From gas tables page no. 105]}$$

$$\frac{T_2}{T_2^*} = 0.783$$

$$\frac{c_2}{c_2^*} = 0.460$$

$$\Rightarrow \qquad p_2 = p_2^* \times 1.702$$

$$= 0.156 \times 10^5 \times 1.702$$

$$\boxed{p_2 = 0.265 \times 10^5\ \text{N/m}^2}$$

$$\Rightarrow \qquad T_2 = T_2^* \times 0.783$$

$$= 3163.26 \times 0.783$$

$$\boxed{T_2 = 2476.83\ \text{K}}$$

$$\Rightarrow \qquad c_2 = c_2^* \times 0.460$$

$$= 1363.64 \times 0.460$$

$$\boxed{c_2 = 627.27\ \text{m/s}}$$

We know

At sonic condition, $M = 1$

Refer Isentropic flow table for $M = 1$ and $\gamma = 1.3$

$$\Rightarrow \qquad \frac{T^*}{T_0^*} = 0.870 \qquad \textit{[From gas tables page no.22]}$$

$$\frac{p^*}{p_0^*} = 0.546$$

$$\Rightarrow \quad T^* = T_0^* \times 0.870$$

$$= 3615.46 \times 0.870 \qquad [\because T_0^* = T_{01}^* = T_{02}^*]$$

$$\boxed{T^* = 3145.45 \text{ K}}$$

$$\Rightarrow \quad p^* = p_0^* \times 0.546$$

$$= 0.286 \times 10^5 \times 0.546 \qquad [\because p_{01}^* = p_{02}^* = p_0^*]$$

$$\boxed{p^* = 0.156 \times 10^5 \text{ N/m}^2}$$

***Result***

1. $p_2 = 0.265 \times 10^5$ N/m$^2$
2. $T_2 = 2476.83$ K
3. $M_2 = 0.52$
4. $c_2 = 627.27$ m/s.
5. At sonic condition

   $T^* = 3145.45$ K

   $p^* = 0.156 \times 10^5$ N/m$^2$

**6** *The condition of a gas in a combustion chamber at entry are $T_1 = 375$ K, $p_1 = 0.50$ bar, $c_1 = 70$ m/s. The air–fuel ratio is 29 and the calorific value of the fuel is 42 MJ/kg. Calculate*

*1. The initial and final Mach number*

*2. Final pressure, temperature and velocity of the gas*

*3. Percentage of stagnation pressure loss*

*4. Maximum stagnation temperature.*

*Take $\gamma = 1.4$ and $R = 0.287$ kJ/kg K*

***Given :***

$T_1 = 375$ K

$p_1 = 0.50$ bar $= 0.50 \times 10^5$ N/m$^2$

$c_1 = 70$ m/s.

Air fuel ratio = 29

Calorific value = 42 MJ/kg = $42 \times 10^6$ J/kg

$\gamma = 1.4$

R = 0.287 kJ/kg K = 287 J/kg K.

***To find :***

1. The initial and final Mach number ($M_1$, $M_2$)
2. Final pressure, temperature and velocity of the gas ($p_2$, $T_2$, $c_2$)
3. Percentage of stagnation pressure loss
4. Maximum stagnation temperature ($T_{0max}$).

***Solution :***

We know that,

$$\text{Mach number at entry, } M_1 = \frac{c_1}{a_1}$$

$$= \frac{c_1}{\sqrt{\gamma R T_1}}$$

$$= \frac{70}{\sqrt{1.4 \times 287 \times 375}}$$

$$\boxed{M_1 = 0.180}$$

Refer Isentropic flow table for $\gamma = 1.4$ and $M_1 = 0.180$

$$\frac{T_1}{T_{0_1}} = 0.994 \qquad \textit{[From gas tables page no.28]}$$

$$\frac{p_1}{p_{0_1}} = 0.978$$

$$T_{01} = \frac{T_1}{0.994}$$

$$= \frac{375}{0.994}$$

$$\boxed{T_{01} = 377.26 \text{ K}}$$

$$\Rightarrow \quad p_{01} = \frac{p_1}{0.978}$$

$$= \frac{0.50 \times 10^5}{0.978}$$

$$\boxed{p_{01} = 0.5112 \times 10^5 \text{ N/m}^2}$$

Refer Rayleigh flow table for $\gamma = 1.4$ and $M_1 = 0.18$

$$\frac{p_1}{p_1^*} = 2.295$$ *[From gas tables page no.111]*

$$\frac{p_{01}}{p_{01}^*} = 1.241$$

$$\frac{T_1}{T_1^*} = 0.171$$

$$\frac{T_{01}}{T_{01}^*} = 0.143$$

$$\frac{c_1}{c_1^*} = 0.074$$

$$\Rightarrow \quad p_1^* = \frac{p_1}{2.295}$$

$$= \frac{0.50 \times 10^5}{2.295}$$

$$\boxed{p_1^* = 0.218 \times 10^5 \text{ N/m}^2 = p_2^*} \qquad [\because p_1^* = p_2^*]$$

$$\Rightarrow \quad p_{01}^* = \frac{p_{01}}{1.241}$$

$$= \frac{0.5112 \times 10^5}{1.241}$$

$$\boxed{p_{01}^* = 0.4119 \times 10^5 \text{ N/m}^2 = p_{02}^*} \qquad [\because p_{01}^* = p_{02}^*]$$

$$\Rightarrow \quad T_1^* = \frac{T_1}{0.171}$$

$$= \frac{375}{0.171}$$

$$\boxed{T_1^* = 2192.98 \text{ K} = T_2^*} \qquad [\because T_1^* = T_2^*]$$

$$\Rightarrow \quad T_{01}^* = \frac{T_{01}}{0.143}$$

$$= \frac{377.26}{0.143}$$

$$\boxed{T_{01}^* = 2638.18 \text{ K} = T_{02}^*} \qquad [\because T_{01}^* = T_{02}^*]$$

$$\Rightarrow \quad c_1^* = \frac{c_1}{0.074}$$

$$= \frac{70}{0.074}$$

$$\boxed{c_1^* = 945.95 \text{ m/s} = c_2^*} \qquad [\because c_1^* = c_2^*]$$

Stagnation enthalpy rise due to the combustion of one kg of fuel is

$$\Delta h_0 = \frac{\text{Calorific vlaue of fuel}}{\text{Air fuel ratio} + 1}$$

$$= \frac{42 \times 10^6}{29 + 1}$$

$$\Delta h_0 = 1.4 \times 10^6 \text{ J/kg}$$

We know that

$$\Delta h_0 = h_{02} - h_{01}$$

$$= c_p T_{02} - c_p T_{01} \qquad [\because h_0 = c_p T_0]$$

$$1.4 \times 10^6 = c_p [T_{02} - T_{01}]$$

$$= \frac{\gamma R}{\gamma - 1} [T_{02} - T_{01}] \qquad \left[\because c_p = \frac{\gamma R}{\gamma - 1}\right]$$

$$\Rightarrow \quad 1.4 \times 10^6 = \frac{1.4 \times 287}{1.4 - 1} [T_{02} - 377.26]$$

$$\Rightarrow \quad [T_{02} - 377.26] = 1393.73$$

$$\Rightarrow \quad \boxed{T_{02} = 1770.99 \text{ K}}$$

$$\Rightarrow \quad \frac{T_{02}}{T_{02}^*} = \frac{1770.99}{2638.18} = 0.67$$

Refer Rayleigh flow table for $\gamma = 1.4$ and $\frac{T_{02}}{T_{02}^*} = 0.67 \approx 0.661$

$$M_2 = 0.48 \qquad \textbf{[Note : } M_1 < 1 \Rightarrow M_2 < 1\textbf{]}$$

$$\frac{p_2}{p_2^*} = 1.815$$

$$\frac{p_{02}}{p_{02}^*} = 1.122 \qquad \textit{[ From gas tables page no. 111]}$$

$$\frac{T_2}{T_2^*} = 0.758$$

$$\frac{c_2}{c_2^*} = 0.418$$

$$\Rightarrow \quad p_2 = p_2^* \times 1.815$$

$$= 0.218 \times 10^5 \times 1.815$$

$$\boxed{p_2 = 0.3956 \times 10^5 \text{ N/m}^2}$$

$$\Rightarrow \quad T_2 = T_2^* \times 0.758$$

$$= 2192.98 \times 0.758$$

$$\boxed{T_2 = 1662.27 \text{ K}}$$

$$\Rightarrow \quad c_2 = c_2^* \times 0.418$$

$$= 945.95 \times 0.418$$

$$\boxed{c_2 = 395.41 \text{ m/s}}$$

$$\Rightarrow \quad p_{02} = 1.122 \times p_{02}^*$$

$$= 1.122 \times 0.4119 \times 10^5$$

$$\boxed{p_{02} = 0.462 \times 10^5 \text{ N/m}^2}$$

Percentage of stagnation pressure loss

$$= \frac{p_{01} - p_{02}}{p_{01}} \times 100$$

$$= \frac{0.5112 \times 10^5 - 0.462 \times 10^5}{0.5112 \times 10^5} \times 100$$

$$= 9.62\ \%$$

Maximum stagnation temperature

$$T_{0max} = T_{01}^* = T_{02}^* = T_0^*$$

$$\boxed{T_{0max} = 2638.18 \text{ K}}$$

***Result***

1. $M_1 = 0.180$, $M_2 = 0.48$
2. $p_2 = 0.3956 \times 10^5$, $T_2 = 1662.27$ K, $c_2 = 395.41$ m/s.

3. Percentage of stagnation pressure loss = 9.62 %

4. Maximum stagnation temperature, $T_{0max} = 2638.18$ K

**7** ***The data for a gas ($\gamma = 1.3$, $c_p = 2.144$ kJ/kg K) at entry of combustion chamber are $c_1 = 150$ m/s, $p_1 = 4$ bar and $T_1 = 395$ K. If the exit Mach number is 0.78, calculate the following***

***1. The initial Mach number***

***2. Final pressure, temperature and velocity of the gas***

***3. Stagnation pressure loss***

***4. Air fuel ratio required.***

***Take calorific value of fuel is 42 MJ/kg***

***Given***

$\gamma = 1.3$

$c_p = 2.144$ kJ/kg K $= 2.144 \times 10^3$ J/kg K

$c_1 = 150$ m/s.

$p_1 = 4$ bar $= 4 \times 10^5$ N/m$^2$

$T_1 = 395$ K

$M_2 = 0.78$

Calorific value = 42 MJ/kg = $42 \times 10^6$ J/kg

***To find***

1. Entry Mach number, $M_1$
2. Pressure, temperature and velocity of the gas at exit, $(p_2, T_2, c_2)$
3. Stagnation pressure loss, $(\Delta p_0)$
4. Air fuel ratio

***Solution :***

Mach number at entry, $M_1 = \dfrac{c_1}{a_1}$

$$= \frac{c_1}{\sqrt{\gamma R T_1}}$$

$$= \frac{c_1}{\sqrt{\gamma \times \dfrac{c_p(\gamma-1)}{\gamma} \times T_1}} \qquad \left[\because R = \frac{c_p(\gamma-1)}{\gamma}\right]$$

$$= \frac{150}{\sqrt{1.3 \times \dfrac{2.144 \times 10^3\,(1.3-1)}{1.3} \times 375}}$$

$$\boxed{M_1 = 0.305}$$

Refer Isentropic flow table for $M_1 = 0.305 \approx 0.3$ and $\gamma = 1.3$

$$\frac{T_1}{T_{0_1}} = 0.987 \qquad \textit{[From gas tables page no.21]}$$

$$\frac{p_1}{p_{0_1}} = 0.944$$

$$\Rightarrow \quad T_{01} = \frac{T_1}{0.987} = \frac{395}{0.987}$$

$$\boxed{T_{01} = 400.20 \text{ K}}$$

$$\Rightarrow \quad p_{01} = \frac{p_1}{0.944}$$

$$= \frac{4 \times 10^5}{0.944}$$

$$\boxed{p_{01} = 4.23 \times 10^5 \text{ N/m}^2}$$

Refer Rayleigh flow table for $\gamma = 1.3$ and $M_1 = 0.297 \approx 0.30$

$$\frac{p_1}{p_1^*} = 2.059 \qquad \text{[From gas tables page no.105]}$$

$$\frac{p_{01}}{p_{01}^*} = 1.191$$

$$\frac{T_1}{T_1^*} = 0.382$$

$$\frac{T_{01}}{T_{01}^*} = 0.336$$

$$\frac{c_1}{c_1^*} = 0.185$$

$$\Rightarrow \qquad p_1^* = \frac{p_1}{2.059} = \frac{4 \times 10^5}{2.059} = 1.94 \times 10^5 \text{ N/m}^2$$

$$\boxed{p_1^* = 1.94 \times 10^5 \text{ N/m}^2 = p_2^*}$$

$$\Rightarrow \qquad p_{01}^* = \frac{p_{01}}{1.191}$$

$$= \frac{4.23 \times 10^5}{1.191} = 3.55 \times 10^5 \text{ N/m}^2$$

$$\boxed{p_{01}^* = 3.55 \times 10^5 \text{ N/m}^2 = p_{02}^*}$$

$$\Rightarrow \qquad T_1^* = \frac{T_1}{0.382}$$

$$= \frac{395}{0.382}$$

$$\boxed{T_1^* = 1034.03 \text{ K} = T_2^*}$$

$$\Rightarrow \qquad T_{01}^* = \frac{T_{01}}{0.336}$$

$$= \frac{400.20}{0.336}$$

$$\boxed{T_{01}^* = 1191.07 \text{ K} = T_{02}^*}$$

$$\Rightarrow \qquad c_1^* = \frac{c_1}{0.185}$$

$$= \frac{150}{0.185} = 810.81 \text{ m/s}$$

$$\boxed{c_1^* = 810.81 \text{ m/s} = c_2^*}$$

Refer Rayleigh flow table for $\gamma = 1.3$ and $M_2 = 0.78$

$$\frac{p_2}{p_2^*} = 1.284 \qquad \text{[From gas tables page no.106]}$$

$$\frac{p_{02}}{p_{02}^*} = 1.023$$

$$\frac{T_2}{T_2^*} = 1.003$$

$$\frac{c_2}{c_2^*} = 0.781$$

$$\frac{T_{02}}{T_{02}^*} = 0.952$$

$$\Rightarrow \quad p_2 = p_2^* \times 1.284$$

$$= 1.94 \times 10^5 \times 1.284$$

$$\boxed{p_2 = 2.49 \times 10^5 \text{ N/m}^2}$$

$$\Rightarrow \quad p_{02} = p_{02}^* \times 1.023$$

$$= 3.55 \times 10^5 \times 1.023$$

$$\boxed{p_{02} = 3.63 \times 10^5 \text{ N/m}^2}$$

$$\Rightarrow \quad T_2 = T_2^* \times 1.003$$

$$= 1034.03 \times 1.003$$

$$\boxed{T_2 = 1037.13 \text{ K}}$$

$$\Rightarrow \quad T_{02} = T_{02}^* \times 0.952$$

$$= 1191.07 \times 0.952$$

$$\boxed{T_{02} = 1133.89 \text{ K}}$$

$$\Rightarrow \quad c_2 = c_2^* \times 0.781$$

$$= 810.81 \times 0.781$$

$$\boxed{c_2 = 633.24 \text{ m/s}}$$

Stagnation pressure loss

$$\Delta p_0 = p_{01} - p_{02}$$

$$= 4.23 \times 10^5 - 3.63 \times 10^5$$

$$\boxed{\Delta p_0 = 0.6 \times 10^5 \text{ N/m}^2}$$

Stagnation enthalpy rise,

$$\Delta h_0 = \frac{\text{Calorific value of fuel}}{\text{Air fuel ratio} + 1}$$

$$\Rightarrow \quad c_p T_{02} - c_p T_{01} = \frac{42 \times 10^6}{\text{Air fuel ratio} + 1} \qquad [\because h_0 = c_p T_0]$$

$$\Rightarrow \quad c_p [T_{02} - T_{01}] = \frac{42 \times 10^6}{\text{Air fuel ratio} + 1}$$

$$\Rightarrow \quad 2.144 \times 10^3 [1133.89 - 400.20] = \frac{42 \times 10^6}{\text{Air fuel ratio} + 1}$$

$$\Rightarrow \quad \text{Air fuel ratio} + 1 = 26.70$$

$$\Rightarrow \quad \text{Air fuel ratio} = 25.70$$

$$\Rightarrow \quad \text{Air fuel ratio} = \underset{\text{Air}}{\underset{\downarrow}{25.70}} : \underset{\text{Fuel}}{\underset{\downarrow}{1}}$$

***Result:***

1. $M_1 = 0.297$
2. $p_2 = 2.44 \times 10^5 \text{ N/m}^2$

   $T_2 = 1037.13 \text{ K}$

   $c_2 = 633.24 \text{ m/s}$
3. $\Delta p_0 = 0.6 \times 10^5 \text{ N/m}^2$
4. Air fuel ratio $= 25.70 : 1$

## 3.2 FLOW IN CONSTANT AREA DUCTS WITH FRICTION AND WITHOUT HEAT TRANSFER (FANNO FLOW)

### 3.2.1. Introduction

Flow in a constant area duct with friction and without heat transfer and work transfer is known as Fanno flow.

