27-07-16.

UNIT - I

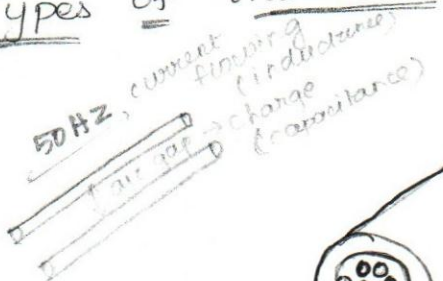
TRANSMISSION LINE THEORY

Electrical lines which are used to transmit the electrical ~~H~~ waves along them called as transmission lines. It consists of pair of wires which are uniformly distributed to the entire length.

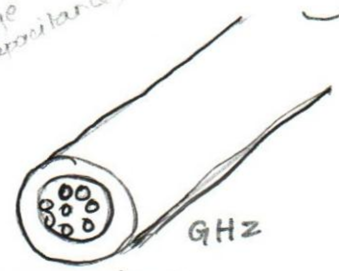
The transmission lines parameters are R, L, C and conductance (G)

Types of transmission lines :

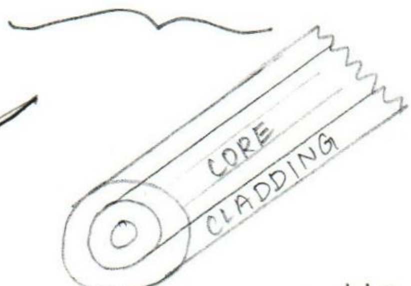
KHZ - MHZ



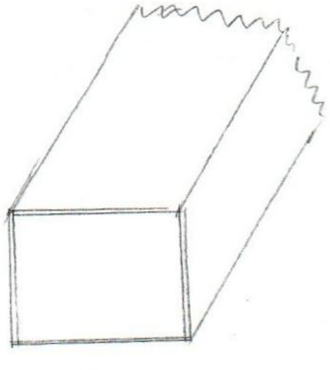
1) Open wire



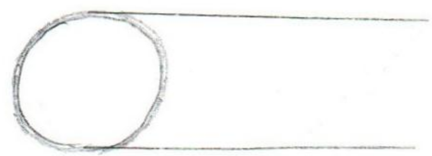
2) Telephone cable



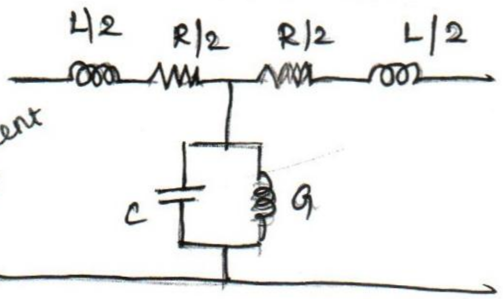
3) coaxial cable



4) Rectangular waveguide



5) Circular waveguide



equivalent ckt

X 2m

TLW

Parameters of transmission lines :

Resistance (R) - Cross section area of the conductor (Ω/m)

Inductance (L) - When the conductor consist current magnetic flux produced around the conductor (H/m)

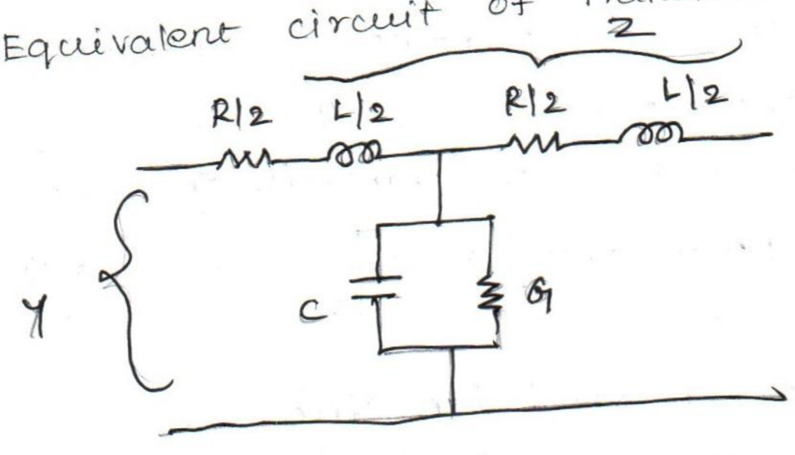
capacitance (C) - Two parallel conductors ~~parallet~~ separated by dielectric (air) produce the capacitive effect. (F/m)

Conductance (G) - Dielectric inbetween conductor is no perfect, small amount of current flow through dielectric is leakage current gives rise to leakage conductance. ($℧/m$)

R, L, C are primary components / constant

α, β, γ are secondary components / constant.

Equivalent circuit of Transmission Line :



Series - R & L

||el - C & G

$$Z = R + j\omega L$$

$$Y = G + j\omega C$$

$$\left(Z = 1/Y, \; R = 1/G \right)$$

$$Z_1 = Z = R + j\omega L$$

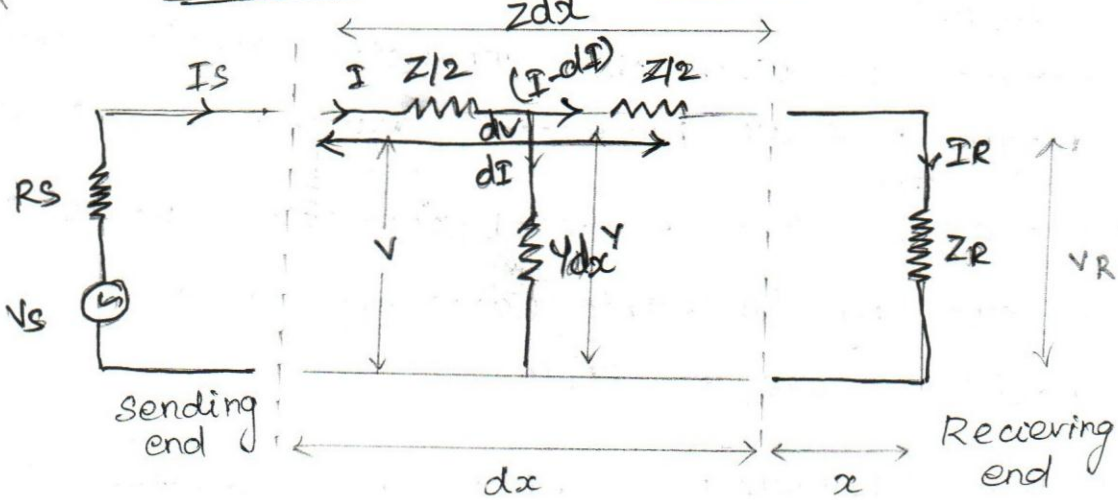
$$Z_2 = \frac{1}{Y} = \frac{1}{G + j\omega C}$$

$$Z_R = \sqrt{Y/Z} \; (A-B)$$

TRANSMISSION LINE GENERAL SOLUTION (OR) SOLUTION OF WAVE EQUATION (OR) VOLTAGE & CURRENT AT ANY POINT OF TRANSMISSION LINE

16m

internal-1



→ Consider a ~~[illegible]~~ transmission line that consists of cascade decimal infinite T-section

→ where R = series resistance

L = " inductance

C = Shunt capacitance

G = " Conductance.

→ 'x' is the distance to the point of observation measured from the recieving end.

→ 'I' ⇒ current at point of transmission line

→ 'V' is voltage ~~btw~~ drop across two conductors at any point

→ 'L' is length of transmission line

→ 'dV' drop across series impedance

→ 'dI' current through shunt admittance.

Current Equation

Voltage Equation

Voltage Equation

ohm's law $V = IR$

$dV = IZdx$

$$\frac{dV}{dx} = IZ \quad \text{—— (1a)}$$

diff w.r.to 'x'

$$\frac{d^2V}{dx^2} = Z\frac{dI}{dx}$$

$$\frac{d^2V}{dx^2} = ZVY \quad \text{—— (2a)}$$

$$\frac{d^2V}{dx^2} = \nu^2 V$$

By linear diff equation

$$\boxed{V = Ae^{\nu x} + Be^{-\nu x}} \quad \text{—— (3a)}$$

Current Equation

$I = V/R$ ohm's law

$dI = VYdx$

$$\frac{dI}{dx} = VY \quad \text{—— (1b)}$$

diff w.r.to 'x'

$$\frac{d^2I}{dx^2} = Y\frac{dV}{dx}$$

$$\frac{d^2I}{dx^2} = YIZ \quad \text{—— (2b)}$$

$$\frac{d^2I}{dx^2} = \nu^2 I$$

By linear diff equation

$$\boxed{I = Ce^{\nu x} + De^{-\nu x}} \quad \text{—— (3b)}$$

$$Z_0 = \sqrt{\frac{Z}{Y}}$$

$$\nu = \sqrt{ZY}$$

A, B, C, D are arbitary constants

(3a) and (3b) are voltage and current equation of transmission line at distance x from the reciering end of TL w.k.t

At load, $x=0$, $V=V_R$, $I=I_R$

$$\boxed{V_R = A+B} \quad \text{—— (4a)}$$

From (3a)

$V = Ae^{\gamma x} + Be^{-\gamma x}$

$V = Ae^{\sqrt{zy}x} + Be^{-\sqrt{zy}x}$

diff w.r.to 'x',

$\frac{dV}{dx} = \sqrt{zy}\,Ae^{\sqrt{zy}x} - \sqrt{zy}\,Be^{-\sqrt{zy}x}$

$\downarrow$

$Iz = \sqrt{zy}\,Ae^{\sqrt{zy}x} - \sqrt{yz}\,Be^{-\sqrt{zy}x}$

$$\boxed{I = \sqrt{\frac{y}{z}}\,Ae^{\sqrt{zy}x} - \sqrt{\frac{y}{z}}\,Be^{-\sqrt{zy}x}} \quad \text{—— (5a)}$$

at $x=0$, $V=V_R$, $I=I_R$

$I_R = \sqrt{y/z}\,A - \sqrt{y/z}\,B$

At load, $x=0$, $I=I_R$, $V=V_R$

$$\boxed{I_R = C+D} \quad \text{—— (4b)}$$

From (3b)