In the previous chapter, frictionless flow in a constant area duct was discussed. But in many engineering applications where the effect of friction may not be neglected. In this chapter, the frictional phenomena will be discussed, in a simplified manner.

Similar to that Rayleigh flow, the following assumptions are made for fanno flow.

1. One dimensional steady flow.
2. Flow takes place in constant sectional area.
3. There is no heat transfer.
4. The gas is perfect with constant specific heats.
5. Absence of work transfer across the boundaries.

### 3.2.2. Fanno Line (or) Curve

Flow in a constant area duct with friction and without heat transfer is described by a curve is known as Fanno line or Fanno curve.

We know that

Mass flow rate, $m = \rho A c$

$$\Rightarrow \quad \frac{m}{A} = \rho c$$

$$\Rightarrow \quad G = \frac{m}{A} = \rho c$$

where

G – Mass flow density

c – velocity of fluid

ρ – Density of fluid

$$\Rightarrow \quad G = \rho c$$

$$\Rightarrow \quad \boxed{c = \frac{G}{\rho}} \qquad ----(3.22)$$

Stagnation enthalpy, $h_0 = h + \frac{1}{2} c^2$

Substitute $c = \frac{G}{\rho}$

$$\Rightarrow \quad h_0 = h + \frac{1}{2}\frac{G^2}{\rho 2}$$

$$\Rightarrow \quad \boxed{h = h_0 - \frac{1}{2}\frac{G^2}{\rho 2}} \qquad ----(3.23)$$

Density (ρ) is a function of entropy and enthalpy.

$$\Rightarrow \quad \rho = f(s, h)$$

Substitute ρ value in Equation no(3.23)

$$\Rightarrow \quad h = h_0 - \frac{1}{2}\frac{G^2}{[f(s, h)]^2}$$

$$\boxed{h = h_0 - \frac{1}{2}\frac{G^2}{[f(s, h)]^2}} \qquad ----(3.24)$$

Equation (3.23) or (3.24) may be used for representing fanno line on the $h$–s diagram as shown in fig. 3.3.

The following assumption are made for Isothermal flow

1. One dimensional frictional flow
2. Flow takes place in constant sectional area
3. The gas is perfect with constant specific heats.
4. Temperature remains constant.

Isothermal flow processes are represented by straight horizontal lines on the temperature – entropy diagram as shown in fig. 3.6.

In acceleration process (heating process), the pressure of the gas decreases up to limiting state where $M = {}^{1}/_{\sqrt{\gamma}}$ and entropy increases.

In deceleration process (cooling process), the pressure of the gas increases and entropy decreases.

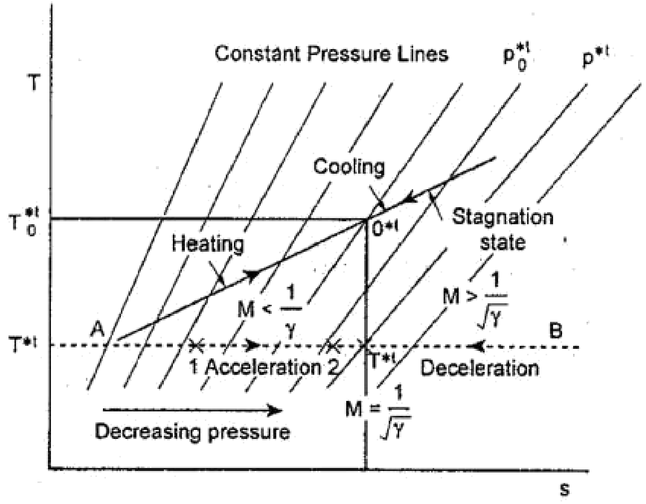


*Fig. 3.6 Isothermal flow with friction*

## 3.2.15 SOLVED PROBLEMS

**1** ***Air at $p_0$ = 11 bar, $T_0$ = 420 K enters a 45 mm diameter pipe at a Mach number of 3 and the friction co-efficient for the pipe surface is 0.001. If the Mach number at exit is 0.8, Determine***

***1. Mass flow rate***

***2. Length of the pipe***

***Given :***

$p_0 = 11 \text{ bar} = 11 \times 10^5 \text{ N/m}^2$

$T_0 = 420 \text{ K}$

$D = 45 \text{ mm} = 0.045 \text{ m}$

$M_1 = 3$

$\bar{f} = 0.001$

$M_2 = 0.8$. For air, $\gamma = 1.4$ and $R = 287$ J/kg–K

***To find :***

1. Mass flow rate (m)
2. Length of the pipe (L)

***Solution :***

Refer Isentropic flow table for $\gamma = 1.4$ and $M_1 = 3$

$$\frac{T_1}{T_{0_1}} = 0.357$$ *[From gas tables (S.M. Yahya, Fifth edition) -page no.37]*

$$\frac{p_1}{p_{0_1}} = 0.0272$$

$$\Rightarrow \quad T_1 = T_{01} \times 0.357$$

$$= 420 \times 0.357 \qquad [\because T_0 = T_{01}]$$

$$\boxed{T_1 = 149.94 \text{ K}}$$

$$\Rightarrow \qquad p_1 = p_{01} \times 0.0272$$

$$= 11 \times 10^5 \times 0.0272 \qquad [\because p_0 = p_{01}]$$

$$\boxed{p_1 = 0.299 \times 10^5 \text{ N/m}^2}$$

Mass flow rate, $\dot{m} = \rho A c = \rho_1 A_1 c_1 = \rho_2 A_2 c_2$

$$\Rightarrow \qquad \dot{m} = \rho_1 A_1 c_1$$

$$= \frac{p_1}{RT_1} \times A_1 \times c_1$$

$$= \frac{p_1}{RT_1} \times \frac{\pi}{4} (D_1{}^2) \times M_1 \times a_1 \qquad [\because M = \frac{c}{a}]$$

$$= \frac{p_1}{RT_1} \times \frac{\pi}{4} (D_1{}^2) \times M_1 \times \sqrt{\gamma RT_1} \qquad [\because a = \sqrt{\gamma RT}]$$

$$= \frac{0.299 \times 10^5}{287 \times 149.94} \times \frac{\pi}{4} [0.045]^2 \times 3 \times \sqrt{1.4 \times 287 \times 149.94}$$

$$\boxed{m = 0.813 \text{ kg/s}}$$

Refer Fanno flow table for $\gamma = 1.4$ and $M_1 = 3$

$$\frac{4 \bar{f} L_{max}}{D} = 0.522 \qquad \textit{[From gas tables page no.85]}$$

Refer Fanno flow table for $\gamma = 1.4$ and $M_2 = 0.8$

$$\frac{4 \bar{f} L_{max}}{D} = 0.073 \qquad \textit{[From gas tables page no.82]}$$

We know that

$$\frac{4 \bar{f} L}{D} = \left[\frac{4 \bar{f} L_{max}}{D}\right]_{M_1} - \left[\frac{4 \bar{f} L_{max}}{D}\right]_{M_2}$$

$$= 0.522 - 0.073$$

$$\frac{4 \bar{f} L}{D} = 0.449$$

$$\Rightarrow \qquad L = \frac{0.449 \times D}{4 \times \bar{f}}$$

$$= \frac{0.449 \times 0.045}{4 \times 0.001}$$

$$\boxed{L = 5.05 \text{ m}}$$

***Result***

1. m = 0.813 kg/s
2. L = 5.05 m

$\boxed{2}$ ***Air at $p_1$ = 3.4 bar, $T_1$ = 35°C enters a circular duct at a Mach number of 0.14. The exit Mach number is 0.6 and co–efficient of friction is 0.004. If the mass flow rate is 8.2 kg/s, determine***

1. ***Pressure, temperature at the exit***
2. ***Diameter of the duct***
3. ***Length of the duct***
4. ***Stagnation pressure loss***
5. ***Verify the exit Mach number through exit velocity and temperature.***

***Given :***

$p_1 = 3.4 \text{ bar} = 3.4 \times 10^5 \text{ N/m}^2$

$T_1 = 35°\text{C} + 273 = 308 \text{ K}$

$M_1 = 0.14$

$M_2 = 0.6$

$\bar{f} = 0.004$

$\dot{m} = 8.2 \text{ kg/s}$

***To find :***

1. Pressure, temperature at exit, $(p_2, T_2)$
2. Diameter of the duct, (D)
3. Length of the duct, (L)
4. Stagnation pressure loss, $(p_{01} - p_{02})$
5. Verify exit Mach number.

***Solution***

Refer Isentropic flow table for $\gamma = 1.4$ and $M_1 = 0.14$

$$\frac{T_1}{T_{0_1}} = 0.996 \qquad \textit{[From gas tables page no.28]}$$

$$\frac{p_1}{p_{0_1}} = 0.986$$

$$\Rightarrow \qquad T_{01} = \frac{T_1}{0.996} = \frac{308}{0.996} = 308.93 \text{ K}$$

$$\boxed{T_{01} = 308.93 \text{ K}}$$

$$\Rightarrow \qquad p_{01} = \frac{p_1}{0.986}$$

$$= \frac{3.4 \times 10^5}{0.986}$$

$$\boxed{p_{01} = 3.44 \times 10^5 \text{ N/m}^2}$$

Refer Fanno flow table for $\gamma = 1.4$ and $M_1 = 0.14$

$$\frac{p_1}{p_1{}^*} = 7.809$$

$$\frac{c_1}{c_1{}^*} = 0.153 \qquad \textit{[From gas tables page no.81]}$$

$$\frac{T_1}{T_1{}^*} = 1.195$$

$$\frac{p_{01}}{p_{01}{}^*} = 4.183$$

$$\frac{4\,\bar{f}\,L_{max}}{D} = 32.511$$

$$\Rightarrow \qquad p_1{}^* = \frac{p_1}{7.809}$$

$$= \frac{3.4 \times 10^5}{7.809}$$

$$\boxed{p_1{}^* = 0.435 \times 10^5 \text{ N/m}^2 = p_2{}^*} \qquad [\because \text{For Fanno flow } p_1{}^* = p_2{}^*]$$

$$\Rightarrow \qquad c_1{}^* = \frac{c_1}{0.153}$$

$$= \frac{M_1 \times a_1}{0.153} \qquad [\because M = \frac{c}{a}]$$

$$= \frac{M_1 \times \sqrt{\gamma R T_1}}{0.153} \qquad [\because a = \sqrt{\gamma RT}]$$

$$= \frac{0.14 \times \sqrt{1.4 \times 287 \times 308}}{0.153}$$

$$\boxed{c_1^* = 321.89 \text{ m/s} = c_2^*}$$

$$\Rightarrow \quad T_1^* = \frac{T_1}{1.195}$$

$$= \frac{308}{1.195}$$

$$\boxed{T_1^* = 257.74 \text{ K} = T_2^*} \qquad [\because T_1^* = T_2^*]$$

$$\Rightarrow \quad p_{01}^* = \frac{p_{01}}{4.183}$$

$$= \frac{3.44 \times 10^5}{4.183}$$

$$\boxed{p_{01}^* = 0.822 \times 10^5 \text{ N/m}^2 = p_{02}^*} \qquad [\because p_{01}^* = p_{02}^*]$$

Refer Fanno flow table for $\gamma = 1.4$ and $M_2 = 0.6$

$$\frac{p_2}{p_2^*} = 1.763$$

$$\frac{c_2}{c_2^*} = 0.635$$

*[From gas tables page no.81]*

$$\frac{T_2}{T_2^*} = 1.119$$

$$\frac{p_{02}}{p_{02}^*} = 1.188$$

$$\frac{4\bar{f}L_{max}}{D} = 0.491$$

$$\Rightarrow \quad p_2 = p_2^* \times 1.763$$

$$= 0.435 \times 10^5 \times 1.763$$

$$\boxed{p_2 = 0.766 \times 10^5 \text{ N/m}^2}$$

$$\Rightarrow \quad T_2 = T_2^* \times 1.119$$

$$= 257.74 \times 1.119$$

$$\boxed{T_2 = 288.41 \text{ K}}$$

$$\Rightarrow \quad c_2 = c_2^* \times 0.635$$

$$= 321.89 \times 0.635$$

$$\boxed{c_2 = 204.40 \text{ m/s}}$$

$$\Rightarrow \quad p_{02} = p_{02}^* \times 1.188$$

$$= 0.822 \times 10^5 \times 1.188$$

$$\boxed{p_{02} = 0.976 \times 10^5 \text{ N/m}^2}$$

**Diameter of the duct (D)**

Mass flow rate, $m = \rho_1 A_1 c_1$

$$= \frac{p_1}{RT_1} \times \frac{\pi}{4}(D_1^2) \times c_1$$

$$\Rightarrow \quad 8.2 = \frac{3.4 \times 10^5}{287 \times 308} \times \frac{\pi}{4}(D_1^2) \times M_1 \times a_1$$

$$[\because M = \frac{c}{a}]$$

$$\Rightarrow \quad 8.2 = \frac{3.4 \times 10^5}{287 \times 308} \times \frac{\pi}{4}(D_1^2) \times M_1 \times \sqrt{\gamma RT}]$$

$$[\because a = \sqrt{\gamma RT}]$$

$$\Rightarrow \quad 8.2 = \frac{3.4 \times 10^5}{287 \times 308} \times \frac{\pi}{4}(D_1^2) \times 0.14 \times \sqrt{1.4 \times 287 \times 308}$$

$$\Rightarrow \quad D_1^2 = 0.0551$$

$$\Rightarrow \quad \boxed{D_1 = 0.234 \text{ m}}$$

Area is constant. so,

$$\boxed{D_1 = D_2 = D = 0.234 \text{ m}}$$

**Length of the duct**

We know that,

$$\frac{4 fL}{D} = \left[\frac{4 \bar{f} L_{max}}{D}\right]_{M_1} - \left[\frac{4 \bar{f} L_{max}}{D}\right]_{M_2}$$

$$= 32.511 - 0.491$$

$$\frac{4 \bar{f} L}{D} = 32.02$$

$$\Rightarrow \quad L = \frac{32.02 \times D}{4 \times \bar{f}}$$

$$= \frac{32.02 \times 0.234}{4 \times 0.004}$$

$$\boxed{L = 468.29 \text{ m}}$$

**Stagnation pressure loss**

$$\Delta p = p_{01} - p_{02}$$

$$= 3.44 \times 10^5 - 0.976 \times 10^5$$

$$\boxed{\Delta p = 2.464 \times 10^5 \text{ N/m}^2}$$

**To verify exit Mach number**

Exit Mach number,

$$M_2 = \frac{c_2}{a_2}$$

$$= \frac{204.40}{\sqrt{\gamma RT_2}}$$

$$= \frac{204.40}{\sqrt{1.4 \times 287 \times 288.41}}$$

$$\boxed{M_2 = 0.60} \quad \text{verified}$$

***Result***

1. (a) $p_2 = 0.766 \times 10^5 \text{ N/m}^2$

   (b) $T_2 = 288.41$ K

2. $D = 0.234$ m

3. $L = 468.29$ m

4. $\Delta p_0 = 2.464 \times 10^5 \text{ N/m}^2$

**3** ***Air enters a pipe of 25 mm diameter, at a Mach number of 2.4 stagnation temperature of 300 K and static pressure of 0.5 bar. If the co-efficient of friction is 0.003, determine the following for a section at which the Mach number reaches 1.2.***

1. ***Static pressure and temperature***
2. ***Stagnation pressure and temperature***
3. ***Velocity of air***
4. ***Distance of this section from the inlet***
5. ***Mass flow rate***

***Given***

$D = 25\text{ mm} = 0.025\text{ m}$

$M_1 = 2.4$

$T_{01} = 300\text{ K}$

$p_1 = 0.5\text{ bar} = 0.5 \times 10^5\text{ N/m}^2$

$\bar{f} = 0.003$

$M_2 = 1.2$

***To find***

1. Static pressure and temperature at exit $(p_2, T_2)$
2. Stagnation pressure and temperature at exit $(p_{02}, T_{02})$
3. Velocity of air at exit $(c_2)$
4. Distance of this section from inlet (L)
5. Mass flow rate (m)