$I = Ce^{\gamma x} + De^{-\gamma x}$

$I = Ce^{\sqrt{zy}x} + De^{-\sqrt{zy}x}$

Diff w.r.to 'x'

$\frac{dI}{dx} = \sqrt{zy}\,Ce^{\sqrt{xyz}} - \sqrt{zy}\,De^{-\sqrt{xyz}}$

$Vy = \sqrt{zy}\,Ce^{\sqrt{xyz}} - \sqrt{zy}\,De^{-\sqrt{xyz}}$

$V = \sqrt{z/y}\,Ce^{\sqrt{xyz}} - \sqrt{z/y}\,De^{-\sqrt{xyz}}$

$$\boxed{V = \sqrt{z/y}\,De^{\sqrt{xyz}} - \sqrt{z/y}\,De^{-\sqrt{xyz}}} \quad \text{—— (5b)}$$

at $x=0$, $I=I_R$, $V=V_R$

$V_R = \sqrt{z/y}\,C - \sqrt{z/y}\,D$

$I_R = \sqrt{y}$ ...

$$I_R = \sqrt{y/z}\,(A-B)$$

$$A-B = I_R\sqrt{z/y} \quad \text{—— (6a)}$$

(4a) + (6a)

$$A+\cancel{B}+A-\cancel{B} = V_R + I_R\sqrt{z/y}$$

$$2A = V_R + I_R\sqrt{z/y}$$

$$\boxed{A = \frac{V_R}{2} + \frac{I_R}{2}\sqrt{\frac{z}{y}}} \quad \text{—— (7a)}$$

$$A+B = V_R \Rightarrow B = V_R - A$$

$$\boxed{B = \frac{V_R}{2} - \frac{I_R}{2}\sqrt{\frac{z}{y}}} \quad \text{—— (8a)}$$

$$V_R = \sqrt{z/y}\,(C-D)$$

$$C-D = V_R\sqrt{y/z} \quad \text{—— (6b)}$$

(4b) + (6b)

$$C+\cancel{D}+C-\cancel{D} = I_R + V_R\sqrt{y/z}$$

$$2C = I_R + V_R\sqrt{\frac{y}{z}}$$

$$\boxed{C = \frac{I_R}{2} + \frac{V_R}{2}\sqrt{\frac{y}{z}}} \quad \text{—— (7b)}$$

$$C+D = I_R \Rightarrow D = I_R - C$$

$$\boxed{D = \frac{I_R}{2} - \frac{V_R}{2}\sqrt{\frac{y}{z}}} \quad \text{—— (8b)}$$

2 eff

Sub A, B in (3a)

$$V = \left(\frac{V_R}{2} + \frac{I_R}{2}\sqrt{\frac{Z}{y}}\right)e^{\gamma x} + \left(\frac{V_R}{2} - \frac{I_R}{2}\sqrt{\frac{Z}{y}}\right)e^{-\gamma x}$$

$$V = \frac{V_R}{2}e^{\gamma x} + \frac{I_R}{2}\sqrt{\frac{Z}{y}}\,e^{\gamma x} + \frac{V_R}{2}e^{-\gamma x} - \frac{I_R}{2}\sqrt{\frac{Z}{y}}\,e^{-\gamma x}$$

$$V = \frac{V_R}{2}(e^{\gamma x} + e^{-\gamma x}) + \frac{I_R}{2}\sqrt{\frac{Z}{y}}\,(e^{\gamma x} - e^{-\gamma x})$$

$$V = \frac{V_R}{2}(e^{\gamma x} + e^{-\gamma x}) + \frac{I_R}{2}Z_0\,(e^{\gamma x} - e^{-\gamma x}) \quad \text{— (9a)}$$

i) Voltage equ in terms of cos & sin :

$$\cosh\gamma x = \frac{e^{\gamma x} + e^{-\gamma x}}{2} \;;\; \sinh\gamma x = \frac{e^{\gamma x} - e^{-\gamma x}}{2}$$

$$\boxed{V = V_R\cosh\gamma x + I_R Z_0 \sinh\gamma x}$$

Sub C, D in (3b)

$$I = \left(\frac{I_R}{2} + \frac{V_R}{2}\sqrt{\frac{y}{Z}}\right)e^{\gamma x} + \left(\frac{I_R}{2} - \frac{V_R}{2}\sqrt{\frac{y}{Z}}\right)e^{-\gamma x}$$

$$I = \frac{I_R}{2}e^{\gamma x} + \frac{V_R}{2}\sqrt{\frac{y}{Z}}\,e^{\gamma x} + \frac{I_R}{2}e^{-\gamma x} - \frac{V_R}{2}\sqrt{\frac{y}{Z}}\,e^{-\gamma x}$$

$$I = \frac{I_R}{2}(e^{\gamma x} + e^{-\gamma x}) + \frac{V_R}{2}\sqrt{\frac{y}{Z}}\,(e^{\gamma x} - e^{-\gamma x})$$

$$I = \frac{I_R}{2}(e^{\gamma x} + e^{-\gamma x}) + \frac{V_R}{2}\frac{1}{Z_0}(e^{\gamma x} - e^{-\gamma x}) \quad \text{— (9b)}$$

i) current equ in terms cos & sin :

$$\cosh\gamma x = \frac{e^{\gamma x} + e^{-\gamma x}}{2} \;;\; \sinh\gamma x = \frac{e^{\gamma x} - e^{-\gamma x}}{2}$$

$$\boxed{I = I_R\cosh\gamma x + \frac{V_R}{Z_0}\sinh\gamma x}$$

ii) voltage equation interms k-reflection co-eff :

ii) Current equ interms of K-reflection co-eff

ii) Voltage equation interms k-reflection co-eff:

$$V = \frac{V_R}{2}(e^{\gamma x}+e^{-\gamma x}) + \frac{I_R Z_0}{2}(e^{\gamma x}-e^{-\gamma x})$$

Sub $I_R = \frac{V_R}{Z_R}$

$$V = \frac{V_R}{2}(e^{\gamma x}+e^{-\gamma x}) + \frac{V_R Z_0}{2Z_R}(e^{\gamma x}-e^{-\gamma x})$$

$$V = \frac{V_R}{2}e^{\gamma x} + \frac{V_R}{2}e^{-\gamma x} + \frac{V_R Z_0}{2Z_R}e^{\gamma x} - \frac{V_R Z_0}{2Z_R}e^{-\gamma x}$$

$$V = \frac{V_R}{Z_R 2}\underbrace{(Z_R+Z_0)}_{\substack{E_i \\ \text{incident voltage}}} e^{\gamma x} + \frac{V_R}{2Z_R}\underbrace{(Z_R-Z_0)}_{E_r \text{ Recieved Vtg}} e^{-\gamma x}$$

$$V = \frac{V_R}{2Z_R}(Z_R+Z_0)\left[e^{\gamma x}+\left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\gamma x}\right]$$

$$\boxed{V = \frac{V_R}{2Z_R}(Z_R+Z_0)\left[e^{\gamma x}+k e^{-\gamma x}\right]}$$

$$K = \frac{Z_R-Z_0}{Z_R+Z_0}$$

ii) Current equ interms of K-reflection co-eff

$$I = \frac{I_R}{2}(e^{\gamma x}+e^{-\gamma x}) + \frac{V_R}{2Z_0}(e^{\gamma x}-e^{-\gamma x})$$

Sub $V_R = I_R Z_R$

$$I = \frac{I_R}{2}(e^{\gamma x}+e^{-\gamma x}) + \frac{I_R Z_R}{2Z_0}(e^{\gamma x}-e^{-\gamma x})$$

$$I = \frac{I_R}{2}e^{\gamma x} + \frac{I_R}{2}e^{-\gamma x} + \frac{I_R Z_R}{2Z_0}e^{\gamma x} - \frac{I_R Z_R}{2Z_0}e^{-\gamma x}$$

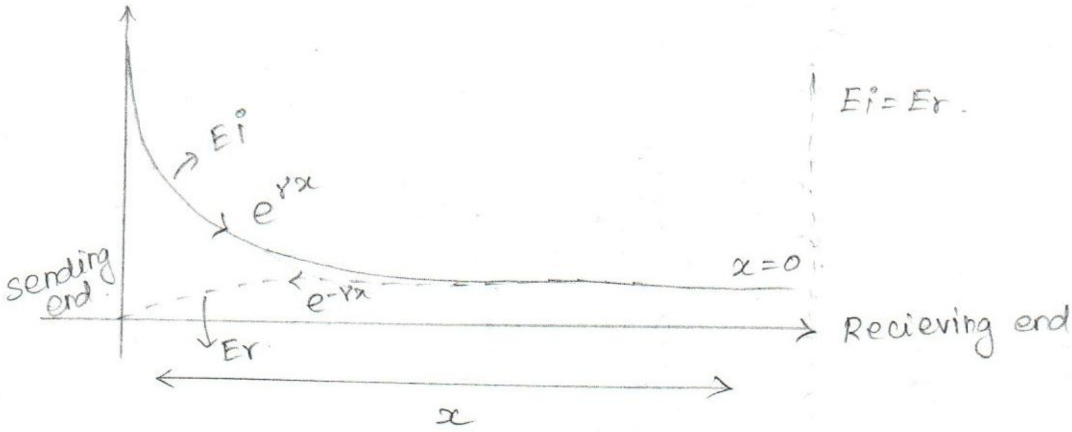
$$I = \frac{I_R}{2Z_0}(Z_0+Z_R)e^{\gamma x} + \frac{I_R}{2Z_0}(Z_0-Z_R)e^{-\gamma x}$$

$$I = \frac{I_R}{2Z_0}(Z_0+Z_R)\left[e^{\gamma x}+\left(\frac{Z_0-Z_R}{Z_0+Z_R}\right)e^{-\gamma x}\right]$$

$$I = \frac{I_R}{2Z_0}(Z_0+Z_R)\left[e^{\gamma x}-\left(\frac{Z_0+Z_R}{Z_0-Z_R}\right)e^{-\gamma x}\right]$$

$$\boxed{I = \frac{I_R}{2Z_0}(Z_0+Z_R)\left[e^{\gamma x}-k e^{-\gamma x}\right]}$$

· Physical significance of voltage and current equation.