***Solution***

Refer Isentropic flow table for $\gamma = 1.4$ and $M_1 = 2.4$

$$\frac{T_1}{T_{0_1}} = 0.464 \qquad \textit{[From gas tables page no.34]}$$

$$\frac{p_1}{p_{0_1}} = 0.0684$$

$$\Rightarrow \quad T_1 = T_{01} \times 0.464$$

$$= 300 \times 0.464$$

$$\boxed{T_1 = 139.2\text{ K}}$$

$$\Rightarrow \quad p_{01} = \frac{p_1}{0.0684}$$

$$= \frac{0.5 \times 10^5}{0.0684}$$

$$\boxed{p_{01} = 7.309 \times 10^5\text{ N/m}^2}$$

Refer Fanno flow table for $\gamma = 1.4$ and $M_1 = 2.4$

$$\frac{p_1}{p_1^*} = 0.311$$

$$\frac{T_1}{T_1^*} = 0.557 \qquad \textit{[From gas tables page no.84]}$$

$$\frac{p_{01}}{p_{01}^*} = 2.403$$

$$\frac{4\bar{f}L_{max}}{D} = 0.409$$

$$\Rightarrow \quad p_1^* = \frac{p_1}{0.311}$$

$$= \frac{0.5 \times 10^5}{0.311}$$

$$\boxed{p_1^* = 1.607 \times 10^5\text{ N/m}^2 = p_2^*} \qquad [\because p_1^* = p_2^*]$$

$$\Rightarrow \quad T_1^* = \frac{T_1}{0.557}$$

$$= \frac{139.2}{0.557}$$

$$\boxed{T_1^* = 249.91\text{ K} = T_2^*} \qquad [\because T_1^* = T_2^*]$$

$$\Rightarrow \quad p_{01}^* = \frac{p_{01}}{2.403}$$

$$= \frac{7.309 \times 10^5}{2.403}$$

$$\boxed{p_{01}^* = 3.042 \times 10^5\text{ N/m}^2 = p_{02}^*}$$

Refer Fanno flow table for $\gamma = 1.4$ and $M_2 = 1.2$

$$\frac{p_2}{p_2^*} = 0.804$$

$$\frac{T_2}{T_2^*} = 0.932 \qquad \textit{[From gas tables page no.82]}$$

$$\frac{p_{02}}{p_{02}^*} = 1.030$$

$$\frac{4\bar{f}L_{max}}{D} = 0.034$$

$$\Rightarrow \quad p_2 = p_2^* \times 0.804$$

$$= 1.607 \times 10^5 \times 0.804$$

$$\boxed{p_2 = 1.292 \times 10^5\text{ N/m}^2}$$

$$\Rightarrow \quad T_2 = T_2^* \times 0.932$$

$$= 249.91 \times 0.932$$

$$\boxed{T_2 = 232.92\text{ K}}$$

$$\Rightarrow \quad p_{02} = p_{02}^* \times 1.030$$

$$= 3.042 \times 10^5 \times 1.030$$

$$\boxed{p_{02} = 3.133 \times 10^5\text{ N/m}^2}$$

We know that

Stagnation temperature remains constant

$$\Rightarrow \quad \boxed{T_0 = T_{01} = T_{02} = 300\text{ K}}$$

Velocity of air exit, $c_2 = M_2 \times a_2$

$$\Rightarrow \quad c_2 = M_2 \times \sqrt{\gamma R T_2}$$

$$= 1.2 \times \sqrt{1.4 \times 287 \times 232.92}$$

$$\boxed{c_2 = 367.10\text{ m/s}}$$

We know

$$\frac{4fL}{D} = \left[\frac{4\bar{f}L_{max}}{D}\right]_{M_1} - \left[\frac{4\bar{f}L_{max}}{D}\right]_{M_2}$$

$$= 0.409 - 0.034$$

$$\frac{4\bar{f}L}{D} = 0.375$$

$$\Rightarrow \quad L = \frac{32.02 \times D}{4 \times \bar{f}}$$

$$= \frac{0.375 \times 0.025}{4 \times 0.003}$$

$$\boxed{L = 0.781 \text{ m}}$$

Mass flow rate, $m = \rho_1 A_1 c_1$

$$= \frac{p_1}{RT_1} \times \frac{\pi}{4}(D_1^2) \times c_1$$

$$\Rightarrow \quad = \frac{p_1}{RT_1} \times \frac{\pi}{4}(D_1^2) \times M_1 \times a_1$$

$$[\because M = \frac{c}{a}]$$

$$\Rightarrow \quad = \frac{p_1}{RT_1} \times \frac{\pi}{4}(D_1^2) \times M_1 \times \sqrt{\gamma R T_1}]$$

$$[\because a = \sqrt{\gamma R T}]$$

$$\Rightarrow \quad = \frac{0.5 \times 10^5}{287 \times 139.2} \times \frac{\pi}{4}[0.025]^2 \times 2.4 \times \sqrt{1.4 \times 287 \times 139.2}$$

$$\Rightarrow \quad \boxed{m = 0.348 \text{ kg/s}}$$

***Result***

1. $p_2 = 1.292 \times 10^5$ N/m²
   $T_2 = 232.92$ K
2. $p_{02} = 3.133 \times 10^5$ N/m²
   $T_{02} = 300$ K
3. $c_2 = 367.10$ m/s
4. $L = 0.781$ m
5. $m = 0.348$ kg/s

**4** ***Air enters 20mm diameter, 11m long pipe at a Mach number of 0.24, pressure of 2 bar and temperature of 300K. If the friction factor is 0.003, Determine the following***

1. ***Mass flow rate***
2. ***Exit pressure***
3. ***Exit temperature***
4. ***Exit Mach number***

***Given :***

$D = 20$ mm $= 0.020$ m

$L = 11$m

$M_1 = 0.24$

$p_1 = 2$ bar $= 2 \times 10^5$ N/m²

$T_1 = 300$ K

$\overline{f} = 0.003$

***To find :***

1. Mass flow rate, m
2. Exit Pressure, $p_2$
2. Exit temeprature, $T_2$
4. Exit Mach number, $M_2$

***Solution :***

Refer Isentropic flow table for $\gamma = 1.4$ and $M_1 = 0.24$

$$\frac{T_1}{T_{0_1}} = 0.988$$ *[From gas tables page no.28]*

$$\frac{p_1}{p_{0_1}} = 0.961$$

$$\Rightarrow \qquad T_{01} = \frac{T_1}{0.988}$$

$$= \frac{300}{0.988}$$

$$\boxed{T_{01} = 303.64 \text{ K}}$$

$$\Rightarrow \qquad p_{01} = \frac{p_1}{0.961}$$

$$= \frac{2 \times 10^5}{0.961}$$

$$\boxed{p_{01} = 2.08 \times 10^5 \text{ N/m}^2}$$

Refer Fanno flow table for $\gamma = 1.4$ and $M_1 = 0.24$

$$\frac{p_1}{p_1^*} = 4.538$$

$$\frac{T_1}{T_1^*} = 1.186$$ *[From gas tables page no.81]*

$$\frac{p_{01}}{p_{01}^*} = 2.496$$

$$\frac{4\bar{f}L_{max}}{D} = 9.387$$

$$\Rightarrow \qquad p_1^* = \frac{p_1}{4.538}$$

$$= \frac{2 \times 10^5}{4.538}$$

$$\boxed{p_1^* = 0.440 \times 10^5 \text{ N/m}^2 = p_2^*}$$

$$\Rightarrow \qquad T_1^* = \frac{T_1}{1.186}$$

$$= \frac{300}{1.186}$$

$$\boxed{T_1^* = 252.95 \text{ K} = T_2^*} \qquad [\because T_1^* = T_2^*]$$

We know that

$$\frac{4fL}{D} = \left[\frac{4\bar{f}L_{max}}{D}\right]_{M_1} - \left[\frac{4\bar{f}L_{max}}{D}\right]_{M_2}$$

$$\Rightarrow \qquad \frac{4 \times 0.003 \times 11}{0.020} = 9.387 - \left[\frac{4\bar{f}L_{max}}{D}\right]_{M_2}$$

$$\Rightarrow \qquad 6.6 = 9.387 - \left[\frac{4\bar{f}L_{max}}{D}\right]_{M_2}$$

$$\Rightarrow \qquad \left[\frac{4\bar{f}L_{max}}{D}\right]_{M_2} = 9.387 - 6.6$$

$$\Rightarrow \qquad \left[\frac{4\bar{f}L_{max}}{D}\right]_{M_2} = 2.787$$

Refer Fanno flow table for $\frac{4\bar{f}L_{max}}{D} = 2.787 \approx 2.705$ and $\gamma = 1.4$

$$M_2 = 0.38$$

$$\frac{p_2}{p_2^*} = 2.842$$

$$\frac{T_2}{T_2^*} = 1.166$$ *[From gas tables page no.81]*

$$\frac{p_{02}}{p_{02}^*} = 1.658$$

$$\Rightarrow \quad p_2 = p_2^* \times 2.842$$

$$= 0.440 \times 10^5 \times 2.842$$

$$\boxed{p_2 = 1.25 \times 10^5 \text{ N/m}^2}$$

$$\Rightarrow \quad T_2 = T_2^* \times 1.166$$

$$= 252.95 \times 1.166$$

$$\boxed{T_2 = 294.94 \text{ K}}$$

We know that,

Mass flow rate, $m = \rho A c = \rho_1 A_1 c_1 = \rho_2 A_2 c_2$

$$= \frac{p_1}{RT_1} \times \frac{\pi}{4}(D_1^2) \times c_1$$

$$\Rightarrow \quad = \frac{p_1}{RT_1} \times \frac{\pi}{4}(D_1^2) \times M_1 \times a_1$$

$$[\because M = \frac{c}{a}]$$

$$\Rightarrow \quad = \frac{p_1}{RT_1} \times \frac{\pi}{4}(D_1^2) \times M_1 \times \sqrt{\gamma RT_1}$$

$$[\because a = \sqrt{\gamma RT}]$$

$$\Rightarrow \quad = \frac{2 \times 10^5}{287 \times 300} \times \frac{\pi}{4}[0.020]^2 \times 0.24 \times \sqrt{1.4 \times 287 \times 300}$$

$$\Rightarrow \quad \boxed{m = 0.060 \text{ kg/s}}$$

***Result***

1. $m = 0.060$ kg/s
2. $p_2 = 1.25 \times 10^5$ N/m$^2$
3. $T_2 = 294.94$ K
4. $M_2 = 0.38$

**5** ***A convergent–divergent nozzle is provided with a pipe of constant cross section at its exit. The exit diameter of the nozzle and that of the pipe is 50 cm. The mean co-efficient of friciton for the pipe is 0.002. The stagnation pressure and temperature of air at the nozzle entry are 10 bar and 620K. The Mach numbers at the entry and exit of the pipe are 1.6 and 1.0 respectively.***

***Determine***

1. ***The length of the pipe***
2. ***Diameter of the nozzle throat***
3. ***Pressure and temperature at the pipe exit.***

***Given***

$D = 50 \text{ cm} = 0.50 \text{ m}$

$\bar{f} = 0.002$

$p_{01} = 10 \text{ bar} = 10 \times 10^5 \text{ N/m}^2$

$T_{01} = 620 \text{ K}$

$M_1 = 1.6$

$M_2 = 1.0$

***To find***

1. Length of the pipe, L
2. Diameter of the nozzle throat, D*
3. Pressure and temperature at the pipe exit ($p_2$, $T_2$).

***Solution :***

Refer Isentropic flow table for $\gamma = 1.4$ and $M_1 = 1.6$

$$\frac{T_1}{T_{0_1}} = 0.661$$ *[From gas tables page no.33]*

$$\frac{p_1}{p_{0_1}} = 0.235$$

$$\frac{A_1}{A^*} = 1.250$$

$$\Rightarrow \quad T_1 = T_{01} \times 0.661$$

$$= 620 \times 0.661$$

$$\boxed{T_1 = 409.82 \text{ K}}$$

$$\Rightarrow \quad p_1 = p_{01} \times 0.235$$

$$= 10 \times 10^5 \times 0.235$$

$$\boxed{p_1 = 2.35 \times 10^5 \text{ N/m}^2}$$

$$\Rightarrow \quad A_1^* = \frac{A_1}{1.250}$$

$$= \frac{\frac{\pi}{4}(D_1)^2}{1.250}$$

$$= \frac{\frac{\pi}{4}[0.50]^2}{1.250}$$

$$\boxed{A_1^* = 0.157 \text{ m}^2}$$

We know that

$$A_1^* = A^* = \frac{\pi}{4} D^{*2}$$

$$\Rightarrow \quad D^{*2} = \frac{4 \times A^*}{\pi}$$

$$= \frac{4 \times 0.157}{\pi}$$

$$D^{*2} = 0.1998$$

$$\Rightarrow \quad \boxed{\text{Throat diameter, } D^* = 0.447\text{m}}$$

Refer Fanno flow table for $\gamma = 1.4$ and $M_1 = 1.6$

$$\frac{p_1}{p_1^*} = 0.557$$

$$\frac{T_1}{T_1^*} = 0.794$$ *[From gas tables page no.83]*

$$\frac{4 \bar{f} L_{max}}{D} = 0.172$$

$$\Rightarrow \quad p_1^* = \frac{p_1}{0.557}$$

$$= \frac{2.35 \times 10^5}{0.557}$$

$$\boxed{p_1^* = 4.219 \times 10^5 \text{ N/m}^2 = p_2^*} \qquad [\because p_1^* = p_2^*]$$

$$\Rightarrow \quad T_1^* = \frac{T_1}{0.794}$$

$$= \frac{409.82}{0.794}$$

$$\boxed{T_1^* = 516.15 \text{ K} = T_2^*} \qquad [\because T_1^* = T_2^*]$$

Refer Fanno flow table for $M_2 = 1$ [sonic flow] and $\gamma = 1.4$

$$\frac{p_2}{p_2^*} = 1$$

$$\frac{T_2}{T_2^*} = 1 \qquad \text{[From gas tables page no.82]}$$

$$\frac{4\bar{f}L_{max}}{D} = 0$$

$$\Rightarrow \quad \boxed{p_2 = p_2^* = 4.219 \times 10^5 \text{ N/m}^2}$$

$$\Rightarrow \quad \boxed{T_2 = T_2^* = 516.15 \text{ K}}$$

We know that

$$\frac{4\bar{f}L}{D} = \left[\frac{4\bar{f}L_{max}}{D}\right]_{M_1} - \left[\frac{4\bar{f}L_{max}}{D}\right]_{M_2}$$

$$= 0.172 - 0$$

$$\frac{4\bar{f}L}{D} = 0.172$$

$$\Rightarrow \quad L = \frac{0.172 \times D}{4 \times \bar{f}}$$

$$= \frac{0.172 \times 0.50}{4 \times 0.002}$$

$$\boxed{L = 10.75 \text{ m}}$$

***Result :***

1. $L = 10.75$ m
2. $D^* = 0.447$ m
3. $p_2 = p_2^* = 4.219 \times 10^5$ N/m$^2$

   $T_2 = T_2^* = 516.15$ K

**6** ***A circular duct of 35 cm diameter passes gas at a Mach number of 2.0. The static pressure and temperature are 1 bar and 410 K respectively. A normal shock occurs at a Mach number of 1.4 and the exit Mach number is 1. If the co–efficient of friction is 0.002, calculate***

1. ***Length of the duct upstream and down stream of the shock wave***
2. ***Mass flow rate of the gas***
3. ***Change of entropy for upstream of the shock, across the shock and down stream of the shock.***

***Take $\gamma = 1.3$ and $R = 0.285$ kJ/kg K.***

***Given***

$D = 35$ cm $= 0.35$ m

$M_1 = 2$

$p_1 =$ 1 bar $= 1 \times 10^5$ N/m$^2$

$T_1 = 410$ K

Mach number at shock, $M_x = 1.4$

Exit Mach number, $M_2 = 1$

$\bar{f} = 0.002$

$\gamma = 1.3$, $R = 0.285$ kJ/kg K

$= 285$ J/kg K

***To find***

1. Length of the duct upstream and down stream of the shock wave $(L_1, L_2)$
2. Mass flow rate, m
3. Change of entropy for upstream of the shock $(\Delta s_{1-x})$, across the shock $(\Delta s_{x-y})$, and downstream of the shock$(\Delta s_{y-2})$.

# UNIT-V

## INTRODUCTION:

The principle of jet propulsion is obtained from the application Newton's third law. For every action there is an equal and opposite reaction.

We know that when a fluid is to be accelerated, a force is required to produce this acceleration in the fluid.at the same time, there is an Equal and opposite reaction force acting on this fluid. This opposite Reaction force of the fluid on the engine is known as thrust. Hence it may state that the principle of jet propulsion is based on the reaction Principle.

Any fluid can be used to achieve the jet propulsion principle. Thus water, steam, and combustion gases are used to propel a body in a fluid. But there are limitations imposed upon the choice of the suitable fluid when is applied to the propulsion bodies.

## CLASSIFICATION OF JET PROPULSION

Jet propulsion engines may be classified broadly into two groups.

(a) Air breathing engines – combustion takes place by using atmospheric air

(b) Rocket engines - combustion takes place by using its own oxygen supply

## CLASSIFICATION OF AIR BREATHING ENGINES

Air breathing engines can be further classified as follows;

1. Ramjet engine
2.pulise jet engine

3. Turbojet engine
4.turbo prop engine

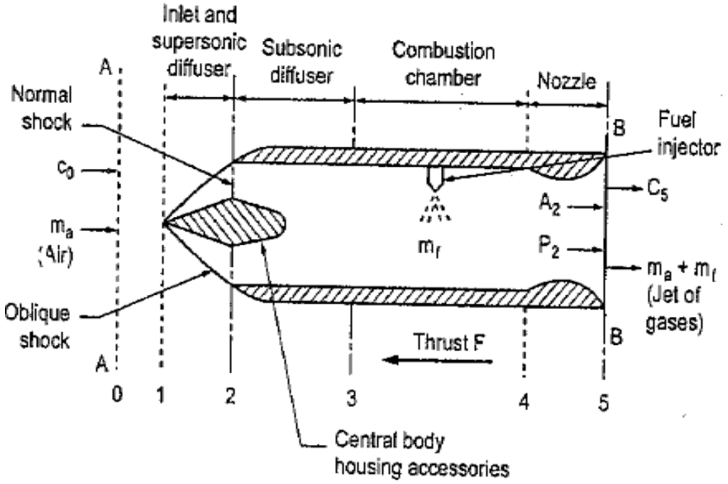
5. Turbo fan engine

## RAMJET ENGINE

Construction

- The construction of ramjet engine is shown in fig.5.1 which is Simplest types of air-breathing engine.
- It consists of;

1. Supersonic diffuser (1-2)
2. Subsonic diffuser (2-3)
3. Combustion chamber (3-4)
4. Discharge nozzle section (4-5)



- The functions of supersonic and subsonic diffusers are to convert the kinetic energy of the entering air into pressure energy. This energy transformation is called ram effect and the pressure rise.is called the ram pressure.
- The function of nozzle is to convert pressure energy of gas into kinetic energy.

**Working**

- Air from the atmosphere enters the engine at a very high speed and its velocity gets reduced and its and its static pressure is increased by supersonic diffuser.
- Then the air passes through the subsonic diffuser and its Velocity further reduces to subsonic value. Due to this, the Pressure of air increases to ignition pressure.
- Then the high pressure air flows into the combustion chamber. In the combustion chamber, the fuel is injected by suitable injectors and the air fuel mixture is burnt.
- The highly heated products of combustion gases are then allowed to expand in the exhaust nozzle section.
- In the nozzle pressure energy of the gas is converted into Kinetic energy.so the gases coming out from the unit with very high velocity.
- Due to high velocity of gases coming out from the unit, a Reaction or thrust is produced in the opposite direction. This Thrust propels he air craft.
- Ramjet produces very high thrust with high efficiency at Supersonic speeds. So, it is best suitable for high speed aircrafts.
- The air enters the engine with a supersonic speed must be reduced to subsonic speed. This is necessary to prevent the blow out of the flame in the combustion chamber. The velocity must be small enough to make it possible to add the required quantity of fuel for stable combustion.
- Both theory and experiment indicate that the speed of the air entering the combustion chamber should not be higher than that corresponding to a local match number of 0.2 approximately

Advantages

- Ramjet engine is very simple and does not have any moving part.
- cost is low
- Less maintenance.
- The specific fuel consumption is better than other gas turbine power plants at high speed.
- There is no upper limit to flight speed.
- Light weight when compared with turbojet engine.

## Disadvantages

- Since the take-off thrust is zero, it is not possible to start a ramjet engine without an external launching device.

- The combustion chamber required flame holder to stabilize the combustion due to high speed of air.
- It is very difficult to design a diffuser which will give good pressure recovery over a wide range of air.
- It has low thermal efficiency.

## Applications

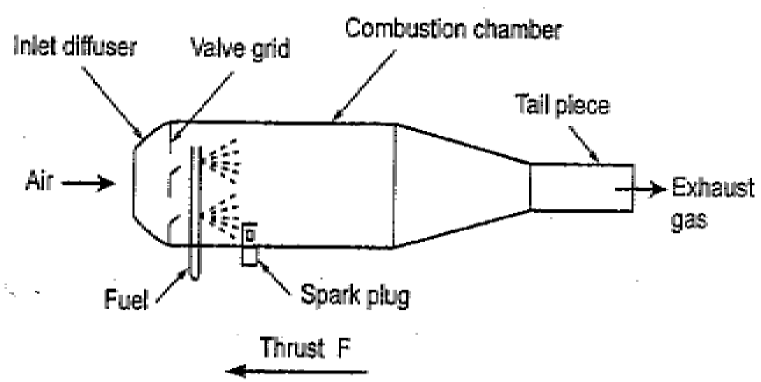
- It is widely used in high speed aircrafts and missiles due to its high thrust and high operational speed.
- Subsonic ramjets are used in target weapons.

## PULSE JET ENGINE (OR) FLYING BOMB:

The constructions of pulse jet engine is shown in fig.5.4 which is similar to ramjet engine.

It consists of;

(1) A diffuser
(2) A valve grid which contains springs that close on their own spring pressure.
(3) Combustion chamber.
(4) Spark plug.
(5) A tail pipe (or discharge nozzle)



- The function of diffuse is to convert the kinetic energy of the entering air into air into pressure energy.

- The function of nozzle is to convert pressure energy of gas into kinetic energy.

## Working

- Air from the atmosphere enters into pulse jet engine. The air velocity gets reduced and its static pressure is increased by diffuser.
- When a certain pressure difference exists across the valve grid, the valve will open and allow the air to enter into the combustion chamber.
- In the combustion chamber, fuel is mixed with air and combustion starts by the use of spark plug.

- Once the combustion starts it proceeds at constant volume.so there is a rapid increase in pressure, which causes the valve to close rapidly.
- The highly heated products of combustion gases are then allowed to expand in the exhaust nozzle(tail pipe)section
- In the nozzle pressure energy of the gas is converted into kinetic energy. So the gases coming out from the unit with very high velocity.
- Due to high velocity of gases coming out from the unit, a reaction (or) thrust is produced in the opposite direction. This thrust propels the air craft.
- Since the combustion causes the pressure to increase, the engine can operate even at static condition once it gets started.
- When the combustion products accelerate from the chamber, they leave a slight vacuum in the combustion chamber. This, in turn, produces sufficient pressure drop across the valve, grid allowing the valves to open again and new charge of air enters the combustion chamber.

## Advantages

- Pulse jet engine is very simple device next to ramjet engine.
- Less maintenance.
- Cost is low.
- Light weight when compared with turbojet engine.
- Unlike the ramjet engine, the pulse jet engine develops thrust at zero speed.

## Disadvantages

- High rates of fuel consumption.
- The maximum flight speed of the pulse jet engine is limited to 750 km/h.
- Low propulsive efficiency than turbojet engines.
- High degree of vibration leads to noise pollution.

## Application

- It is used in subsonic flights, German v-1 missiles, target aircraft missiles, pilotless air craft,etc.
- A factor practically restricting the use of the pulse jet engine to pilotless air craft is its severe vibrations and high intensity of noise.

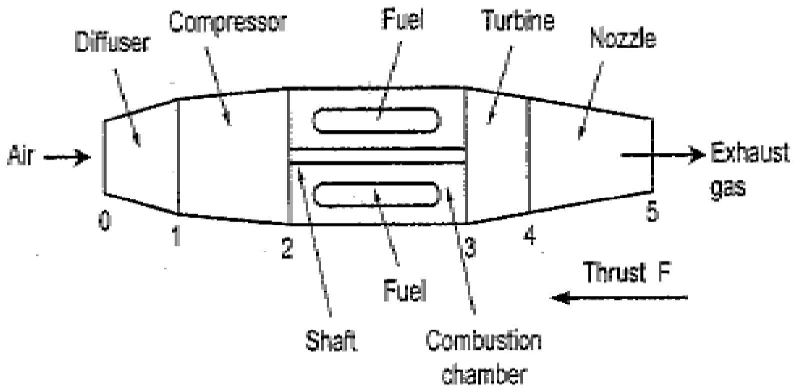
## TURBOJET ENGINE:

- The two air-breathing engines described so far are simple in construction and they have not been used very extensively.
- The most common type of air- breathing engines is the turbojet engine.

## Construction

- The construction of turbojet engine is shown in fig
- It consists of
    1. Diffuser
    2. Rotary compressors
    3. Combustion chamber
    4. Turbine and

5. Exhaust nozzle



- The function of the diffuser is to convert the kinetic energy of the entering air into pressure energy
- The function of the nozzle is to convert the pressure energy of the combustion gases into kinetic energy.

**Working**

- Air from the atmosphere enters into turbojet engine. The air velocity gets reduced and its static pressure is increased by diffuser.
- Then the air passes through the rotary compressor in which the air is further compressed.
- Then the high pressure air flows into the combustion chamber, in the combustion chamber, the fuel is injected by suitable injectors and the air –fuel mixture is burnt. Heat is supplied at constant pressure.
- The highly heated product of combustion gases are then enters the turbine and partially expanded.
- The power produced by the turbine is just sufficient to drive the compressor, fuel pump and other auxiliaries.
- The hot gases from the turbine are then allowed to expand in the exhaust nozzle section.
- In the nozzle, pressure energy of the gas is converted into kinetic energy .so the gases coming out from the unit with very high velocity.
- Due to high velocity of gases coming out from the unit, a reaction or thrust produced in the opposite direction .this thrust propels the air craft.
- Like ramjet engine, the turbojet engine is a continuous flow engine.
- Because of turbine material limitations, only a limited amount of fuel can be burnt in the combustion chamber.

**Advantages**

- Construction is simple
- Less wear and tear
- Less maintenance cost
- It runs smoothly because continuous thrust is produced by continuous combustion of fuel.
- The speed of a turbojet is not limited by the propeller and it can attain higher flight speed than turbo propeller air crafts.
- Low grade fuels like kerosene, paraffin, ect., can be used. This reduced the fuel cost.

**Disadvantages**

- It has low take-off thrust and hence poor stating characteristics.
- Fuel consumption is high.
- Costly materials are used.
- The fuel economy at low operational speed is extremely poor.
- Sudden decrease of speed is difficult.

**Applications**

It is best suited for piloted air-crafts, military aircrafts,etc.

## TURBO-PROP ENGINE:

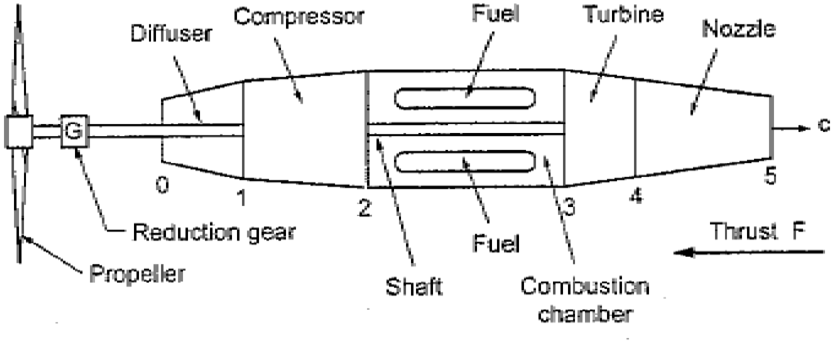
It is very similar to turbojet engine. In this type, the turbine drives the compressor and propeller.

**Construction**

- The construction of turbo-prop engine is shown in fig

It consists of:

(i)Diffuser (ii) Compressor
(iii) combustion chamber (iv) Turbine
(v) Exhaust nozzle (vi) Reduction gear and
(vii) Propeller



- The function of diffuser is to convert the kinetic energy of the entering air into pressure energy.
- The function of nozzle is to convert the pressure energy of the combustion gases into kinetic energy.
- The angular velocity of the shaft is very high. But the propeller cannot run at higher angular velocity. So a reduction gear box is provided before the power is transmitted to the propeller.

**WORKING**

- Air from the atmosphere enters into turbo prop engine. The air velocity gets reduced and its static pressure is increased by diffuser.
- Then the air passes through the rotary in which the air is further compressed. So, the static pressure of the air is further increased.
- Then the high pressure air flows into the combustion chamber. In the combustion chamber, the fuel is

injected by suitable injectors and the air-fuel mixture is burnt. Heat is supplied at constant pressure.

- The highly heated products of combustion gases are then enters the turbine and partially (about 80 to 90%) expanded.
- The power produced by the turbine is used to drive the compressor and propeller.
- Propeller is used to increase the flow rate of air which result in better fuel economy.
- The hot gases from the turbine are then allowed to expand in the exhaust nozzle section.
- In the nozzle, pressure energy of the gas is converted into kinetic energy. So the gases coming out from the unit with very high velocity.
- Due to high velocity of gases coming out from the unit, a reaction or thrust is produced in the opposite direction
- The total thrust produced in this engine is the sum of the thrust produced by the propeller and the thrust produced by the nozzle. This total thrust propels the air craft.

**Advantages**

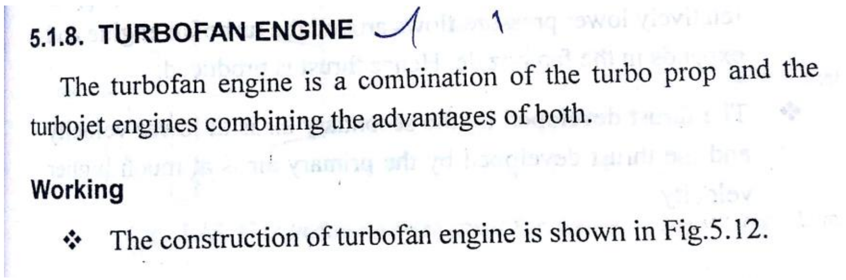
- High take-off thrust
- Good propeller efficiency at a speed below 800km/hr.
- Reduced vibration and noise
- Better fuel economy
- Easy maintenance
- The power output is not limited
- Sudden decrease of speed is possible by thrust reversal.

**Disadvantages**

- The main disadvantage is, the propeller efficiency is rapidly decreases at high speeds due to shocks and flow separation.
- It requires a reduction gear which increases the cost of the engine.
- More space needed than turbojet engine.
- Engine construction is more complicated.

**Application**

- The turbo prop engine is best suited for commercial and military air-craft operation due to its high flexibility of operation and good fuel economy.

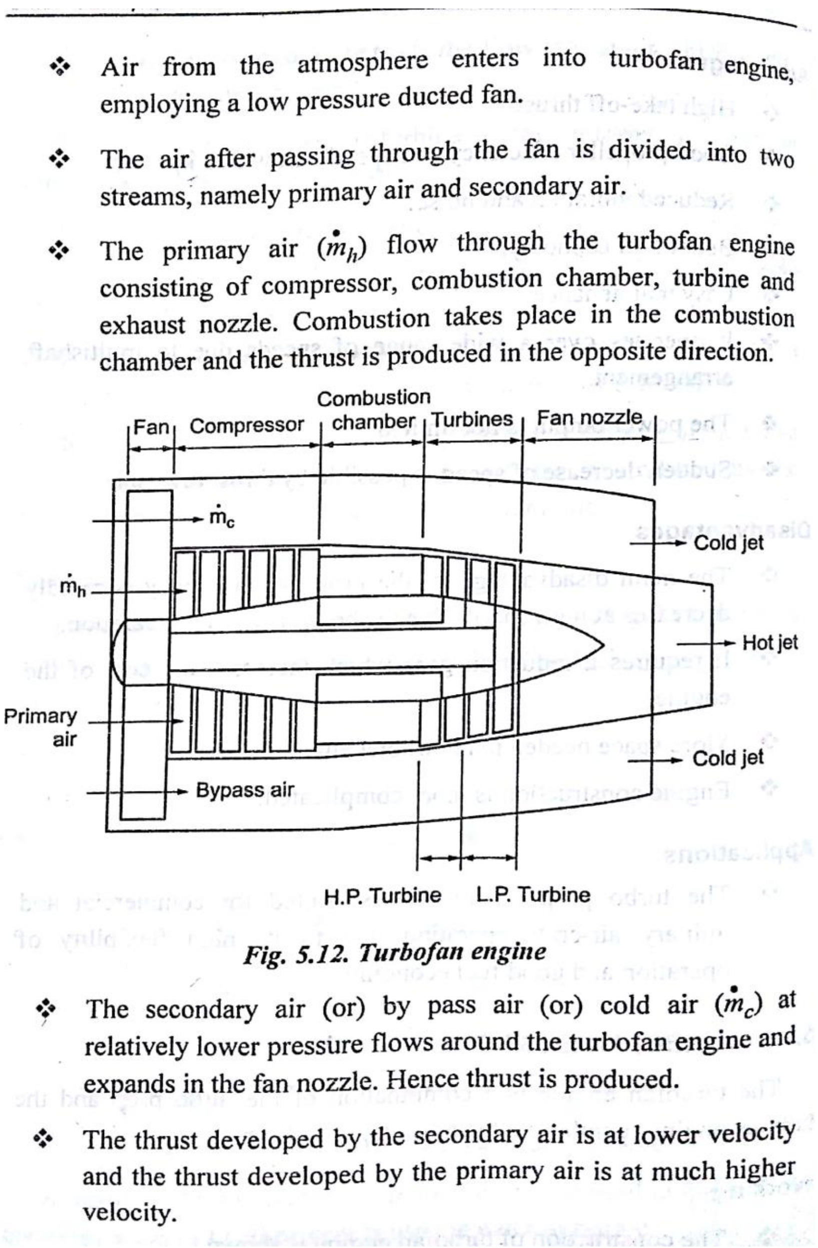


### 5.1.8. TURBOFAN ENGINE

The turbofan engine is a combination of the turbo prop and the turbojet engines combining the advantages of both.