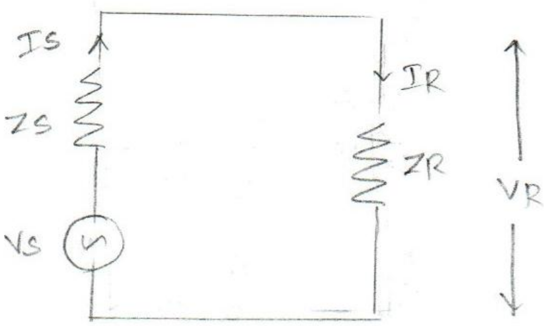


$$V = \frac{V_R}{2Z_R}\underbrace{(Z_R+Z_0)}_{E_i}e^{\gamma x} + \frac{V_R}{2Z_R}\underbrace{(Z_R-Z_0)}_{E_r}e^{-\gamma x}$$

$E_i \rightarrow$ Incident Vtg $E_r \rightarrow$ Reflected Vtg.

(Input Impedance (Z_S (or) Z_{in}) of transmission line :

internal-1



w.k.t,

$$V = V_R \cosh\gamma x + I_R Z_0 \sinh\gamma x$$

$$I = I_R \cosh\gamma x + \frac{V_R}{Z_0}\sinh\gamma x$$

$$Z_R = \frac{V_R}{I_R}$$

$$V_R = I_R Z_R$$

$$V_R = Z_R \left(I_R \cosh\gamma x + \frac{V_R}{Z_0} \sinh\gamma x \right)$$

$$\cancel{V_R} \quad Z_R = \frac{V_R}{I_R}$$

$$Z_R = \frac{Z_R \left(I_R \cosh\gamma x + \frac{V_R}{Z_0} \sinh\gamma x \right)}{\left(I_R \cosh\gamma x + \frac{V_R}{Z_0} \sinh\gamma x \right)}$$

$$Z_S = Z_0 \left(\frac{Z_R \cosh\gamma x + Z_0 \sinh\gamma x}{Z_0 \cosh\gamma x + Z_R \sinh\gamma x} \right)$$

$$Z_S = \frac{Z_0 \cancel{\cosh\gamma x} \left(Z_R + Z_0 \frac{\sinh\gamma x}{\cosh\gamma x} \right)}{\cancel{\cosh\gamma x} \left(Z_0 + Z_R \frac{\sinh\gamma x}{\cosh\gamma x} \right)}$$

$$Z_S = \frac{Z_0 \left(Z_R + Z_0 \tanh\gamma x \right)}{\left(Z_0 + Z_R \tanh\gamma x \right)}$$

w.k.t,

$$V = \frac{V_R}{2Z_R} (Z_0 + Z_R) \left[e^{\gamma x} + K e^{-\gamma x} \right]$$

$$I = \frac{I_R}{2Z_0} (Z_0 + Z_R) \left[e^{\gamma x} - K e^{-\gamma x} \right]$$

$$Z_S = \frac{V_S}{I_S} \Rightarrow \frac{V_R/2Z_R \, (Z_0 + Z_R) \, (e^{\gamma x} + K e^{-\gamma x})}{I_R/2Z_0 \, (Z_0 + Z_R) \, (e^{\gamma x} - K e^{-\gamma x})}$$

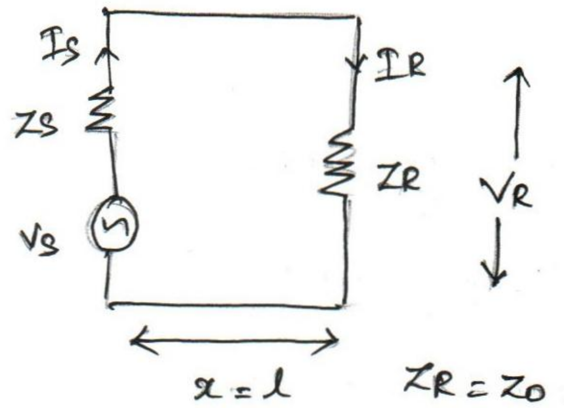
$[V_R = I_R Z_R]$

$$Z_S = \frac{\cancel{I_R} \cancel{Z_R} / \cancel{2} \cancel{Z_R} \, (Z_0 + \cancel{Z_R}) \, (e^{\gamma x} + K e^{-\gamma x})}{\cancel{I_R} / \cancel{2} Z_0 \, (Z_0 \cancel{+} Z_R) \, (e^{\gamma x} - K e^{-\gamma x})}$$

$$Z_S = \frac{Z_0 \left(e^{\gamma x} + K e^{-\gamma x} \right)}{\left(e^{\gamma x} - K e^{-\gamma x} \right)}$$

i) Input Impedance at transmission line at finite line :

8m



$$Z_S = \frac{Z_0 (Z_R + Z_0 \tanh \gamma x)}{(Z_0 + Z_R \tanh \gamma x)}$$

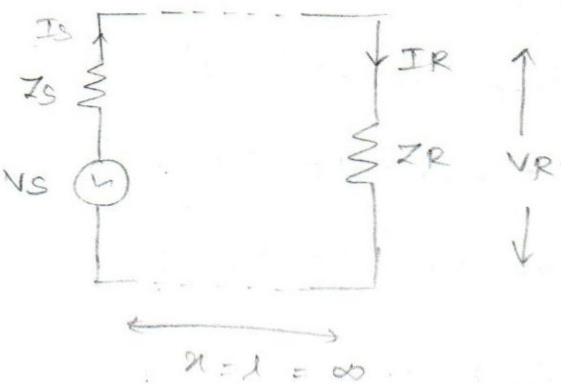
Replace $x = l$

$$Z_R = Z_0 . \quad \Rightarrow \quad \frac{Z_0 (\cancel{Z_R} + \cancel{Z_0} \tanh \gamma l)}{(\cancel{Z_0} + Z_R \cancel{\tanh \gamma l})}$$

$$\boxed{Z_S = Z_0}$$

It shows that finite length of transmission line terminating with load Z_0 conforms that Z_{in} (or) $Z_S = Z_0$ as symmentrical n/w.

ii) Input Impedance at infinite line :



$$Z_S = \frac{Z_0 (Z_R + Z_0 \tanh \gamma x)}{(Z_0 + Z_R \tanh \gamma x)}$$

Replace $x = l = \infty$ $[\tanh \infty = 1]$

matching load $Z_R = Z_0$.

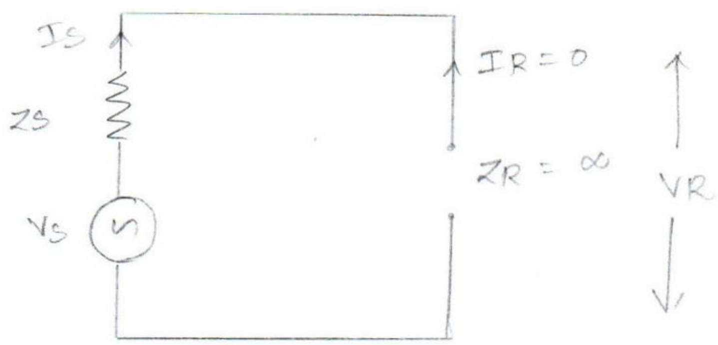
$$Z_S = \frac{Z_0 (\cancel{Z_R} + Z_0 \tanh \infty)}{(Z_0 + \cancel{Z_R} \tanh \infty)}$$

$$\boxed{Z_S = Z_0}$$

finite line terminated by matching load is act as infinite line.

Open ckt Impedance (Z_{oc}) of transmission line :

2m



$$Z_{oc} = \frac{V_{oc}}{I_{oc}}$$

i) $V \to V_{oc}$ when $I_R = 0$; $Z_R = \infty$

$$V = V_R \cosh \gamma x + \cancel{I_R Z_0} \sinh \gamma x$$

$$\boxed{V_{oc} = V_R \cosh \gamma x}$$

ii) $I \rightarrow I_{oc}$ when $I_R = 0$, $Z_R = \infty$

$$I = I_R \cancel{\cosh \gamma x}^{\,0} + \frac{V_R}{Z_0} \sinh \gamma x$$

$$\boxed{I_{oc} = \frac{V_R}{Z_0} \sinh \gamma x}$$

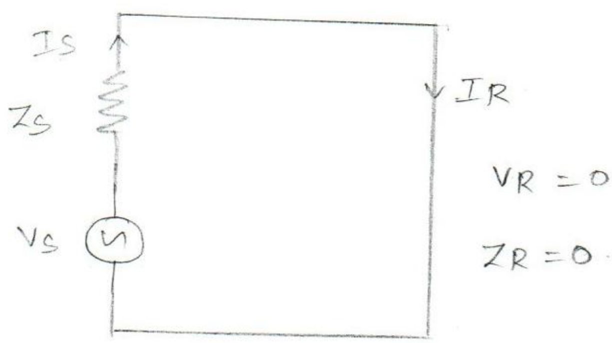
$$Z_{oc} = \frac{V_{oc}}{I_{oc}}$$

$$Z_{oc} = \frac{\cancel{V_R} \cosh \gamma x}{\frac{\cancel{V_R}}{Z_0} \sinh \gamma x}$$

$$\boxed{Z_{oc} = Z_0 \coth \gamma x}$$

Short ckt Impedance (Z_{sc}) at tran^m line :

2m



$$Z_{sc} = \frac{V_{sc}}{I_{sc}}$$

i) $V \rightarrow V_{sc}$ when ~~IR~~ $V_R = Z_R = 0$

$$V = V_R \cancel{\cosh \gamma x}^{\,0} + I_R Z_0 \sinh \gamma x$$

$$V_{sc} = I_R Z_0 \sinh \gamma x$$

ii) $I \rightarrow I_{sc}$ when $V_R = Z_R = 0$

$$I = I_R \cosh \gamma x + \frac{V_R}{Z_0} \cancel{\sinh \gamma x}^{\,0}$$

$$I_{sc} = I_R \cosh \gamma x$$

$$Z_{sc} = \frac{V_{sc}}{I_{sc}}$$

$$Z_{sc} = \frac{\cancel{I_R} Z_0 \sinh\gamma x}{\cancel{I_R} \cosh\gamma x}$$

$$\boxed{Z_{sc} = Z_0 \tanh\gamma x}$$

Relation between Z_{oc} and Z_{sc} :