**Working**

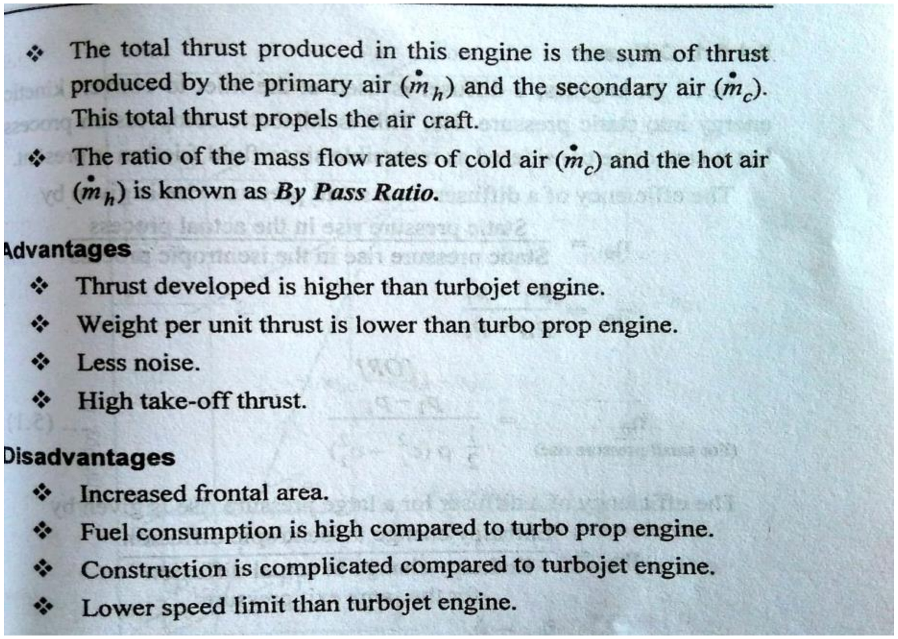
- The construction of turbofan engine is shown in Fig.5.12.



- ❖ Air from the atmosphere enters into turbofan engine, employing a low pressure ducted fan.
- ❖ The air after passing through the fan is divided into two streams, namely primary air and secondary air.
- ❖ The primary air ($\dot{m}_h$) flow through the turbofan engine consisting of compressor, combustion chamber, turbine and exhaust nozzle. Combustion takes place in the combustion chamber and the thrust is produced in the opposite direction.

*Fig. 5.12. Turbofan engine*

- ❖ The secondary air (or) by pass air (or) cold air ($\dot{m}_c$) at relatively lower pressure flows around the turbofan engine and expands in the fan nozzle. Hence thrust is produced.
- ❖ The thrust developed by the secondary air is at lower velocity and the thrust developed by the primary air is at much higher velocity.



- ❖ The total thrust produced in this engine is the sum of thrust produced by the primary air ($\dot{m}_h$) and the secondary air ($\dot{m}_c$). This total thrust propels the air craft.
- ❖ The ratio of the mass flow rates of cold air ($\dot{m}_c$) and the hot air ($\dot{m}_h$) is known as ***By Pass Ratio.***

**Advantages**

- ❖ Thrust developed is higher than turbojet engine.
- ❖ Weight per unit thrust is lower than turbo prop engine.
- ❖ Less noise.
- ❖ High take-off thrust.

**Disadvantages**

- ❖ Increased frontal area.
- ❖ Fuel consumption is high compared to turbo prop engine.
- ❖ Construction is complicated compared to turbojet engine.
- ❖ Lower speed limit than turbojet engine.

### 5.1.10. THRUST

The force which propels the air craft forward at a given speed is called as thrust or propulsive force. This thrust mainly depends on the velocity of gases at the exit of the nozzle.

### 5.1.11. JET THRUST

The control surface of a turbojet engine between section 1 and 2 is shown in Fig.5.16.

Air from atmospheric ($\dot{m}_a$) enters the Turbojet engine at a pressure $p_a$ and velocity $u$.

The gases leaves the nozzle at a pressure of $p_e$ and velocity of $c_e$.

The mass flow rate of gases at exit of the nozzle is $\dot{m}_a + \dot{m}_f$.

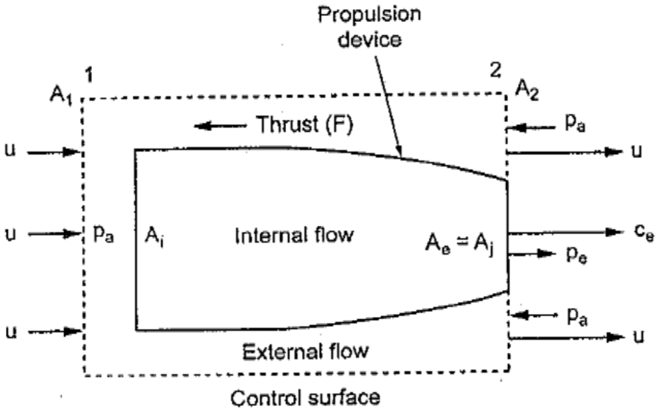


*Fig. 5.16. Internal and external flows for a propulsion device*

We know that

$$\text{Net thrust of the engine (F)} = \begin{Bmatrix} \text{Momentum} \\ \text{thrust} \\ (\text{F}_{mom}) \end{Bmatrix} + \begin{Bmatrix} \text{Pressure} \\ \text{thrust} \\ (\text{F}_{pre}) \end{Bmatrix}$$

$$\text{Momentum thrust } (\text{F}_{mom}) = (\dot{m}_a + \dot{m}_f)\, c_e - \dot{m}_a \times u$$

$$\text{Pressure thrust } (\text{F}_{pr}) = (p_e - p_a) \times \text{A}_e$$

$$\Rightarrow \quad \text{Net thrust (F)} = (\dot{m}_a + \dot{m}_f)\, c_e - \dot{m}_a \times u + (p_e - p_a) \times \text{A}_e \qquad \ldots (5.10)$$

For complete expansion, $p_a = p_e$.

$$\Rightarrow \quad \text{Net thrust F} = (\dot{m}_a + \dot{m}_f)\, c_e - \dot{m}_a \times u$$

where $\dot{m}_a$ – Mass of air (kg / s)

$\dot{m}_f$ – Mass of fuel (kg / s)

$c_e$ – Exit velocity of gases

(or) (or)

$c_j$ Jet velocity

[ ***Note :*** For complete expansion, $c_e = c_j$ ]

$u$ – Flight speed

### 5.1.12. PROPELLER THRUST

The control surface of a turbo prop engine between section 1 and 2 is shown in Fig.5.17.

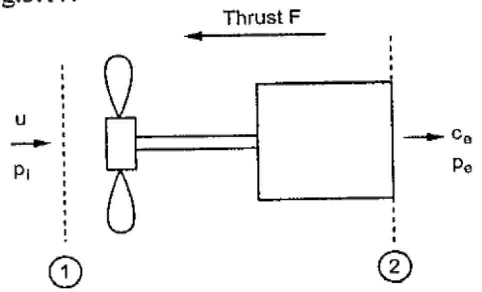


***Fig. 5.17.***

We know that,

$$\left.\begin{matrix} \text{Net thrust} \\ \text{considering} \\ \text{mass of fuel} \end{matrix}\right\} \text{F} = (\dot{m}_a + \dot{m}_f)\; c_e - \dot{m}_a \times u$$

$$= \dot{m}\, c_e - \dot{m}_a \times u \qquad [\because\ \dot{m} = \dot{m}_a + \dot{m}_f]$$

For complete expansion,

$$c_e = c_j$$

$$\Rightarrow \quad \boxed{F = \dot{m}\, c_j - \dot{m}_a \times u} \qquad \dots (5.11)$$

Since $\dot{m}_f$ is very small compared to $\dot{m}_a$, it is neglected.

$$\left.\begin{array}{r}\text{Net thrust without}\\ \text{considering mass of fuel}\end{array}\right\} F = \dot{m}\, c_j - \dot{m}\, u$$

$$[\because \ \dot{m} = \dot{m}_a + \dot{m}_f \Rightarrow \dot{m} = \dot{m}_a]$$

$$\boxed{\begin{array}{l} F = \dot{m}\,(c_j - u) \\ \quad (\text{or}) \\ F = \dot{m}_a\,(c_j - u) \end{array}} \qquad \dots (5.12)$$

### 5.1.13. EFFECTIVE SPEED RATIO (σ)

The ratio of flight speed to jet velocity is known as effective speed ratio (σ).

$$\sigma = \frac{\text{Flight speed}}{\text{Jet velocity (or) Velocity of exit gases}}$$

$$\boxed{\sigma = \frac{u}{c_j}} \qquad \dots (5.13)$$

We know that, Thrust $F = \dot{m}_a\,[\,c_j - u\,]$

$$= \dot{m}_a \times c_j \left[\,1 - \frac{u}{c_j}\,\right]$$

$$\boxed{F = \dot{m}_a \times c_j\,[\,1 - \sigma\,]} \qquad \dots (5.14)$$

### 5.1.14. SPECIFIC THRUST ($F_{sp}$)

The thrust developed per unit mass flow rate is known as specific thrust.

$$\boxed{F_{sp} = \frac{F}{\dot{m}}} \qquad \dots (5.15)$$

### 5.1.15. THRUST SPECIFIC FUEL CONSUMPTION (TSFC)

The fuel consumption rate per unit thrust is known as Thrust Specific Fuel Consumption

$$\boxed{\text{TSFC} = \frac{\dot{m}_f}{F}} \qquad \dots (5.16)$$

### 5.1.16. SPECIFIC IMPULSE ($I_{sp}$)

The thrust developed per unit weight flow rate is known as specific impulse.

$$I_{sp} = \frac{F}{W}$$

$$= \frac{\dot{m}\,(c_j - u)}{\dot{m} \times g} \qquad [\because m \approx \dot{m}_a]$$

$$= \frac{c_j - u}{g} = \frac{u}{g}\left[\frac{c_j}{u} - 1\right]$$

$$\boxed{I_{sp} = \frac{u}{g}\left[\frac{1}{\sigma} - 1\right]} \qquad \dots (5.17)$$

where, $\sigma$ – Effective speed ratio $= \dfrac{u}{c_j}$

### 5.1.17. PROPULSIVE EFFICIENCY

It is defined as the ratio of Propulsive power (or) Thrust power to the power output of the engine.

$$\eta_P = \frac{\text{Propulsive power (or) Thrust power}}{\text{Power output of the engine}}$$

We know that

$$\text{Thrust power} = \text{Thrust (F)} \times \text{Flight speed } (u)$$

$$\boxed{\text{Thrust power} = \dot{m}\,[\,c_j - u\,] \times u}$$

At the outlet of the engine, the power is available in the form of kinetic energy. So the power output of the engine is $\frac{1}{2}\,\dot{m}\,[\,c_j^2 - u^2\,]$.

$$\boxed{\text{Power output} = \frac{1}{2}\,\dot{m}\,[\,c_j^2 - u^2\,]}$$

$$\Rightarrow \qquad \eta_P = \frac{\dot{m}\,[\,c_j - u\,] \times u}{\frac{1}{2}\,\dot{m}\,[\,c_j^2 - u^2\,]} \qquad \dots (5.18)$$

$$\eta_P = \frac{[\,c_j - u\,] \times u}{\frac{1}{2}\,[\,c_j^2 - u^2\,]} = \frac{2\,u\,[\,c_j - u\,]}{c_j^2 - u^2}$$

$$= \frac{2\,u\,[\,c_j - u\,]}{(c_j + u)\,(c_j - u)}$$

$$\boxed{\eta_P = \frac{2\,u}{c_j + u}} \qquad \dots (5.19)$$

Divide the numerator and denominator by $c_j$.

$$\Rightarrow \qquad \eta_P = \frac{\dfrac{2\,u}{c_j}}{\dfrac{c_j + u}{c_j}} = \frac{\dfrac{2\,u}{c_j}}{1 + \dfrac{u}{c_j}}$$

$$\boxed{\eta_P = \frac{2\,\sigma}{1 + \sigma}} \qquad \dots (5.20)$$

where $\sigma$ – Effective speed ratio $= \frac{u}{c_j}$

### 5.1.18. THERMAL EFFICIENCY

It is defined as the ratio of power output of the engine to the power input to the engine.

$$\eta_t = \frac{\text{Power output of the engine}}{\text{Power input to the engine through fuel}}$$

Power is given as the input by burning the fuel.

So, Power Input $= \dot{m}_f \times \mathrm{C \cdot V}$

We know that

$$\text{Power output} = \frac{1}{2}\,\dot{m}\,[\,c_j^2 - u^2\,]$$

$$\Rightarrow \qquad \eta_t = \frac{\frac{1}{2}\,\dot{m}\,[\,c_j^2 - u^2\,]}{\dot{m}_f \times \mathrm{C \cdot V}} \qquad \dots (5.21)$$

where $\dot{m}$ – Mass of air fuel mixture
$c_j$ – Velocity of jet
$u$ – Flight velocity
$\dot{m}_f$ – Mass of fuel
$\mathrm{C \cdot V}$ – Calorific value of fuel

If efficiency of combustion is considered,

$$\eta_t = \frac{\frac{1}{2}\,\dot{m}\,[\,c_j^2 - u^2\,]}{\eta_B \times \dot{m}_f \times \mathrm{C.V}} \qquad \dots (5.22)$$

### 5.1.19. OVERALL EFFICIENCY

It is defined as the ratio of propulsive power to the power input to the engine.

$$\eta_0 = \frac{\text{Propulsive Power (or) Thrust Power}}{\text{Power input to the engine}}$$

We know that,

$$\text{Thrust power} = \dot{m}\,[\,c_j - u\,] \times u$$

$$\text{Power input} = \dot{m}_f \times \mathrm{C \cdot V}$$

$$\Rightarrow \qquad \eta_0 = \frac{\dot{m}\,[\,c_j - u\,] \times u}{\dot{m}_f \times \mathrm{C \cdot V}} \qquad \dots (5.23)$$

$$= \frac{\dot{m}\,[\,c_j - u\,] \times u}{\frac{1}{2}\,\dot{m}\,[\,c_j^2 - u^2\,]} \times \frac{\frac{1}{2}\,\dot{m}\,[\,c_j^2 - u^2\,]}{\dot{m}_f \times \mathrm{C \cdot V}}$$

$$= \eta_P \times \eta_t$$

$$\Rightarrow \qquad \boxed{\eta_0 = \eta_P \times \eta_t} \qquad \dots (5.24)$$

## FORMULAE USED

1. Velocity of flight (or) Aircraft $= u$
2. Velocity of jet (or) Velocity of exit gases $= c_j$ (or) $c_e$
3. Diffuser efficiency, $\eta_D = \dfrac{\left(\dfrac{p_1}{p_i}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\dfrac{\gamma-1}{2} M_i^2}$
4. Compressor efficiency, $\eta_C = \dfrac{T_{01}\left[(R_{0C})^{\frac{\gamma-1}{\gamma}} - 1\right]}{T_{02} - T_{01}}$

   where $R_{0C}$ – Compressor ratio $= \dfrac{p_{02}}{p_{01}}$
5. Combustion efficiency

$$\eta_B = \frac{\dfrac{\dot{m}_a}{\dot{m}_f}[c_p T_{03} - c_p T_{02}] + c_p T_{03}}{C \cdot V}$$

where $C \cdot V$ – Calorific value of fuel.

6. Turbine efficiency

$$\eta_T = \frac{T_{03} - T_{04}}{T_{03}\left[1 - \dfrac{1}{(R_{0T})^{\frac{\gamma-1}{\gamma}}}\right]}$$

where $R_{0T}$ = Turbine pressure ratio $= \dfrac{p_{03}}{p_{04}}$

7. Nozzle efficiency

$$\eta_N = \frac{T_{04} - T_e}{T_{04}\left[1 - \dfrac{1}{(R_{0N})^{\frac{\gamma-1}{\gamma}}}\right]}$$

where $R_{0N}$ = Nozzle pressure ratio $= \dfrac{p_{04}}{p_e}$

8. Velocity of exit gas (or) Velocity of jet

$$c_j \text{ or } c_e = \sqrt{2 \times \eta_N c_p T_{04}\left[1 - \left(\frac{p_e}{p_{04}}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

(or)

$$c_j \text{ or } c_e = \sqrt{2\,\Delta h_0}$$

$$c_e = \sqrt{2\,c_p\,(T_{04} - T_e)}$$

If nozzle efficiency is considered, $c_e = \sqrt{2 \times \eta_N \times \Delta h_0}$

9. Thrust, $F = \dot{m}\,c_j - \dot{m}_a \times u$

   [considering mass of fuel]
10. Thrust, $F = \dot{m}_a\,[c_j - u]$

   [without considering mass of fuel]
11. Effective speed ratio, $\sigma = \dfrac{u}{c_j}$
12. Specific thrust, $F_{sp} = \dfrac{F}{\dot{m}}$
13. Thrust specific fuel consumption (TSFC)

   (or) $= \dfrac{\dot{m}_f}{F}$

   Specific fuel consumption (SFC)
14. Specific Impulse, $I_{sp} = \dfrac{F}{W}$

$$= \frac{F}{\dot{m} \times g} = \frac{u}{g}\left[\frac{1}{\sigma} - 1\right]$$

15. Thrust power, $P = F \times u$

$$= \dot{m}\,[c_j - u] \times u$$

16. Propulsive efficiency, $\eta_P = \dfrac{2u}{c_j + u}$

**17.** Thermal efficiency, $\eta_t = \dfrac{\frac{1}{2}\,\dot{m}\,[\,c_j^2 - u^2\,]}{\dot{m}_f \times C\cdot V}$

[If efficiency of combustion is considered, $\eta_t = \dfrac{\frac{1}{2}\,\dot{m}\,(c_j^2 - u^2)}{\eta_B \times \dot{m}_f \times C.V}$]

**18.** Overall efficiency, $\eta_0 = \dfrac{F \times u}{\dot{m}_f \times C\cdot V}$

(or)

$$= \eta_P \times \eta_t$$

**19.** Velocity of air flow at the propeller

$$c = \frac{1}{2}\,[\,u + c_j\,]$$

**20.** Air standard efficiency, $\eta_a = 1 - \dfrac{1}{(R_{0C})^{\frac{\gamma-1}{\gamma}}}$

where $R_{0C}$ – Compressor pressure ratio $= \dfrac{p_{02}}{p_{01}}$

**21.** Mass flow rate $\dot{m} = \rho\,A_1\,u$ or $\rho\,A_j\,c_j$

**22.** Stagnation temperature – Mach number relation

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2}\,M^2$$

**23.** Mach number at entry, $M_i = \dfrac{u}{a_i}$

$$= \frac{u}{\sqrt{\gamma\,R\,T_i}}$$

24. Mach number at exit $M_e = \dfrac{c_e \text{ or } c_j}{a_e}$

$$= \frac{c_j}{\sqrt{\gamma\,R\,T_e}}$$

25. Isentropic relation $\dfrac{T_{02}}{T_{01}} = \left(\dfrac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}$

26. Stagnation temperature, $T_0 = T + \dfrac{c^2}{2\,c_p}$

27. Absolute velocity $c_{abs} = c_j - u$

### For Ramjet Engine

1. Efficiency of Ideal cycle, $\eta_I = \dfrac{1}{1 + \dfrac{2}{\gamma - 1} \times \dfrac{1}{M_1^2}}$

2. Efficiency of diffuser, $\eta_D = \dfrac{(R_{OD})^{\frac{\gamma-1}{\gamma}} - 1}{\dfrac{\gamma-1}{2}\,M_1^2}$

where, $R_{OD}$ = Diffuser pressure ratio $= \dfrac{p_{02}}{p_1}$

3. Combustion efficiency

$$\eta_B = \frac{\dot{m}_a\,c_p\,(T_{03} - T_{02})}{\dot{m}_f \times C.V}$$

where, C.V = Calorific value of fuel.

## 5.1.20. SOLVED PROBLEMS

**Example 1** *An aircraft flies at a speed of 520 kmph at an altitude of 8000 m. The diameter of the propeller of an aircraft is 2.4 m and flight to jet speed ratio is 0.74. Find the following :*

*(i) The rate of air flow through the propeller*

*(ii) Thrust produced* *(iii) Specific thrust*

*(iv) Specific impulse* *(v) Thrust power*

***Given :***

Air craft speed (or) Flight speed $u = 520$ kmph

$$= \frac{520 \times 10^3}{3600 \text{ s}}$$

$$= 144.44 \text{ m/s}$$

Altitude $z = 8000$ m

Diameter of the propeller $d = 2.4$ m

Flight to jet speed ratio $\sigma = \frac{u}{c_j} = 0.74$

where $c_j$ – Jet speed (or)

Speed of exit gases from the engine

***To find :*** (*i*) Flow rate of air, $\dot{m}_a$

(*ii*) Thrust produced, F

(*iii*) Specific thrust, $F_{sp}$

(*iv*) Specific impulse, $I_{sp}$

(*v*) Thrust power P

***Solution :*** Area of the propeller disc A $= \frac{\pi}{4} d^2$

$$= \frac{\pi}{4} (2.4)^2$$

$$\boxed{A = 4.52 \text{ m}^2}$$

**From gas tubes at** $Z = 8000$ **m**

$$\rho = 0.525 \text{ kg / m}^3$$

[Gas Tables, S.M. Yahya, fifth edition, page no. 20]

**Effective speed ratio,** $\sigma = \frac{u}{c_j}$

$$0.74 = \frac{144.44}{c_j}$$

$$\Rightarrow \quad \boxed{\text{Velocity of Jet, } c_j = 195.19 \text{ m/s}}$$

**Velocity of air flow at the propeller**

$$c = \frac{1}{2} [u + c_j]$$

$$= \frac{1}{2} [144.44 + 195.19]$$

$$\boxed{c = 169.81 \text{ m/s}}$$

**Mass flow rate of air-fuel mixture**

$$\dot{m} = \rho A c$$

$$= 0.525 \times 4.52 \times 169.81$$

$$\boxed{\dot{m} = 402.96 \text{ kg / s}}$$

**We know that** $\dot{m} = \dot{m}_a + \dot{m}_f$

**Since mass flow rate of fuel ($\dot{m}_f$) is not given, let us take**

$$\dot{m} = \dot{m}_a$$

$$\boxed{\text{Mass flow rate of air } \dot{m}_a = 402.96 \text{ kg / s}}$$

**Thrust produced,** $F = \dot{m}_a [c_j - u]$ **[From equation no. 5.12]**

$$= 402.96 [195.19 - 144.44]$$

$$\boxed{F = 20.45 \times 10^3 \text{ N}}$$

$$\text{Specific thrust, } F_{sp} = \frac{F}{\dot{m}} \qquad \text{[From equation no. 5.15]}$$

$$= \frac{F}{\dot{m}_a} \qquad [\because \dot{m} = \dot{m}_a]$$

$$= \frac{20.45 \times 10^3}{402.96}$$

$$\boxed{F_{sp} = 50.75 \text{ N / (kg / s)}}$$

$$\text{Specific Impulse } (I_{sp}) = \frac{F}{W}$$

$$= \frac{F}{\dot{m}\, g}$$

$$= \frac{F}{\dot{m}_a \times g} \qquad [\because \dot{m} = \dot{m}_a]$$

$$= \frac{20.45 \times 10^3}{402.96 \times 9.81}$$

$$\boxed{I_{sp} = 5.17 \text{ s}}$$

$$\text{Thrust power, } P = \text{Thrust (F)} \times \text{Flight speed } (u)$$

$$= 20.45 \times 10^3 \times 144.44$$

$$\boxed{P = 2.95 \times 10^6 \text{ W}}$$

***Result :*** *(i)* $\dot{m}_a = 402.96$ kg / s

*(ii)* $F = 20.45 \times 10^3$ N

*(iii)* $F_{sp} = 50.75$ N / (kg / s)

*(iv)* $I_{sp} = 5.17$ s

*(v)* $P = 2.95 \times 10^6$ W

**Example 2** ***An aircraft propeller flies at a speed of 440 kmph. The diameter of the propeller is 4.1 m and the speed ratio is 0.8. The ambient conditions of air at the flight altitude are T = 255 K and p = 0.55 bar.***

***Find the following :*** ***(i) Thrust***

***(ii) Thrust power***

***(iii) Propulsive efficiency***

***Given :*** Air craft speed, $u = 440$ kmph

$$= \frac{440 \times 10^3 \text{ m}}{3600 \text{ s}}$$

$$u = 122.22 \text{ m/s}$$

Diameter of the propeller, $d = 4.1$ m

Speed ratio, $\sigma = 0.8$

Ambient temperature, $T = 255$ K

Pressure, $p = 0.55 \text{ bar} = 0.55 \times 10^5 \text{ N / m}^2$

***To find :*** *(i)* Thrust (F)

*(ii)* Thrust power (P)

*(iii)* Propulsive efficiency ($\eta_P$)

***Solution :*** Area of the propeller disc, $A = \frac{\pi}{4} d^2$

$$= \frac{\pi}{4} (4.1)^2$$

$$\boxed{A = 13.20 \text{ m}^2}$$

$$\text{Speed ratio, } \sigma = \frac{u}{c_j}$$

$$0.8 = \frac{122.22}{c_j}$$

$$\boxed{\text{Velocity of jet } c_j = 152.77 \text{ m/s}}$$

Velocity of air flow at the propeller

$$c = \frac{1}{2} [u + c_j]$$

$$= \frac{1}{2} [122.22 + 152.77]$$

$$\boxed{c = 137.49 \text{ m / s}}$$

Mass flow rate of air fuel mixture, $\dot{m} = \rho \times A \times c$

$$= \frac{p}{RT} \times A \times c$$

$$= \frac{0.55 \times 10^5}{287 \times 255} \times 13.20 \times 137.49$$

$$\boxed{\dot{m} = 1363.9 \text{ kg/s}}$$

We know that $\dot{m} = \dot{m}_a + \dot{m}_f$

Since mass flow rate of fuel $\dot{m}_f$ is not given, let us take

$$\dot{m} = \dot{m}_a$$

$$\Rightarrow \quad \boxed{\dot{m}_a = 1363.9 \text{ kg/s}}$$

$$\text{Thrust, } F = \dot{m}_a [c_j - u]$$

$$= 1363.9 [152.77 - 122.22]$$

$$\boxed{F = 41{,}667 \text{ N}}$$

$$\text{Thrust power, } P = \text{Thrust (F)} \times \text{Flight speed } (u)$$

$$= 41{,}667 \times 122.22$$

$$\boxed{P = 5.09 \times 10^6 \text{ W}}$$

Propulsive efficiency, $\eta_P = \dfrac{2u}{c_j + u}$ [From equation no. 5.19]

$$= \frac{2 \times 122.22}{152.77 + 122.22}$$

$$\boxed{\eta_P = 0.88 \text{ (or) } 88\%}$$

***Result :*** (*i*) $F = 41{,}667$ N

(*ii*) $P = 5.09 \times 10^6$ W

(*iii*) $\eta_P = 88\%$

***Example 3*** ***An aircraft takes 45 kg/s of air from the atmosphere and flies at a speed of 950 kmph. The air fuel ratio is 50 and the calorific value of the fuel is 42 MJ/kg. For maximum thrust power, find :***

***(i) Jet velocity*** ***(ii) Thrust***

***(iii) Specific thrust*** ***(iv) Thrust power***

***(v) Propulsive efficiency*** ***(vi) Thermal efficiency***

***(vii) Overall efficiency***

***(viii) Thrust specific fuel consumption (TSFC)***

***Given :*** Mass of air, $\dot{m}_a = 45$ kg/s

Air craft speed, $u = 950$ kmph

$$= \frac{950 \times 10^3 \text{ m}}{3600 \text{ s}}$$

$$\boxed{u = 263.88 \text{ m/s}}$$

$$\text{Air fuel ratio } \frac{\dot{m}_a}{\dot{m}_f} = 50$$

Calorific value of the fuel, $C \cdot V = 42$ MJ / kg

$$= 42 \times 10^6 \text{ J/kg}$$

***Note :*** At maximum thrust power, $\sigma = 0.5$

***To find :*** (*i*) Jet velocity, $c_j$ (*ii*) Thrust, F

(*iii*) Specific thrust, $F_{sp}$ (*iv*) Thrust power, P

(*v*) Propulsive efficiency, $\eta_P$

(*vi*) Thermal efficiency, $\eta_t$

(*vii*) Overall efficiency, $\eta_0$

(*viii*) Thrust specific fuel consumption (TSFC)

***Solution :*** At the maximum thrust power, effective speed ratio

$$\sigma = 0.5$$

$$\Rightarrow \qquad \sigma = \frac{u}{c_j} = 0.5$$

$$\Rightarrow \qquad \frac{263.88}{c_j} = 0.5$$

$$\boxed{\text{Jet velocity } c_j = 527.76 \text{ m/s}}$$

Mass of air fuel mixture, $\dot{m} = \dot{m}_a + \dot{m}_f$

$$= \dot{m}_a \left[ 1 + \frac{\dot{m}_f}{\dot{m}_a} \right]$$

$$= 45 \left[ 1 + \frac{1}{50} \right]$$

$$\boxed{\dot{m} = 45.9 \text{ kg / s}}$$

We know that,

Thrust, $F = \dot{m}\, c_j - \dot{m}_a\, u$ [From equation no. 5.11]

$$= 45.9 \; [\, 527.76 \,] - 45 \times 263.88$$

$$\boxed{F = 12.35 \times 10^3 \text{ N}}$$

Specific thrust, $F_{sp} = \dfrac{F}{m}$

$$= \frac{12.35 \times 10^3}{45.9}$$

$$\boxed{F_{sp} = 269.06 \text{ N / (kg / s)}}$$

Thrust power, $P = F \times u$

$$= 12.35 \times 10^3 \times 263.88$$

$$\boxed{P = 3.2 \times 10^6 \text{ W}}$$

Propulsive efficiency, $\eta_P = \dfrac{2\,u}{c_j + u}$ [From equation no. 5.19]

$$= \frac{2 \times 263.88}{527.76 + 263.88}$$

$$\boxed{\eta_P = 0.66 \text{ (or) } 66\%}$$

Thermal efficiency, $\eta_t = \dfrac{\frac{1}{2}\, \dot{m}\, [\, c_j^2 - u^2 \,]}{\dot{m}_f \times C \cdot V}$

[From equation no. 5.21]

$$\dot{m} = \dot{m}_a + \dot{m}_f$$

$$\Rightarrow \qquad \dot{m}_f = \dot{m} - \dot{m}_a$$

$$= 45.9 - 45$$

$$\boxed{\dot{m}_f = 0.9}$$

$$\Rightarrow \qquad \eta_t = \frac{\frac{1}{2} \times 45.9 \times [\, (527.76)^2 - (263.88)^2 \,]}{0.9 \times 42 \times 10^6}$$

$$\boxed{\eta_t = 0.126 \text{ (or) } 12.6\%}$$

Overall efficiency, $\eta_0 = \eta_P \times \eta_t$

$$= 0.66 \times 0.126$$

$$\boxed{\eta_0 = 0.083 \text{ (or) } 8.3\%}$$

Thrust specific fuel consumption (TSFC) $= \dfrac{\dot{m}_f}{F}$

[From equation no. 5.16]

$$= \frac{0.9}{12.35 \times 10^3}$$

$$\boxed{\text{TSFC} = 7.29 \times 10^{-5} \, \frac{\text{kg}}{\text{s - N}}}$$

***Result :***

(*i*) $c_j = 527.76$ m/s

(*ii*) $F = 12.35 \times 10^3$ N

(*iii*) $F_{sp} = 269.06$ N / (kg / s)

(*iv*) $P = 3.2 \times 10^6$ W

(*v*) $\eta_P = 66.6\%$

(*vi*) $\eta_t = 12.6\%$

(*vii*) $\eta_0 = 8.3\%$

(*viii*) TSFC $= 7.29 \times 10^{-5} \, \dfrac{\text{kg}}{\text{s - N}}$

$\boxed{\textit{Example 4}}$ *A turbojet flies at a speed of 870 kmph at an altitude of 10,000 m. The data for the engine is given below.*