2m (or) problems

$$Z_{oc} \times Z_{sc} = Z_0 \cot h\gamma x \times Z_0 \tanh\gamma x$$

$$= Z_0 \frac{1}{\cancel{\tanh\gamma x}} \times Z_0 \cancel{\tanh\gamma x}$$

$$Z_0^2 = Z_{oc} Z_{sc}$$

$$\boxed{Z_0 = \sqrt{Z_{oc} Z_{sc}}}$$

$$\frac{Z_{sc}}{Z_{oc}} = \frac{\cancel{Z_0}\tanh\gamma x}{\frac{\cancel{Z_0}}{\tanh\gamma x}} \Rightarrow \tanh^2\gamma x$$

$$\gamma x = \tanh^{-2} \frac{Z_{sc}}{Z_{oc}}$$

(OR)

$$\boxed{\gamma x = \tanh^{-1} \sqrt{\frac{Z_{sc}}{Z_{oc}}}}$$

Transfer Impedance (Z_T) :

4m

$$Z_T = \frac{V_S}{I_R}$$

$$V_S = V_R \cosh\gamma x + I_R Z_0 \sinh\gamma x$$

$$V_S = I_R Z_R \cosh\gamma x + I_R Z_0 \sinh\gamma x$$

$$V_S = I_R (Z_R \cosh\gamma x + Z_0 \sinh\gamma x)$$

$$\frac{V_S}{I_R} = \boxed{Z_S = Z_R \cosh\gamma x + Z_0 \sinh\gamma x}$$

$$V_S = \frac{V_R}{2Z_R}(Z_0+Z_R)(e^{\gamma x}+ke^{-\gamma x})$$

$$V_S = \frac{I_R \cancel{Z_R}}{2\cancel{Z_R}}(Z_0+Z_R)(e^{\gamma x}+ke^{-\gamma x})$$

$$\frac{V_S}{I_R} = \boxed{Z_S = \left(\frac{Z_0+Z_R}{2}\right)(e^{\gamma x}+ke^{-\gamma x})}$$

Properties of infinite line :

2m

* As line has infinite length no waves will reach the recieving end and there is no possibility of reflection at the recieving end. Therefore, complete power applied at the sending end is observed by the line.

* As the reflected waves are absent the characteristic impedance Z_0 at recieving end will decide the current flow as voltage is applied at the sending end. This current will not be affected by terminating load impedance Z_R

Wavelength, velocity of propagation, phase constant, Attenuation constant.

Wavelength (λ) :

Distance travelled by the wave along the line for phase shift of 2π radians.

$$\boxed{\lambda = \frac{2\pi}{\beta}} \text{ meter.}$$

Velocity of propagation (v) :

$$v = \frac{\text{distance travelled}}{\text{time}} = \frac{\lambda}{1/f} = \lambda f$$

$$v = \lambda f$$

$$v = \left(\frac{2\pi}{R}\right) f$$

$$\boxed{v = \omega/\beta} \text{ m/s}$$

Attenuation constant and Phase constant

8m Internal-1

$$\gamma = \alpha + j\beta \quad \text{—— (1)}$$

$$\gamma = \sqrt{zy} = \sqrt{(R+j\omega L)(G+j\omega C)} \quad \text{—— (2)}$$

$$\alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$(\alpha+j\beta)^2 = (R+j\omega L)(G+j\omega C)$$

$$\alpha^2 - \beta^2 + 2\alpha\beta j = RG + j\omega RC + j\omega GL - \omega^2 LC$$

Real part : $\alpha^2 - \beta^2 = RG - \omega^2 LC$ —— (3)

Img part : $2\alpha\beta = \omega(RC + LG)$ —— (4)

Squaring equ (4)

$$4\alpha^2\beta^2 = \omega^2(RC+LG)^2$$

$$\alpha^2 = \frac{\omega^2(RC+LG)^2}{4\beta^2}$$

Sub α^2 in (3)

$$\frac{\omega^2(RC+LG)^2}{4\beta^2} - \beta^2 = RG-\omega^2LC$$

$$\omega^2(RC+LG)^2 - 4\beta^4 = 4\beta^2(RG-\omega^2LC)$$

$$4\beta^4 + 4\beta^2(RG-\omega^2LC) - \omega^2(RC+LG)^2 = 0.$$

$$x^2 = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$a = 4$; $b = 4(RG-\omega^2LC)$; $c = -\omega^2(RC+LG)^2$

$$\therefore \beta^2 = \frac{-4(RG-\omega^2LC) \pm \sqrt{16(RG-\omega^2LC)^2 - 4(4)(-\omega^2(RC+LG)^2)}}{2\times 4}$$

$$\beta^2 = \frac{(\omega^2LC-RG) \pm \sqrt{(RG-\omega^2LC)^2 + \omega^2(RC+LG)^2}}{2}$$

$$\boxed{\beta = \sqrt{\frac{(\omega^2LC-RG) + \sqrt{(RG-\omega^2LC)^2 + \omega^2(RC+LG)^2}}{2}}}$$

$$\alpha^2 - \beta^2 = RG - \omega^2 LC$$

$$\alpha^2 = (RG - \omega^2 LC) + \beta^2$$

$$\alpha^2 = (RG - \omega^2 LC) + \left[\frac{-(RG-\omega^2 LC) + \sqrt{(RG-\omega^2 LC)^2 + \omega^2 (RC+LG)^2}}{2}\right]$$

$$\boxed{\alpha = \sqrt{\frac{(RG-\omega^2 LC) + \sqrt{(RG-\omega^2 LC)^2 + \omega^2 (RC+LG)^2}}{2}}}$$

Conclusion :

For a perfect transmission line the elements

$R = G = 0$.

$$\alpha = \sqrt{\frac{(\cancel{RG}-\omega^2 LC) + \sqrt{(\cancel{RG}-\omega^2 LC)^2 + \omega^2 (\cancel{RC}+LG)^2}}{2}}$$

$$\alpha = \sqrt{\frac{-\omega^2 LC + \sqrt{(\omega^2 LC)^2}}{2}} = \sqrt{\frac{-\omega^2 LC + \omega^2 LC}{2}}$$

$$\boxed{\alpha = 0}$$

$$\beta = \sqrt{\frac{(\omega^2 LC - \cancel{RG}) + \sqrt{(\cancel{RG}-\omega^2 LC)^2 + \omega^2 (\cancel{RC}+LG)^2}}{2}}$$

$$\beta = \sqrt{\frac{\omega^2 LC + \sqrt{(\omega^2 LC)^2}}{2}} = \sqrt{\frac{\omega^2 LC + \omega^2 LC}{2}}$$

$$\beta = \sqrt{\omega^2 LC}$$

$$\boxed{\beta = \omega\sqrt{LC}}$$

Distortion in Transmission Line :

16m

When the recieved signal is not exact replica of transmitted signal, then the signal is said to be distorted.

Types :

1) Selective power absorption ⇒ due to variations of characteristic impedance (Z_0) with frequency.

2) Frequency distortion ⇒ due to variation of attenuation constant (α) with frequency.

3) Phase (or) Delay distortion ⇒ due to variation of phase (β) with frequency.

1) **Selective Power absorption :**

The characteristic impedance (Z_0) with line varies with frequency while the line is terminated by the impedance which does not vary with frequency. So the is maximum power absorped at certain freq and reflected at certain frequency.

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

condition for distortionless tran. line,

2m $\boxed{LG = CR}$

$$Z_0 = \sqrt{\frac{R\left(1+\cancel{\frac{j\omega L}{R}}\right)}{G\left(1+\cancel{\frac{j\omega C}{G}}\right)}} \quad (R=G)$$

$$\boxed{\begin{aligned} LG &= CR \\ \frac{L}{R} &= \frac{C}{G} \end{aligned}}$$

$$\boxed{Z_0 = \sqrt{\frac{R}{G}}}$$

conclusion:

characteristic imp Z_0 doesnot depend on frequency but depend on primary constants R and G.

~~Attenu~~

2) <u>Frequency distortion</u>:

Attenuation constant (α) is function of frequency. Hence different freq transmitted along the line will be attenuated to a different extent.

Ex: i) In Radio broadcasting freq distortion is eliminated by an equalizer

ii) voice s/l consists of many freq and all the frequencies will not be attenuated equally along the transmission Line.

$$\alpha = \sqrt{\frac{(RG-\omega^2LC) + \sqrt{(RG-\omega^2LC)^2 + \omega^2(RC+LG)^2}}{2}}$$

condition for distortionless Line,

$$LG = RC$$

$$\gamma = \sqrt{zy} = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{R\left(1+\frac{j\omega L}{R}\right) G\left(1+\frac{j\omega C}{G}\right)}$$

$$\gamma = \sqrt{RG\left(1+\frac{j\omega L}{R}\right)^2} \qquad [L = C] \quad R = G$$

$$\gamma = \sqrt{RG}\left(1+\frac{j\omega L}{R}\right)$$

$$\gamma = \sqrt{RG} + j\frac{\sqrt{RG}\,\omega L}{R}$$

$$\gamma = \sqrt{RG} + j\omega L\sqrt{\frac{G}{R}}$$

$$\frac{G}{R} = \frac{C}{L}$$

$$\gamma = \sqrt{RG} + j\omega L\sqrt{\frac{C}{L}}$$

$$\gamma = \underbrace{\sqrt{RG}}_{\alpha} + j\underbrace{\omega\sqrt{LC}}_{\beta}$$

$$\boxed{\alpha = \sqrt{RG}} \; ; \; \beta = \omega\sqrt{LC}$$

Conclusion:

The α is independent of frequency only depend on primary constant.

3) Phase (or) Delay Distortion :

Some waves will reach the recieving end very fast and some will slow / delayed, bcz all the waves doest not propagate with same velocity.

$$\beta = \omega\sqrt{LC}$$

$$V = \frac{\omega}{\beta}$$

$$V = \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{LC}}$$

$$\boxed{V = \frac{1}{\sqrt{LC}}} \quad m/s$$

The velocity v doesnot depend on frequency only depends on L & C.

Distortionless transmission line :

The line with no frequency or phase distortion which satisfies $LG = CR$ and also it is currently terminated with Z_R.

~~PROBLEMS :~~

FORMULAS :

$$Z_0 = \sqrt{Z/Y}$$

$$\gamma = \sqrt{ZY}$$

$$Z_0 \times \gamma = \sqrt{\frac{Z}{Y}} \times \sqrt{ZY}$$

$$= \sqrt{Z^2}$$

$$\boxed{Z_0 \gamma = Z}$$

$$\frac{\gamma}{Z_0} = \frac{\sqrt{ZY}}{\sqrt{Z/Y}} = \sqrt{Y^2}$$

$$\boxed{\frac{\gamma}{Z_0} = Y}$$

PROBLEMS :

1) The transmission line has the following parameter ~~for~~ per unit length $L = 0.1\mu H$, $R = 5\Omega$, $G = 0.01\Omega^{-1}$, $C = 300pF$. Calculate γ and Z_0 at 500 MHz

Given data : $L = 0.1\mu H$

$R = 5\Omega$

$G = 0.01\Omega^{-1}$

$C = 300pF$

(i) $\gamma = \sqrt{zy}$

$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$

$\omega = 2\pi f \Rightarrow 2\pi \times 500 \times 10^6$

$\omega = 3141.59 \times 10^6$

$\omega = 3141.6 \times 10^6$

$z = R + j\omega L$

$z = 5 + (3141.6 \times 10^6)(0.1\mu)$

$z = 5 + j\,314.16\ \Omega$

$z = 314.19 \angle 1.55$ → Alpha (tan) = F = (Ans)

$y = G + j\omega C$

$y = (0.01) + j(3141.6M)(300p)$

$y = 0.01 + j\,0.9424$ → Rect

$y = 0.9424 \angle 1.560$ → Polar

$$\gamma = \sqrt{(314.19 \angle 1.55^\circ)(0.9424 \angle 1.56^\circ)}$$

$$\gamma = 17.20 \angle \left(\frac{1.55 + 1.56}{2}\right)$$

$$\boxed{\gamma = 17.20 \angle 1.55^\circ} \ m$$

* $\sqrt{r_1 \angle \theta_1 \, r_2 \angle \theta_2}$

$\sqrt{r_1 r_2} \angle \frac{\theta_1+\theta_2}{2}$

* $\sqrt{\frac{r_1 \angle \theta_1}{r_2 \angle \theta_2}}$

$\sqrt{\frac{r_1}{r_2}} \angle \frac{\theta_1-\theta_2}{2}$

* $(r_1 \angle \theta_1 \times r_2 \angle \theta_2)^2$

$(r_1 r_2)^2 \angle 2(\theta_1+\theta_2)$

* $\left(\frac{r_1 \angle \theta_1}{r_2 \angle \theta_2}\right)^2$

$\left(\frac{r_1}{r_2}\right)^2 \ 2 \angle \theta_1 - \theta_2$

$\sqrt{\ } \angle Ans$

$\frac{1}{2} \times \angle Ans$

$(\angle Ans)^2$

$2 \times \angle Ans$

(ii) $Z_0 = \sqrt{\frac{Z}{Y}}$

$$Z_0 = \sqrt{\frac{314.19 \angle 1.55}{0.9424 \angle 1.56}}$$

$$Z_0 = 18.25 \angle \frac{1.55^\circ - 1.56^\circ}{2}$$

$$Z_0 = 18.25 \angle -0.005^\circ \ \Omega$$

2. The characteristic imp of uniform transmission line is 2309.6 Ω at freq of 800 Hz. At this freq propagation is $\gamma = 0.054\,(0.0366 + j0.999)$. Determine R, L, G, C (primary constants).

* U.Q

Gn: $\gamma = 0.054\,(0.0366 + j0.999)$

$Z_0 = 2309.6 \,\Omega$

$f = 800\,Hz$

Sol: $Z_0 \times \gamma = Z = R + j\omega L$

$= 2309.6 \times 0.054\,(0.0366 + j0.999)$

$= 2309.6 \times (0.0019 + j0.053)$

$Z = 2309.6\ (0.0019 + j\ 0.053)$

~~$Z = 4.388 + j\ 122.4$~~

$4.388 + j\ 122.4 \Rightarrow Z = R + j\omega L$

$R = 4.388\ \Omega$

$\omega L = 122.4$

$L = \frac{122.4}{\omega} = \frac{122.4}{2\pi f}$

$L = \frac{122.4}{2\pi \times 800}$

$L = 24.35\,mH$

$\frac{v}{Z_0} = Y = G + j\omega C$

$\frac{0.054\,(0.0366 + j\ 0.999)}{2309.6} = Y$

$\frac{0.0019 + j\ 0.053}{2309.6} = Y$

V.Q 16r

$Z = 2309.6\ (0.0019 + j\ 0.0539)$

$Z = 4.388 + j\ 124.4$

$Z = 124.47\ \underline{\lfloor 1.53^{\circ}}$

$Z = 5.08 + j\ 124.36$

$Z = R + j\omega L$

$5.08 + j\,124.36 = R + j\omega L$

$R = 5.08\,\Omega$

$\omega L = 124.36$

$L = \dfrac{124.36}{2\pi \times 800}$

$L = 24.74\,mH$

$y = \dfrac{\gamma}{Z_0} = \dfrac{0.054\,(0.0366 + j0.999)}{2309.6}$ $\quad 8.55 \times 10^{-7} + j\,2.32 \times 10^{-5}$

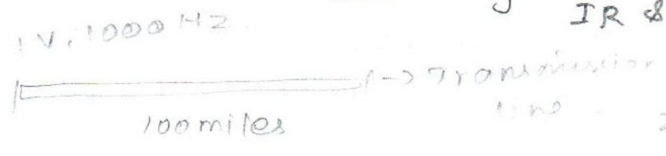
$y = 8.55 \times 10^{-7} + j\,2.33 \times 10^{-5}$ $\quad G = 0.855\,\mu$

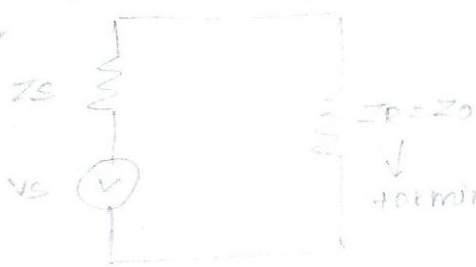
$2.33 \times 10^{-5} \angle 1.534^\circ$ $\quad C = 14.6\,nf$

$8.571 \times 10^{-7} + j\,2.328 \times 10^{-5}$

$0.855\mu + j\,0.0233\,m$

V.Q. (16m) Generatar produce 1V, 1000Hz freq which is supplied to 100 miles, open wire line terminated by Z_0 ($Z_R = Z_0$) having $R = 10.4\,\Omega/km$, $L = 0.00367\,H/km$, $G = 0.8 \times 10^{-6}\,\Omega^{-1}/km$, $C = 0.00835\,\mu F/km$. calculate Z_0, γ, α, β, v, λ (secondary constant), sending end I (I_S), recieving end I & V (I_R & V_R), recieved power (P_R).





$\omega = 2\pi f$

$\omega = 2\pi \times 1000$

$\omega = 6.283\,KHZ$

$R = 10.4\,\Omega/km$

$L = 0.00367H/km$

$G = 0.8 \times 10^{-6}\,\Omega^{-1}/km$

$C = 0.0085\,\mu F/km$.

1) $Z_0 = \sqrt{Z/Y} = \sqrt{\dfrac{R+j\omega L}{G+j\omega C}}$

$$Z_0 = \sqrt{\frac{(10.4) + j(6.283k)(0.00367)}{(0.8\times10^{-6}) + j(6.283k)(0.0085\mu)}}$$

$$Z_0 = \sqrt{\frac{10.4 + j\,23.058}{(0.8\times10^{-6}) + j\,0.0534m}}$$

$$Z_0 = \sqrt{\frac{25.294 \angle 1.147^\circ}{0.0534m \angle 1.555^\circ}}$$

$$\boxed{\begin{aligned} Z_0 &= 688.23 \angle -0.204^\circ\ \Omega. \\ Z_0 &= 673.95 - j\,139.427\ \Omega. \end{aligned}}$$

2) $\gamma = \sqrt{ZY} = \sqrt{(R+j\omega L)(G+j\omega C)}$

$Z = R + j\omega L$

$Z = 10.4 + j(6.283k)(0.00367)$

$Z = 10.4 + j\,23.058$

$Y = 0.8\mu + j\,0.0534m$

$$\gamma = \sqrt{(10.4 + j\,23.058)(0.8\mu + j\,0.0534m)}$$

~~$\gamma = 2.884m \angle$~~

$$\gamma = \sqrt{(25.294 \angle 1.147^\circ)(0.0534m \angle 1.555^\circ)}$$

$$\boxed{\begin{aligned} \gamma &= 0.0367 \angle 1.351^\circ \\ \gamma &= 7.936 \times 10^{-3} + j\,0.0355 \end{aligned}}$$

3) $\gamma = \alpha + j\beta$

$7.936 \times 10^{-3} + j\,0.0355 = \alpha + j\beta$

comparying both sides,

$$\boxed{\begin{aligned} \alpha &= 7.936 \times 10^{-3} \\ \beta &= 0.0355 \end{aligned}}$$