*Diameter at inlet section* = *0.74 m*

*Velocity of the gases at the exit of the jet pipe* = *505 m / s*

*Pressure at the exit of the jet pipe* = *0.28 bar*

*Air-fuel ratio* = *42*

***Determine :*** *(i) Air flow rate through the engine*

*(ii) Thrust* *(iii) Specific thrust*

*(iv) Specific impulse* *(v) Thrust power*

*(vi) TSFC*

***Given :*** Flight speed, $u = 870$ kmph

$$= \frac{870 \times 10^3}{3600}$$

$$\boxed{u = 241.66 \text{ m/s}}$$

Altitude, $Z = 10{,}000$ m

Diameter at inlet, $d_i = 0.74$ m

$$\left.\begin{array}{r}\text{Velocity of the gas at exit of}\\ \text{jet pipe } c_e \text{ (or) } c_j\end{array}\right\} = 505 \text{ m/s}$$

Pressure at exit, $p_e = 0.28$ bar

$$= 0.28 \times 10^5 \text{ N/m}^2$$

$$\text{Air fuel ratio} = \frac{\dot{m}_a}{\dot{m}_f} = 42$$

***To find :*** (*i*) Air flow rate through the engine ($\dot{m}_a$)

(*ii*) Thrust F (*iii*) Specific thrust, $F_{sp}$

(*iv*) Specific impulse, $I_{sp}$ (*v*) Thrust power, P

(*vi*) TSFC

***Solution :*** Area at inlet, $A_i = \frac{\pi}{4} d_i^2$

$$= \frac{\pi}{4} (0.74)^2$$

$$\boxed{A_i = 0.43 \text{ m}^2}$$

From gas tables at $z = 10{,}000$ m

$T_i = 223.15$ K [Gas tables, page no. 20]

$p_i = 0.264 \text{ bar} = 0.264 \times 10^5 \text{ N/m}^2$

$\rho_i = 0.413$

$$\left.\begin{array}{r}\text{Mass flow rate of mixture}\\ \text{through the engine}\end{array}\right\} = \rho_i A_i u$$

$$= 0.413 \times 0.43 \times 241.66$$

$$\boxed{\dot{m} = 42.9 \text{ kg/s}}$$

We know that $\dot{m} = \dot{m}_a + \dot{m}_f$

$$\dot{m} = \dot{m}_a \left[1 + \frac{\dot{m}_f}{\dot{m}_a}\right]$$

$$42.9 = \dot{m}_a \left[1 + \frac{1}{42}\right]$$

$$\Rightarrow \boxed{\dot{m}_a = 41.9 \text{ kg/s}}$$

$$\boxed{\text{Air flow rate through the engine } \dot{m}_a = 41.9 \text{ kg/s}}$$

We know that

Thrust, $F = \dot{m} c_j - \dot{m}_a u$ [From equation no. 5.11]

$$= 42.9 \times 505 - 41.9 \times 241.66$$

$$\boxed{F = 11{,}538 \text{ N}}$$

Thrust power, $P = F \times u$

$$= 11{,}538 \times 241.66$$

$$\boxed{P = 2.78 \times 10^6 \text{ W}}$$

$$\text{Specific thrust, } F_{sp} = \frac{F}{\dot{m}}$$

$$= \frac{11,538}{42.9}$$

$$\boxed{F_{sp} = 268.95 \text{ N / (kg / s)}}$$

$$\text{Specific Impulse, } I_{sp} = \frac{F}{W} = \frac{F}{\dot{m}\, g}$$

$$= \frac{11,538}{42.9 \times 9.81}$$

$$\boxed{I_{sp} = 27.41 \text{ s}}$$

$$\text{Thrust specific fuel consumption (TSFC)} = \frac{\dot{m}_f}{F}$$

We know that, $\dot{m} = \dot{m}_a + \dot{m}_f$

$\Rightarrow$ $42.9 = 41.9 + \dot{m}_f$

$\Rightarrow$ $\boxed{\dot{m}_f = 1 \text{ kg / s}}$

$\Rightarrow$ $\text{TSFC} = \dfrac{1}{11,538}$

$$= 8.66 \times 10^{-5} \frac{\text{kg}}{\text{s - N}}$$

$$\boxed{\text{TSFC} = 0.312 \frac{\text{kg}}{\text{h - N}}}$$

***Result :*** *(i)* $\dot{m}_a = 41.9 \text{ kg / s}$

*(ii)* $F = 11,538 \text{ N}$

*(iii)* $F_{sp} = 268.95 \text{ N / kg / s}$

*(iv)* $I_{sp} = 27.41 \text{ s}$

*(v)* $P = 2.78 \times 10^6 \text{ W}$

*(vi)* $\text{TSFC} = 0.312 \dfrac{\text{kg}}{\text{h - N}}$

---

✓ **Example 8** ***The flight speed of a turbojet is 800 km/h at an ambient pressure of 1.1 bar. The mass flow rate of air is 15 kg/s. The pressure of the gas entering the nozzle is 4 bar and temperature is 300°C. Determine the following :***

1. ***Thrust***
2. ***Thrust power***
3. ***Propulsive efficiency***

***Take $\gamma = 1.4$ and $R = 287$ J/kg-K***

***Given :*** Flight speed, $u = 800 \text{ km/r}$

$$= \frac{800 \times 10^3 \text{ m}}{3600 \text{ s}}$$

$$\boxed{u = 222.22 \text{ m/s}}$$

Ambient pressure, $p_i = 1.1 \text{ bar} = 1.1 \times 10^5 \text{ N/m}^2$

Mass flow rate of air, $\dot{m}_a = 15 \text{ kg/s}$

Pressure of the gas entering the nozzle, $p_{04} = 4 \text{ bar}$

$$= 4 \times 10^5 \text{ N/m}^2$$

Temperature of the gas entering the nozzle $\}$ $T_{04} = 300° \text{ C} + 273$

$$= 573 \text{ K}$$

$\gamma = 1.4$

$R = 287$ J/kg-k

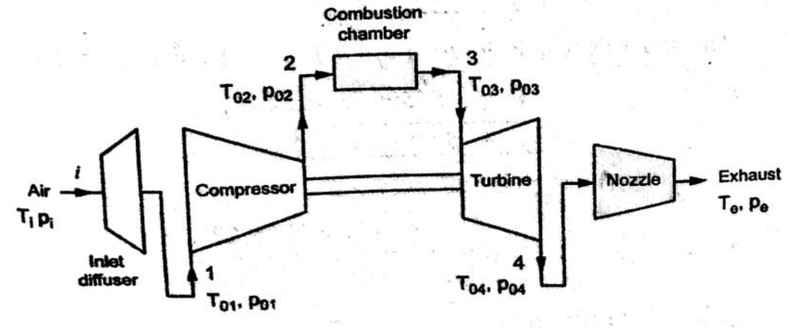


*Fig. 5.20.*

***To find :***

1. Thrust, F.
2. Thrust power, P.
3. Propulsive efficiency, $\eta_P$.

☺ ***Solution :*** We know that,

$$\text{Isentropic relation, } \frac{T_e}{T_{04}} = \left(\frac{p_e}{p_{04}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\Rightarrow \quad \frac{T_e}{573} = \left(\frac{p_i}{4 \times 10^5}\right)^{\frac{1.4-1}{1.4}}$$

[∵ For complete expansion of nozzle, $p_e = p_i$]

$$\Rightarrow \quad \frac{T_e}{573} = \left(\frac{1.1 \times 10^5}{4 \times 10^5}\right)^{\frac{1.4-1}{1.4}}$$

$$\Rightarrow \quad T_e = 573 \times 0.6915$$

$$\boxed{T_e = 396.24 \text{ K}}$$

We know that,

$$\text{Exit velocity of gas, } c_e \text{ (or) } c_j = \sqrt{2\, c_p\, (T_{04} - T_e)}$$

$$\Rightarrow \quad c_j = \sqrt{2 \times 1005 \times (573 - 396.24)}$$

$$\Rightarrow \quad \boxed{c_j = 596.06 \text{ m/s}}$$

$$\text{Thrust developed, } F = \dot{m}_a\, [\, c_j - u\, ] \qquad [\because \dot{m}_f \text{ is not given}]$$

$$\Rightarrow \quad F = 15\, [596.06 - 222.22]$$

$$\boxed{F = 5.60 \times 10^3 \text{ N}}$$

$$\text{Thrust power, } P = F \times u$$

$$= 5.60 \times 10^3 \times 222.22$$

$$\boxed{P = 1.24 \times 10^6 \text{ N}}$$

$$\text{Propulsive efficiency, } \eta_P = \frac{2\,u}{c_j + u}$$

$$= \frac{2 \times 222.22}{596.06 + 222.22} = 0.543$$

$$= 54.3\%$$

***Result :***

1. $F = 5.60 \times 10^3$ N
2. $P = 1.24 \times 10^6$ N
3. $\eta_N = 54.3\%$

***Example 1*** ***A turbojet propels an aircraft at a speed of 900 km/h while taking 3000 kg of air per minute. The isentropic enthalpy drop in the nozzle is 200 kJ / kg and the nozzle efficiency is 90%. The air-fuel ratio is 85 and the combustion efficiency is 95%. The calorific value of the fuel is 42,000 kJ / kg. Calculate :***

***(i) Propulsive power (or) Thrust power***

***(ii) Thermal efficiency***

***(iii) Propulsive efficiency*** *[Dec. 2003, 2004 Anna University]*

***Given :*** Aircraft speed, $u = 900 \text{ km/h}$

$$= \frac{900 \times 10^3 \text{ m}}{3600 \text{ s}} = 250 \text{ m/s}$$

Mass of air, $\dot{m}_a = 3000 \text{ kg/min}$

$$= \frac{3000}{60} \text{ kg/s} = 50 \text{ kg/s}$$

Enthalpy drop, $\Delta h = 200 \text{ kJ/kg} = 200 \times 10^3 \text{ J/kg}$

Nozzle efficiency, $\eta_N = 90\%$

Air fuel ratio, $\dfrac{\dot{m}_a}{\dot{m}_f} = 85$

Combustion efficiency, $\eta_B = 95\%$

Calorific value of the fuel, $C \cdot V = 42{,}000 \dfrac{\text{kJ}}{\text{kg}} = 42{,}000 \times 10^3 \dfrac{\text{J}}{\text{kg}}$

***To find :*** (*i*) Propulsive power, P

(*ii*) Thermal efficiency, $\eta_T$

(*iii*) Propulsive efficiency, $\eta_P$

***Solution :*** Mass flow rate of air-fuel mixture

$$\dot{m} = \dot{m}_a + \dot{m}_f$$

$$= \dot{m}_a \left[ 1 + \frac{\dot{m}_f}{\dot{m}_a} \right] = \dot{m}_a \left[ 1 + \frac{1}{85} \right]$$

$$\dot{m} = 50 \left[ 1 + \frac{1}{85} \right]$$

$$\dot{m} = 50.58 \text{ kg/s}$$

$$\boxed{\text{Mass flow rate of air-fuel mixture } \dot{m} = 50.58 \text{ kg/s}}$$

$$\Rightarrow \quad \dot{m}_f = \dot{m} - \dot{m}_a = 50.58 - 50$$

$$\boxed{\dot{m}_f = 0.58 \text{ kg/s}}$$

$$\boxed{\text{Mass flow rate of fuel } \dot{m}_f = 0.58 \text{ kg/s}}$$

We know that

Velocity of jet (or) Velocity of exit gases } $c_j = \sqrt{2 \times \eta_N \times \Delta h_0}$

$$= \sqrt{2 \times 0.90 \times 200 \times 10^3}$$

$$\boxed{c_j = 600 \text{ m/s}}$$

Thrust, $F = \dot{m}\, c_j - \dot{m}_a u$ [From equation no. 5.11]

$$= 50.58 \times 600 - 50 \times 250$$

$$\boxed{F = 17.84 \times 10^3 \text{ N}}$$

Thrust power (or) Propulsive power $P = F \times u$

$$\Rightarrow \quad P = 17.84 \times 10^3 \times 250$$

$$\boxed{\text{Propulsive power } P = 4.46 \times 10^6 \text{ W}}$$

Propulsive efficiency, $\eta_P = \dfrac{2u}{c_j + u}$ [From equation no. 5.19]

$$= \frac{2 \times 250}{600 + 250}$$

$$\boxed{\eta_P = 0.588 \text{ (or) } 58.8\%}$$

Thermal efficiency, $\eta_t = \dfrac{\frac{1}{2} \dot{m} \left[ c_j^2 - u^2 \right]}{\eta_B \times \dot{m}_f \times C \cdot V}$

[From equation no. 5.22]

$$= \frac{\frac{1}{2} \times 50.58\ [(600)^2 - (250)^2]}{0.95 \times 0.58 \times 42{,}000 \times 10^3}$$

$$\boxed{\eta_t = 0.325\ \text{(or)}\ 32.5\%}$$

**Result :** (i) $P = 4.46 \times 10^6$ W

(ii) $\eta_t = 32.5\%$

(iii) $\eta_P = 58.8\%$

**Example 2** *The flight speed of a turbojet is 800 km/h at 10,000 m altitude. The density of air at that altitude is 0.17 kg / m³. The drag for the plane is 6.8 kN. The propulsive efficiency of the jet is 60%. Calculate the SFC, Air-fuel ratio, Jet velocity. Assume the calorific value of fuel is 45000 kJ / kg and overall efficiency of the turbojet plane is 18%.*

*[April 2003, Madras University, Dec. 2005, Anna University]*

**Given :** Flight speed, $u = 800$ km / h

$$= \frac{800 \times 10^3\ \text{m}}{3600\ \text{s}}$$

$$\boxed{u = 222.22\ \text{m/s}}$$

Altitude, $z = 10{,}000$ m

Density, $\rho = 0.17$ kg / m³

Drag (or) Thrust, $F = 6.8$ kN $= 6.8 \times 10^3$ N

Propulsive efficiency, $\eta_P = 60\%$

Calorific value of the fuel $C \cdot V = 45{,}000$ kJ / kg

$= 45{,}000 \times 10^3$ J / kg

Overall efficiency, $\eta_0 = 18\%$

**To find :** (i) SFC (ii) Air-fuel ratio $\frac{\dot{m}_a}{\dot{m}_f}$

(iii) Jet velocity $c_j$

**Solution :** We know that,

Propulsive efficiency, $\eta_P = \frac{2u}{c_j + u}$

$$0.60 = \frac{2 \times 222.22}{c_j + 222.22}$$

$$\Rightarrow \quad 0.60\,[c_j + 222.22] = 2 \times 222.22$$

$$\Rightarrow \quad 0.60\,c_j + 133.33 = 444.44$$

$$\Rightarrow \quad c_j = 518.51\ \text{m/s}$$

$$\boxed{\text{Velocity of jet, } c_j = 518.51\ \text{m/s}}$$

Overall efficiency $\eta_0 = \frac{\text{Thrust Power}}{\text{Power Input}}$

$$= \frac{\text{Thrust} \times \text{Flight speed}}{\text{Power Input}}$$

$$= \frac{F \times u}{\dot{m}_f \times CV}$$

$$0.18 = \frac{6.8 \times 10^3 \times 222.22}{\dot{m}_f \times 45{,}000 \times 10^3}$$

$$\dot{m}_f = 0.186\ \text{kg/s}$$

$$\boxed{\text{Mass flow rate of fuel, } \dot{m}_f = 0.186\ \text{kg/s}}$$

Thrust, $F = \dot{m}\,c_j - \dot{m}_a\,u$ [From equation no. 5.11]

$$= [\dot{m}_a + \dot{m}_f]\,c_j - \dot{m}_a\,u$$

$$F = \dot{m}_a c_j + \dot{m}_f\,c_j - \dot{m}_a\,u$$

$$6.8 \times 10^3 = \dot{m}_a \times 518.51 + 0.186 \times 518.51 - \dot{m}_a \times 222.22$$

$$6.8 \times 10^3 = 296.29\,\dot{m}_a + 96.44$$

$$\Rightarrow \quad \dot{m}_a = 22.62\ \text{kg/s}$$

$$\boxed{\text{Mass flow rate of air, } \dot{m}_a = 22.62\ \text{kg/s}}$$

$$\text{Air fuel ratio } \frac{\dot{m}_a}{\dot{m}_f} = \frac{22.62}{0.186}$$

$$= 121.61$$

$$\boxed{\frac{\dot{m}_a}{\dot{m}_f} = 121.61}$$

$$\left.\begin{array}{r}\text{Thrust specific fuel consumption}\\ \text{TSFC (or) SFC}\end{array}\right\} = \frac{\dot{m}_f}{F}$$

$$= \frac{0.186}{6.8 \times 10^3}$$

$$\boxed{\text{TSFC} = 2.735 \times 10^{-5} \ \frac{\text{kg}}{\text{s - N}}}$$

***Result :*** (*i*) SFC (or) TSFC $= 2.735 \times 10^{-5} \ \frac{\text{kg}}{\text{s - N}}$

(*ii*) $\frac{\dot{m}_a}{\dot{m}_f} = 121.61$ (*iii*) $c_j = 518.51$ m/s

**Example 5** ***The following data refer to a turbojet flying at an altitude of 9500 m.***

*Speed of turbojet = 850 km / h*

*Propulsive efficiency = 55%*

*Overall efficiency of turbine plant = 17%*

*Density of air = 0.17 kg / m*$^3$

*Drag on plane = 6.10 kN*

***Calculate :*** *(a) Absolute velocity of jet*

*(b) Diameter of jet*

*(c) Propulsive power* *[April 99, MU]*

***Given :*** Altitude, $z = 9500$ m

Speed of turbojet, $u = 850$ km / h

$$= \frac{850 \times 10^3 \text{ m}}{3600 \text{ s}}$$

$$\boxed{u = 236.11 \text{ m/s}}$$

Propulsive efficiency, $\eta_P = 55\%$

Overall efficiency, $\eta_0 = 17\%$

Density of air, $\rho = 0.17 \text{ kg/m}^3$

Drag on the plane (or) Thrust, $F = 6.10 \text{ kN}$

$= 6.10 \times 10^3 \text{ N}$

***To find :*** (*i*) Absolute velocity of jet, $c_{abs}$

(*ii*) Diameter of the jet, $d_j$

(*iii*) Propulsive power, P

***Solution :*** We know that,

Propulsive efficiency, $\eta_P = \frac{2u}{c_j + u}$ [From equation no. 5.19

$$\Rightarrow \quad 0.55 = \frac{2 \times 236.11}{c_j + 236.11}$$

$$\Rightarrow \quad 0.55 [c_j + 236.111] = 2 \times 236.11$$

$$\Rightarrow \quad 0.55 c_j + 129.86 = 472.22$$

$$\Rightarrow \quad c_j = 622.47 \text{ m/s}$$

$$\boxed{\left.\begin{array}{l}\text{Velocity of jet } c_j \text{ (or)} \\ \text{Velocity of exit gases}\end{array}\right\} = 622.47 \text{ m/s}}$$