4) $V = \dfrac{\omega}{\beta} = \dfrac{2\pi f}{\beta}$

$V = \dfrac{6.283k}{0.0355}$

$$\boxed{V = 176.99 \text{ km/s}}$$

5) $\lambda = \dfrac{2\pi}{\beta} = \dfrac{2 \times \pi}{0.0355}$

$$\boxed{\lambda = 176.99m}$$

6) $I_S = \dfrac{V_S}{Z_S} = \dfrac{V_S}{Z_0}$

$I_S = \dfrac{1}{Z_0} \Rightarrow \dfrac{1}{688.23 \angle -0.204}$

$$\boxed{I_S = 1.46 \times 10^{-3} \angle 0.204 \text{ A}}$$

7) $$K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$\because Z_R = Z_0$

$$\boxed{K = 0}$$

$$V = \frac{V_R}{2Z_R}(Z_R + Z_0)(e^{\gamma x} + \cancel{K} e^{-\gamma x})$$

$$V = \frac{I_R}{\cancel{2}}(\cancel{2}Z_0)\, e^{\gamma x}$$

$$V = I_R Z_0 e^{\gamma x}$$

$(\because V = V_S)$

$$V_S = I_R Z_0 e^{\gamma x}$$

$$I_R = \frac{V_S}{Z_0} e^{-\gamma x}$$

$$= I_S e^{-(\alpha + j\beta)x}$$

$$= I_S e^{-\alpha x} \angle -\beta x$$

$$I_R = (1.46 \times 10^{-3} \angle 0.204^\circ)\, e^{-7.936 \times 10^{-3} \times 100} \angle -0.035 \times 100$$

$$I_R = (1.46 \times 10^{-3} \angle 0.204^\circ)(0.4522) \angle -3.5^\circ$$

~~$I_R = 0.6602$~~

$$\boxed{I_R = 660.2 \times 10^{-6} \angle -3.296^\circ \; A}$$

$$V_R = I_R Z_R$$

$$V_R = I_R Z_0$$

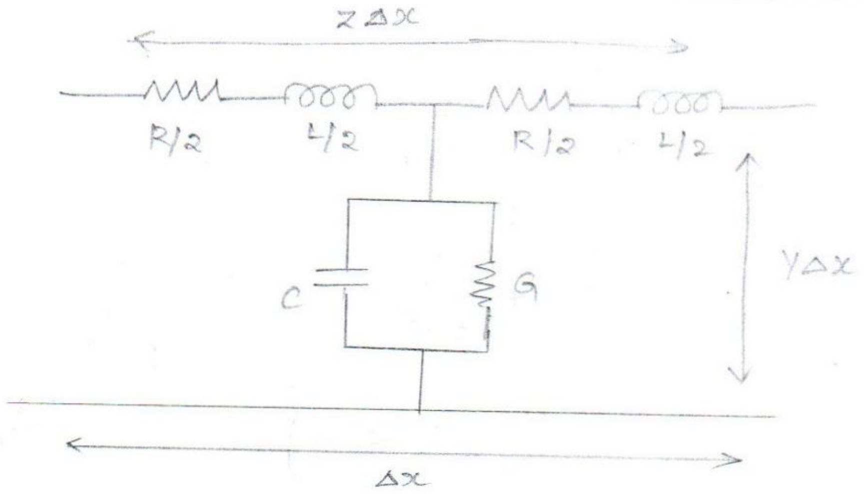
$$V_R = (660.2 \times 10^{-6} \angle -3.296^\circ)(688.23 \angle -0.204^\circ)$$

$$\boxed{V_R = 0.454 \angle -3.549 \; V}$$

8) $P_R = I_R \times V_R$

$$\boxed{P_R = 0.299 \times 10^{-3} \angle -6.89 \; W}$$

Characteristic impedance (Z_0) of the transmission line :



$$Z_1 = Z\Delta x$$

$$Z_2 = 1/Y\Delta x$$

$$Z_0 = \sqrt{Z_1 Z_2 + Z_1^2/4}$$

$$= \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

$$Z_0 = \sqrt{Z\Delta \cancel{x} \times \frac{1}{Y\Delta \cancel{x}} \left(1 + \frac{Z\Delta x}{4/Y\Delta x}\right)}$$

$$Z_0 = \sqrt{Z/Y \left(1 + \frac{Z\Delta x^2 Y}{4}\right)}$$

0.

$\Delta x = 0$.

$$Z_0 = \sqrt{Z/Y} \quad (\text{or}) \quad Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Consider cascaded T - section with number of identical and symmenterical T - n/w which are connected in series, if the final section is terminated by an characteristic impedance Z_0. The input of the first section

$$\boxed{\begin{aligned} Z_{in} &= Z_0 \\ Z_0 &= Z_R \end{aligned}}$$

Each section is terminated by the i/p impedance of section, and last one has Z_0 termination

Propagation Constant (γ) of the txion Line:—

$$\gamma = \ln\left[1 + \frac{Z_1}{2Z_2} + \frac{\boxed{Z_0}}{Z_2}\right]$$

$$e^{\gamma} = 1 + \frac{Z_1}{2Z_2} + \frac{\sqrt{Z_1 Z_2 + Z_1^2/4}}{Z_2}$$

$$e^{\gamma} = 1 + \frac{Z_1}{2Z_2} + \sqrt{\frac{Z_1}{Z_2} + \frac{Z_1^2}{4Z_2^2}}$$

$$= 1 + \frac{Z_1}{2Z_2} + \sqrt{\frac{Z_1}{Z_2}\left(1 + \frac{Z_1}{2Z_2}\right)}$$

$$e^{\gamma} = 1 + \frac{Z_1}{2Z_2} + \sqrt{\frac{Z_1}{Z_2}}\left(1 + \frac{\cancel{Z_1}}{2Z_2}\right)^{1/2}$$

neg

$$(a+b)^n = a^n + nc_1 a^{n-1} b^1 + nc_2 a^{n-2} b^2 + \ldots\ldots nc_n b^n$$

$$e^{\gamma} = 1 + \frac{Z_1}{2Z_2} + \sqrt{\frac{Z_1}{Z_2}}$$

$$e^{x} = 1 + \frac{x}{1!} + \frac{x^2}{2!} \quad \text{—— (1)}$$

$$e^{\gamma} = 1 + \sqrt{\frac{Z_1}{Z_2}} + \frac{1}{2}\left(\sqrt{\frac{Z_1}{Z_2}}\right)^2 \quad \text{—— (2)}$$

$$x = \sqrt{\frac{Z_1}{Z_2}}$$

compare (1) & (2)

$$e^{x} = e^{\gamma}$$

$$\gamma = x$$

$$\gamma = \sqrt{\frac{Z_1}{Z_2}}$$

consider the transmission length Δx of the propagation constant $\gamma = \sqrt{zy}$

$$\gamma\Delta x = \sqrt{\frac{z\Delta x}{1/y\Delta x}}$$

$$\gamma\Delta x = \sqrt{zy\Delta x^2}$$

$$\gamma \cancel{\Delta x} = \sqrt{zy}\,\cancel{\Delta x}$$

$$\gamma = \sqrt{zy} \quad (\text{or}) \quad \gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Telephone cable : v(x) 2marks

6m In oridinary telephone cable, the wires are insulated with papers and twisted in pairs.

For Audio frequency range, inductance L and conductance G are negligible and small (ie)

$L = 0$ & $G = 0$.

$L = 0$, $G = 0$.

$Z = R + j\cancel{\omega L}^{0}$

$Y = G + j\cancel{\omega C}^{0}$

$Z = R$

$Y = j\omega C$

$\gamma = \sqrt{ZY} = \sqrt{R \cdot j\omega C}$

$= \sqrt{\frac{R \times 2j\omega C}{2}}$

$\gamma = \sqrt{2j} \times \sqrt{\frac{\omega RC}{2}}$

$\angle j = \angle 90^{\circ} \Rightarrow 1 \angle 45^{\circ}$

$\gamma = \sqrt{2j} = \sqrt{2} \times 1 \angle 45^{\circ}$

$\sqrt{2j} = \sqrt{2} \angle 45^{\circ}$

$\sqrt{2j} = 1 + j$

$\gamma = (1+j)\sqrt{\frac{\omega RC}{2}}$

$= \sqrt{\frac{\omega RC}{2}} + j\sqrt{\frac{\omega RC}{2}}$

$$\gamma = \alpha + j\beta$$

$$\boxed{\begin{aligned} \alpha &= \sqrt{\frac{\omega RC}{2}} \\ \beta &= \sqrt{\frac{\omega RC}{2}} \end{aligned}}$$

$$V = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\frac{\omega RC}{2}}}$$

$$\boxed{V = \sqrt{\frac{2\omega}{RC}}}$$

Conclusion:

Both α and β are functions of frequency. Hence at the high frequency, high range of attenuation will occur. Therefore, waves travel very fast at high frequency and low at low frequency. Thus in telephone cable both phase and frequency distortion and dominant.

LOADING :-

(defn Inductance (L) is coounded around the txion line to increase the value of Series inductance to achieve distortionless txion line)

(Loading :

16m

v. imp

16m

$$RC = LG$$

$$\frac{R}{G} = \frac{L}{C}$$

condition for distortionless tran-y

Practically,

$$\frac{R}{G} \gg \frac{L}{C}$$

$\left(\frac{R}{G}\right)$ ↓es $\left(\frac{L}{C}\right)$ ↑es

R↓ G↑ L↑ C↓

1) R↓ ⇒ decreasing txion line area of cross section is not possible

2) G↑ ⇒ No possible to increase leakage (conductance)

3) C↓, ⇒ capacitance ie) Space b/n txion line decreasing is not possible

4) L↑ ⇒ only way is to ↑ L-value.

Types :

1. Continuous loading
2. Lumped loading.

Continuous Loading :

→ Iron (or) magnetic metal (μ-metal) which is highly permeability is founded on transmission line.

→ It is very expensive

→ Used on ocean cables

Advantage :

α and β independent of frequency

Propagation constant for continuous Loading cable:

$$z = R + j\omega L$$

$$y = G + j\omega C$$

$$\gamma = \sqrt{zy}.$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$R+j\omega L = \left(\sqrt{R^2+\omega^2L^2}\ \angle\tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

$$G+j\omega C = \sqrt{G^2+\omega^2C^2}\ \left(\angle\tan^{-1}\left(\frac{\omega C}{G}\right)\right)$$

$$\left.\begin{aligned}\sqrt{\omega^2C^2}\left(\tan^{-1}\left(\frac{\omega C}{0}\right)\right) &\Rightarrow \omega C\ (\angle\tan^{-1}\infty) \\ &\Rightarrow \omega C\ \angle\pi/2\end{aligned}\right\}\ G=0$$

$$\gamma = \sqrt{\sqrt{R^2+\omega^2L^2}\ \angle\tan^{-1}(\omega L/R)\quad \omega C\ \angle\pi/2}$$

$$\sqrt{R^2+\omega^2L^2} = \sqrt{\omega^2L^2\left(1+\frac{R^2}{\omega^2L^2}\right)}$$

$$= \omega L\sqrt{\left(1+\frac{R^2}{\omega^2L^2}\right)}$$

$$\tan^{-1}x = \frac{\pi}{2} - \cot^{-1}(x)$$

$$\tan^{-1}\left(\frac{\omega L}{R}\right) = \frac{\pi}{2} - \tan^{-1}\left(\frac{R}{\omega L}\right) \qquad \tan^{-1}x = \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{x}\right)$$

$$\gamma = \sqrt{\omega L\left(1+\cancel{\frac{R^2}{\omega^2L^2}}_{\to 0}\right)\ \angle\pi/2-\tan^{-1}(R/\omega L)\quad \omega C\ \angle\pi/2}$$

$$\omega L \gg R \rightarrow \frac{R^2}{\omega^2L^2} \ll 1 \rightarrow \frac{R^2}{\omega^2L^2} = \text{Img}$$

$$\gamma = \sqrt{\omega^2 LC\ \angle\pi/2-\tan^{-1}(R/\omega L)\ \angle\pi/2}$$

$$\gamma = \omega\sqrt{LC}\ \underbrace{\angle \frac{\pi - \tan^{-1}(R/\omega L)}{2}}_{\theta}$$

γ / γ

$$\theta = \frac{\pi}{2} - \frac{1}{2}\tan^{-1}\left(\frac{R}{\omega L}\right)$$

$\angle\theta = e^{j\theta}$

$$\gamma = \omega\sqrt{LC}\ e^{j\theta}$$

$$\gamma = \omega\sqrt{LC}\ (\cos\theta + j\sin\theta)$$

$$\cos\theta = \cos\left(\overset{90\ -}{\frac{\pi}{2}} - \overset{\theta}{\frac{1}{2}\tan^{-1}\left(\frac{R}{\omega L}\right)}\right)$$

$\cos 90 - \theta = \sin\theta$

$$= \sin\theta \Rightarrow \cancel{\sin}\left(\frac{1}{2}\cancel{\tan^{-1}}\left(\frac{R}{\omega L}\right)\right)$$

$$\cos\theta = \frac{R}{2\omega L}$$

[$\sin\theta = \theta$

$\tan\theta = \theta$]

where θ = small

$$\sin\theta = \sin\left(\frac{\pi}{2} - \frac{1}{2}\tan^{-1}\left(\frac{R}{\omega L}\right)\right)$$

$$\sin\theta = 1$$

$$\gamma = \omega\sqrt{LC}\ \left(R/2\omega L + j\right)$$

$$\gamma = \frac{\omega\sqrt{LC}\,R}{2\omega L} + j\omega\sqrt{LC}$$

$$\gamma = \frac{R}{2}\sqrt{C/L} + j\omega\sqrt{LC}$$

From above

$$\gamma = \alpha + j\beta$$

$$\alpha = R/2\sqrt{C/L} \quad ; \quad \beta = \omega\sqrt{LC}$$

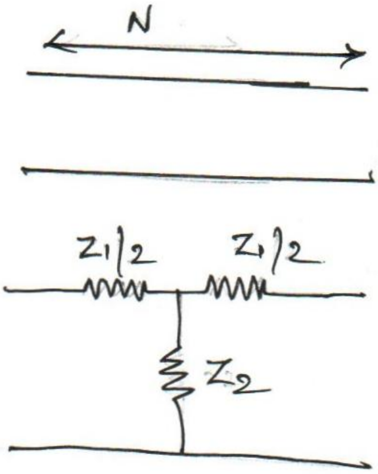
$$v = \omega/\beta = \omega / \omega\sqrt{LC}$$

$$v = 1/\sqrt{LC}$$

CAMBELL'S EQUATION :

To analyse the performance of loaded transmission line, cambell's equation is used.

Unloaded



$Z_1 \rightarrow$ series arm

$Z_2 \rightarrow$ shunt arm

~~For N-length~~

$$\boxed{\begin{aligned} \cosh\gamma &= 1 + \frac{Z_1}{2Z_2} \\ \sinh\gamma &= \frac{Z_0}{Z_2} \end{aligned}}$$

$$Z_2 = \frac{Z_0}{\sinh\gamma}$$

For N-length

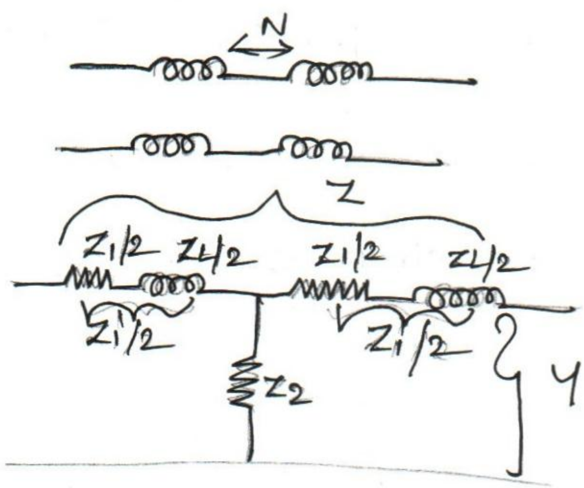
$$\cosh N\gamma = 1 + \frac{Z_1}{2Z_2}$$

$$\sinh N\gamma = \frac{Z_0}{Z_2}$$

$$Z_2 = \frac{Z_0}{\sinh N\gamma}$$

sub Z_2 in $\cosh N\gamma$

Loaded



series arm $= Z_1' = \frac{Z_1}{2} + \frac{Z_L}{2} + \frac{Z_1}{2} + \frac{Z_L}{2}$

shunt arm $= Z_2$

$$Z_1' = Z_1 + Z_L$$

$$\cosh\gamma' N = 1 + \frac{Z_1'}{2Z_2}$$

$$\sinh\gamma' N = \frac{Z_0}{Z_2}$$

$$Z_2 = \frac{Z_0}{\sinh N\gamma'}$$

w.k.T, $Z_1' = Z_1 + Z_L$

$$\frac{Z_1'}{2} = \frac{Z_1 + Z_L}{2}$$

Sub Z_1 from unloaded

$$\frac{Z_1'}{2} = \frac{\frac{2Z_0(\cosh N\gamma - 1)}{\sinh N\gamma} + Z_L}{2}$$

$$\cosh N\gamma = 1 + \frac{Z_1}{2Z_2}$$

$$\cosh N\gamma = 1 + \frac{Z_1}{2\left(\frac{Z_0}{\sinh N\gamma}\right)}$$

$$\cosh N\gamma = 1 + \frac{Z_1 \sinh N\gamma}{2Z_0}$$

$$\cosh N\gamma - 1 = \frac{Z_1 \sinh N\gamma}{2Z_0}$$

$$Z_1 = \frac{(\cosh N\gamma - 1)\, 2Z_0}{\sinh N\gamma}$$

$$\frac{Z_1'}{2} = \frac{2Z_0(\cosh N\gamma - 1)}{2\sinh N\gamma} + \frac{Z_L}{2}$$

$$\frac{Z_1'}{2} = \frac{Z_0(\cosh N\gamma - 1)}{\sinh N\gamma} + \frac{Z_L}{2}$$

wkt. $\cosh N\gamma' = 1 + \frac{Z_1'}{2Z_2}$

$$\cosh N\gamma' = 1 + \frac{\left(\frac{Z_0(\cosh N\gamma - 1)}{\sinh N\gamma} + \frac{Z_L}{2}\right)}{\frac{Z_0}{\sinh N\gamma}}$$

Sub Z_2 in unloaded

$$\cosh N\gamma' = 1 + \frac{\sinh N\gamma}{Z_0}\left[\frac{Z_0(\cosh N\gamma - 1)}{\sinh N\gamma} + \frac{Z_L}{2}\right]$$

$$\cosh N\gamma' = \cancel{1} + (\cosh N\gamma - \cancel{1}) + \frac{Z_L \sinh N\gamma}{2Z_0}$$

$$\boxed{\cosh N\gamma' = \cosh N\gamma + \frac{Z_L \sinh N\gamma}{2Z_0}}$$

Conclusion:

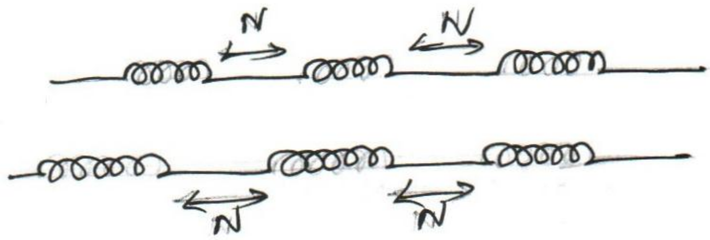
Propagation constant of loaded line interms of propagation constant of unloaded line and cable Inductance.

Hence loaded transmission line act as Low pass filter $f_c = \frac{1}{\pi\sqrt{LC}}$

Lumped Loading :

→ Inductance are used in uniform interval on both lines of transmission line to balance the circuit.

→ Loading coil has its own resistance, thus increasing L value will further increase ~~or~~ R. Therefore, coil must be ~~dissolved~~ designed and installed carefully.