Absolute velocity of jet, $c_{abs} = c_j - u = 622.47 - 236.11$

$$\boxed{c_{abs} = 386.36 \text{ m/s}}$$

We know that, Thrust, $F = \dot{m} [c_j - u]$

[$\because m_f$ is not given]

[From equation no. 5.12]

$$6.10 \times 10^3 = \dot{m} [622.47 - 236.11]$$

$$\boxed{\dot{m} = 15.7 \text{ kg/s}}$$

Mass flow rate, $\dot{m} = \rho \times c_j \times A_j$

$$\Rightarrow \quad 15.7 = 0.17 \times 622.47 \times \frac{\pi}{4} (d_j^2)$$

$$d_j = 0.434 \text{ m}$$

$$\Rightarrow \quad \boxed{\text{Diameter of the jet, } d_j = 0.434 \text{ m}}$$

Propulsive power, $P = F \times u = 6.10 \times 10^3 \times 236.11$

$$\boxed{P = 1.44 \times 10^6 \text{ W}}$$

***Result :*** (*i*) $c_{abs} = 386.36 \text{ m/s}$

(*ii*) $d_j = 0.434 \text{ m}$

(*iii*) $P = 1.44 \times 10^6 \text{ W}$

**Example 6** ***A turbojet has a speed of 750 km/h while flying at an altitude of 10000 m. The propulsive efficiency of the jet is 50% and the overall efficiency of the turbine plant is 16%. The density of air at 10,000 m altitude is 0.173 kg / m³. The drag on the plane is 6250 N. Calorific value of the fuel is 48000 kJ / kg. Calculate :***

***(i) Absolute velocity of the jet***

***(ii) Diameter of the jet***

***(iii) Power output of the unit in kW*** *[April 98, MU]*

***Given :*** Flight speed, $u = 750 \text{ km/h} = \frac{750 \times 10^3}{3600 \text{ s}}$

$$\boxed{u = 208.33 \text{ kg/s}}$$

Altitude, $z = 10{,}000 \text{ m}$

Propulsive efficiency, $\eta_P = 50\%$

Overall efficiency, $\eta_0 = 16\%$

Density of air, $\rho = 0.173 \text{ kg/m}^3$

Drag (or) Thrust, $F = 6250 \text{ N}$

Calorific value $C{\cdot}V = 48000 \text{ kJ/kg}$

$= 48000 \times 10^3 \text{ J/kg}$

***To find :*** *(i)* Absolute velocity $c_{abs}$

*(ii)* Diameter of the jet $d_j$

*(iii)* Power output (P) in kW

***Solution :***

Propulsive efficiency, $\eta_P = \dfrac{2u}{c_j + u}$ [From equation no.

$$\Rightarrow \quad 0.50 = \frac{2 \times 208.33}{c_j + 208.33}$$

$$\Rightarrow \quad 0.50\, c_j + 104.165 = 416.66$$

$$\Rightarrow \quad c_j = 624.99 \text{ m/s}$$

$$\boxed{\text{Velocity of jet } c_j = 624.99 \text{ m/s}}$$

Absolute velocity of jet, $c_{abs} = c_j - u$

$$= 624.99 - 208.33$$

$$\boxed{c_{abs} = 416.66 \text{ m/s}}$$

We know that

Thrust, $F = \dot{m}\,[\,c_j - u\,]$ [From equation no

$$6250 = \dot{m}\,[\,624.99 - 208.33\,]$$

$$\Rightarrow \quad \boxed{\dot{m} = 15 \text{ kg/s}}$$

Mass flow rate, $\dot{m} = \rho \times A_j \times c_j$

$$\Rightarrow$$

$$\Rightarrow \quad 15 = 0.173 \times \frac{\pi}{4}\,(d_j^2) \times 624.99$$

$$d_j = 0.42 \text{ m}$$

$$\boxed{\text{Diameter of the jet, } d_j = 0.42 \text{ m}}$$

Propulsive efficiency, $\eta_P = \dfrac{\text{Propulsive Power (or) Thrust Po}}{\text{Power output of the engine}}$

$$\eta_P = \frac{F \times u}{\text{Power output of the engine}}$$

[∵ Thrust power $P = F \times u$ ]

$$0.50 = \frac{6250 \times 208.33}{\text{Power output}}$$

$$\Rightarrow \quad \boxed{\text{Power output} = 2.6 \times 10^6 \text{ W} = 2.6 \times 10^3 \text{ kW}}$$

***Result :*** *(i)* $c_{abs} = 416.66$ m/s

*(ii)* $d_j = 0.42$ m

*(iii)* $P = 2.60 \times 10^3$ kW

**Example 7** ***A turbojet engine takes in 50 kg/s of air and propels an aircraft with uniform flight speed of 880 km/hr. Isentropic enthalpy change for nozzle is 188 kJ/kg and velocity coefficient is 0.96. The fuel air ratio is 1.2%. Combustion efficiency is 95%. Calorific value of fuel is 44,000 kJ/kg. Find out :***

***(i) Thermal efficiency of the engine***

***(ii) Fuel flow in kg / hr***

***(iii) Propulsive efficiency***

***(iv) Overall efficiency*** *[April 97, MU]*

***Given :*** Air flow rate, $\dot{m}_a = 50$ kg / s

Flight speed, $u = 880$ km / h

$$= \frac{880 \times 10^3}{3600} \text{ m/s}$$

$$= 244.44 \text{ m/s}$$

Isentropic enthalpy change in the nozzle

$$\Delta h_0 = 188 \text{ kJ / kg}$$

$$= 188 \times 10^3 \text{ J / kg}$$

Velocity coefficient = 0.96

Fuel air ratio = 1.2%

$$\Rightarrow \quad \frac{\dot{m}_f}{\dot{m}_a} = 0.012$$

Combustion efficiency, $\eta_B = 95\%$

Calorific value $C \cdot V = 44{,}000 \text{ kJ / kg}$

$$= 44000 \times 10^3 \text{ J / kg}$$

***To find :*** (*i*) Thermal efficiency, $\eta_t$

(*ii*) Fuel flow, $\dot{m}_f$ in kg / *h*

(*iii*) Propulsive efficiency, $\eta_P$

(*iv*) Overall efficiency, $\eta_0$

***Solution :*** Mass flow rate of air fuel mixture

$$\dot{m} = \dot{m}_a + \dot{m}_f$$

$$\dot{m} = \dot{m}_a \left[ 1 + \frac{\dot{m}_f}{\dot{m}_a} \right]$$

$$= 50 [ 1 + 0.012 ]$$

$$\boxed{\dot{m} = 50.6 \text{ kg /s}}$$

$$\dot{m}_f = \dot{m} - \dot{m}_a$$

$$= 50.6 - 50$$

$$\boxed{\dot{m}_f = 0.6 \text{ kg / s}}$$

$$\dot{m}_f = 2160 \text{ kg / h}$$

$$\boxed{\text{Mass flow rate of fuel, } \dot{m}_f = 2160 \text{ kg / h}}$$

$$\left.\begin{array}{r}\text{Velocity of jet (or)}\\ \text{Velocity of exit gas}\end{array}\right\} c_j = \sqrt{2 \times \Delta h_0}$$

$$= \sqrt{2 \times 188 \times 10^3}$$

$$\boxed{c_j = 613.18 \text{ m/s}}$$

Velocity coefficient $= 0.96$ [given]

$$c_j = 0.96 \times 613.18$$

$$\Rightarrow \quad \boxed{\text{Velocity of Jet, } c_j = 588.66 \text{ m/s}}$$

Propulsive efficiency, $\eta_P = \frac{2\,u}{c_j + u}$ [From equation no. 5.19]

$$= \frac{2 \times 244.44}{588.66 + 244.44}$$

$$\boxed{\eta_P = 0.5868 \text{ (or) } 58.6\%}$$

Thermal efficiency, $\eta_t = \frac{\frac{1}{2} \dot{m} \, [ c_j^2 - u^2 ]}{\eta_B \times \dot{m}_f \times C \cdot V}$

[From equation no. 5.22]

$$= \frac{\frac{1}{2} \times 50.6 \, [ (588.66)^2 - (244.44)^2 ]}{0.95 \times 44000 \times 10^3}$$

$$\boxed{\eta_t = 0.289 \text{ (or) } 28.9\%}$$

Overall efficiency, $\eta_0 = \eta_P \times \eta_t$

$$= 0.586 \times 0.289$$

$$\boxed{\eta_0 = 0.169 \text{ (or) } 16.9\%}$$

***Result :*** (*i*) $\eta_t = 28.9\%$

(*ii*) $\dot{m}_f = 2160 \text{ kg / h}$

(*iii*) $\eta_P = 58.6\%$

(*iv*) $\eta_0 = 16.9\%$

**Example 8** *A turbojet plane has two jets of 250 mm diame and the net power at the turbine is 3000 kW. The fuel consumpt per kwhr is 0.42 kg with a fuel of calorific value 49 MJ / kg, wl flying at a speed of 300 m/s in atmospheric having a density of 0. kg/m³. The air fuel ratio is 53. Calculate :*

*(i) Absolute velocity of jet*

*(ii) Resistance (or) Drag of the plane*

*(iii) Overall efficiency of the plane*

*(iv) Efficiency of the thermal* *[Oct. 95, l*

***Given :***

Number of jets, $n = 2$

Diameter, $d = 250 \text{ mm} = 0.25 \text{ m}$

Power, $P = 3000 \text{ kW}$

$= 3000 \times 10^3 \text{ W}$

Fuel consumption, $\dot{m}_f = 0.42 \text{ kg / kwhr}$

Since power is 3000 kW, $\dot{m}_f = 3000 \times 0.42 \text{ kg / h}$

$= 1260 \text{ kg / h}$

$= \frac{1260}{3600} \text{ kg / s}$

$$\boxed{\dot{m}_f = 0.35 \text{ kg / s}}$$

Calorific value $C{\cdot}V = 49 \text{ MJ / kg}$

$C{\cdot}V = 49 \times 10^6 \text{ J / kg}$

Flight speed, $u = 300 \text{ m/s}$

Density, $\rho = 0.168 \text{ kg / m}^3$

Air fuel ratio $\frac{\dot{m}_a}{\dot{m}_f} = 53$

***To find :***

(i) Absolute velocity of jet, $c_{abs}$

(ii) Resistance (or) Drag of the plane, (F)

(iii) Overall efficiency, $\eta_0$

(iv) Thermal efficiency, $\eta_t$

***Solution :*** Mass flow rate of air fuel mixture, $\dot{m} = \dot{m}_a + \dot{m}_f$

$$= \dot{m}_f \left[ \frac{\dot{m}_a}{\dot{m}_f} + 1 \right]$$

$$= 0.35 \, [53 + 1]$$

$$\boxed{\dot{m} = 18.9 \text{ kg / s}}$$

We know that,

Mass flow rate, $\dot{m} = A_j \, c_j \, \rho$

Here, Number of jets, $n = 2$

$$\Rightarrow \quad \dot{m} = 2 \times A_j \times c_j \times \rho$$

$$\Rightarrow \quad \dot{m} = 2 \times \frac{\pi}{4} d^2 \times c_j \times \rho$$

$$18.9 = 2 \times \frac{\pi}{4} (0.25)^2 \times c_j \times 0.168$$

$$\Rightarrow \quad c_j = 1145.91 \text{ m/s}$$

$$\boxed{\text{Velocity of jet, } c_j = 1145.91 \text{ m/s}}$$

Absolute velocity of jet, $c_{abs} = c_j - u$

$$= 1145.91 - 300$$

$$\boxed{c_{abs} = 845.91 \text{ m/s}}$$

Thrust (or) Resistance (or) Drag

$$F = \dot{m} \, c_j - \dot{m}_a \times u$$

$$= 18.9 \times 1145.91 - 18.55 \times 300$$

$$[\because \dot{m}_a = \dot{m} - \dot{m}_f = 18.9 - 0.35 = 18.55 \text{ kg/s}]$$

$$\boxed{F = 16.09 \times 10^3 \text{ N}}$$

Overall efficiency, $$\eta_0 = \frac{\dot{m}[c_j - u] \times u}{\dot{m}_f \times C \cdot V}$$

[From equation no. 5.22]

$$= \frac{18.9[1145.91 - 300] \times 300}{0.35 \times 49 \times 10^6}$$

$$\boxed{\eta_0 = 0.279 \text{ (or) } 27.9\%}$$

Thermal efficiency, $$\eta_t = \frac{\frac{1}{2}\dot{m}[c_j^2 - u^2]}{\dot{m}_f \times C \cdot V}$$

[From equation no. 5.21]

$$= \frac{\frac{1}{2} \times 18.9[(1145.91)^2 - (300)^2]}{0.35 \times 49 \times 10^6}$$

$$\boxed{\eta_t = 0.673 \text{ (or) } 67.3}$$

**Result:** (*i*) $c_{abs} = 845.91$ m/s

(*ii*) $F = 16.09 \times 10^3$ N

(*iii*) $\eta_0 = 27.9\%$

(*iv*) $\eta_t = 67.3\%$

**Example 12.2.** *The diameter of the propeller of an aircraft is 2.5 m; it flies at a speed of 500 kmph at an altitude of 8000 m. For a flight to jet speed ratio of 0.75 determine* (*a*) *the flow rate of air through the propeller,* (*b*) thrust produced, (*c*) *specific thrust,* (*d*) *specific impulse and* (*e*) *the thrust power.*

**Solution.** Area of cross-section of the propeller disc

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4} \times 2.5^2 = 4.908 \text{ m}^2$$

Air density at $Z = 8000$ m is

$$\rho = 0.525 \text{ kg/m}^3$$

Flight speed $u = 500$ kmph $= 138.89$ m/s

$$\sigma = u/c_j = 0.75$$

$$c_j = 138.89/0.75 = 185.18 \text{ m/s}$$

(*a*) Velocity of air flow at the propeller disc is

$$c = \frac{1}{2}(u + c_j)$$

$$c = 0.5\,(138.89 + 185.18) = 162.035 \text{ m/s}$$

Theoretical value of the flow rate is given by

$$\dot{m}_a = \rho A c = 0.525 \times 4.908 \times 162.035$$

$$\dot{m}_a = 417.516 \text{ kg/s} \quad \textbf{Ans.}$$

(*b*) $$F = \dot{m}_a(c_j - u)$$

$$F = 417.516\,(185.18 - 138.89) \times 10^{-3}$$

$$F = 19.3268 \text{ kN} \quad \textbf{Ans.}$$

(*c*) $$F_s = \frac{F}{\dot{m}_a} = \frac{19326.8}{417.516} = 46.29 \text{ N/(kg/s)} \quad \textbf{Ans.}$$

(*d*) $$I_s = \frac{F}{\dot{w}_a} = \frac{F}{\dot{m}_a\, g} = \frac{46.29}{9.81} = 4.718 \text{ s} \quad \textbf{Ans.}$$

(*e*) Thrust power is $P = F \times u$

$$P = 19.3268 \times 138.89 = 2684.3 \text{ kW} \quad \textbf{Ans.}$$