Propagation constant of lumped loaded cable :

$$\gamma = \sqrt{zy} = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$\gamma = \sqrt{j\omega L\left(1+\frac{R}{j\omega L}\right) j\omega C\left(1+\frac{G}{j\omega C}\right)}$$

$$\gamma = \sqrt{j^2\omega^2 LC\left(1+\frac{R}{j\omega L}\right)\left(1+\frac{G}{j\omega C}\right)}$$

$$\gamma = \sqrt{j^2\omega^2 LC\left(1+\frac{R}{j\omega L}+\frac{G}{j\omega C}-\frac{RG}{\omega^2 LC}\right)}$$

high = neglected

$$\gamma = j\omega\sqrt{LC}\left(1+\frac{R}{j\omega L}+\frac{G}{j\omega C}\right)^{1/2}$$

$$(1+a)^{1/2} = 1+\frac{1}{2}a+\frac{1}{4}a^2+\dots$$

$$\gamma = j\omega\sqrt{LC}\left[1 + \frac{1}{2}\left(\frac{R}{j\omega L} + \frac{G}{j\omega C}\right)\right]$$

$$\gamma = j\omega\sqrt{LC} + \frac{1}{2} j\omega\sqrt{LC}\left(\frac{R}{j\omega L} + \frac{G}{j\omega C}\right)$$

$$\gamma = j\omega\sqrt{LC} + \frac{1}{2}\sqrt{LC}\left(\frac{R}{L} + \frac{G}{C}\right)$$

$$\gamma = \alpha + j\beta$$

$$\left.\begin{aligned} \alpha &= \frac{\sqrt{LC}}{2}\left(\frac{R}{L} + \frac{G}{C}\right) \\ \beta &= \omega\sqrt{LC} \; ; \; v = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \end{aligned}\right\}$$

Independent of frequency (f)

⇓

attenuation will be less.

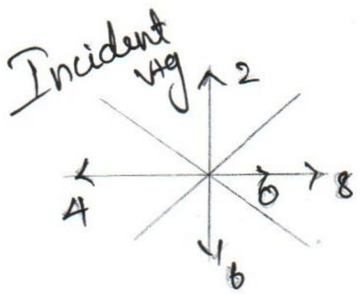
Reflection (or) Line not Terminated with Z_0 :

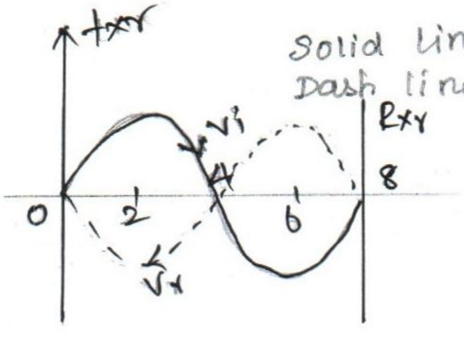
$$V = \frac{V_R}{2Z_R}(Z_R + Z_0)\left[e^{\gamma x} + \underbrace{K e^{-\gamma x}}\right] \rightarrow K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$V = \underbrace{\frac{V}{2Z_R}(Z_R + Z_0)e^{\gamma x}}_{V_i} + \underbrace{\frac{V}{2Z_R}(Z_R - Z_0)e^{-\gamma x}}_{V_R}$$

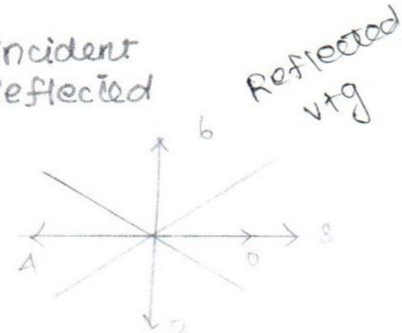
$$I = \underbrace{\frac{I_R}{2Z_0}(Z_R + Z_0)e^{\gamma x}}_{I_i} - \underbrace{\frac{I_R}{2Z_0}(Z_R - Z_0)e^{-\gamma x}}_{I_R}$$

$$V = V_i + V_R \; ; \; I = I_i - I_R$$





Solid line - Incident
Dash line - Reflected



. Reflection coefficient (K) :

2m It is ratio of reflected voltage (or) current to the transmitted voltage (or) current.

Problem Definition

$$K = \frac{V_r}{V_i} \text{ (or) } \frac{I_r}{I_i}$$

$$\cancel{K = \frac{V_R}{2Z_R}}$$

$$K = \frac{V_r}{V_i}$$

$$K = \frac{\frac{\cancel{V_R}}{\cancel{2Z_R}}(Z_R - Z_0)\cancel{e^{-\gamma x}}}{\frac{\cancel{V_R}}{\cancel{2Z_R}}(Z_R + Z_0)\cancel{e^{\gamma x}}}$$ when $x = 0$

$$K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

PROPERTIES :

① when $Z_R = Z_0$, $K = 0 \rightarrow$ no reflection

② when $Z_R = 0$ (short ckt), $K = -1 \rightarrow$ line is shorted

$K = 1 \angle 180°$ $\theta = 180°$ reflection is minimum

③ when $Z_R = \infty$ (open ckt), $K = \frac{Z_R - Z_0}{Z_R + Z_0} \rightarrow$ line is open, reflection max.

$$K = \frac{Z_R(1 - \cancel{Z_0/Z_R})}{Z_R(1 + \cancel{Z_0/Z_R})}$$

$$K = +1 \angle 0°$$ $\theta = 0$, $K = +1$

4. K ranges from, mag $-1 \leq |K| \leq 1$

$0^\circ \leq \theta \leq 180^\circ$

5. Polarity of reflected wave depends on magnitude and phase of Z_0 and Z_R.

Disadvantage:

→ Reduces the efficiency

→ Reduces the output

→ Attenuation is large, if the attenuation is not large then the reflected wave appears as ~~eco~~ 'echo' at the reflecting end.

→ If input impedance Z_S is not equal Z_0 the reflected wave again reflected by the transmitter and it become incidence wave.

Reflection Phenomenon:

The quantity which is actually transmitted along the line is not current or voltage but if it is energy it is transmitted through electric and magnetic field

$$\text{Electric field} = \frac{1}{2} CE^2 \ J/m^2$$

$$\text{Magnetic field} = \frac{1}{2} LI^2 \ J/m^2$$

$$\boxed{E = IZ_0}$$

$$\text{Electric field} = \frac{1}{2} C (I Z_0)^2$$

$$= \frac{1}{2} C I^2 Z_0^2$$

w.k.T $Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$

For perfect line, $R=0$, $G=0$.

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$\text{Electric field} = \frac{1}{2} \cancel{C} I^2 \frac{L}{\cancel{C}}$$

$$= \frac{1}{2} L I^2 \ J/m^2$$

$$\text{Magnetic field} = \frac{1}{2} L I^2 \ J/m^2$$

Thus for perfect line, electric field is equal to magnetic field.

Reflection loss :

Under mismatched condition, $Z_R \neq Z_0$, the part of incidence energy is rejected and reflected by load. Thus the energy is delivered to the load under the mismatch load is always less than the energy which would be ~~deli~~ delivered to the load.

Definition :

It is defined as no.of nepers (or) decibels by which the current in the load under image matched condition which exceeds the current actually flowing in the load.

$$\text{Reflection loss} = \ln\left(\frac{I_R'}{I_R}\right) \text{ np}$$

(or)

$$= 20\log\left(\frac{I_R'}{I_R}\right) \text{ dB}$$

I_R' = matched current $(Z_R = Z_0)$

I_R = mismatched current $(Z_R \neq Z_0)$

$$\text{Reflection loss} = \ln\left|\frac{P_R'}{P_R}\right|^{1/2} \text{ np}$$

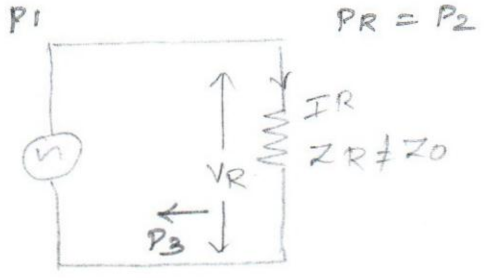
(or)

$$= 10\log\left(\frac{P_R'}{P_R}\right) \text{ dB}$$

P_R' = matched power

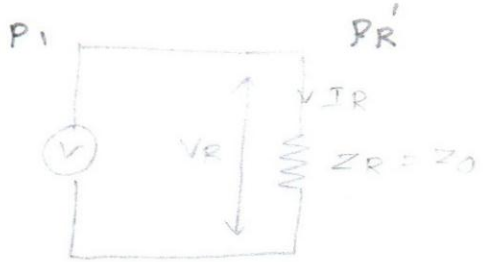
P_R = mismatched power.

Mismatched load $(Z_0 \neq Z_R)$:



(Transmission line diagram)

Matched load $(Z_0 = Z_R)$



$P_1 = P_R'$

Incident vtg = V_1

" current = I_1

Reflected vtg = KV_1

" current = KI_1

Incident power $(P_1) = V_1 I_1$

Reflected power $(P_3) = K V_1 K I_1$

$$= K^2 V_1 I_1$$

$$P_1 = P_2 + P_3$$

$$P_2 = P_1 - P_3$$

$$= V_1 I_1 - K^2 V_1 I_1$$

$$P_2 = V_1 I_1 (1 - K^2)$$

$$P_2 = P_1 (1 - K^2)$$

Reflected loss $= \frac{1}{2} \ln \left(\frac{P_R'}{P_R} \right)$

$$= \frac{1}{2} \ln \left(\frac{P_R'}{P_1 (1-K^2)} \right) \quad \because P_2 = P_R$$

$$= \frac{1}{2} \ln \left(\frac{\cancel{P_1}}{\cancel{P_1}(1-K^2)} \right) \quad \because P_R' = P_1$$

Reflected loss $= \frac{1}{2} \ln \left(\frac{1}{1-K^2} \right)$ np

(or)

$$= 10 \log \left(\frac{1}{1-K^2} \right) \text{ dB.}$$

Reflection coeff

W.K.T $\quad K = \dfrac{Z_R - Z_0}{Z_R + Z_0}$

$$1 - K^2 = 1 - \left[\frac{Z_R - Z_0}{Z_R + Z_0} \right]^2$$

$$= 1 - \frac{(Z_R - Z_0)^2}{(Z_R + Z_0)^2}$$

$$= \frac{(Z_R + Z_0)^2 - (Z_R - Z_0)^2}{(Z_R + Z_0)^2}$$

$$= \frac{\cancel{Z_R^2} + 2Z_RZ_0 + \cancel{Z_0^2} - \cancel{Z_R^2} + 2Z_RZ_0 - \cancel{Z_0^2}}{(Z_R+Z_0)^2}$$

$$= \frac{4Z_RZ_0}{(Z_R+Z_0)^2}$$

$$1-K^2 = \left(\frac{2\sqrt{Z_RZ_0}}{Z_R+Z_0}\right)^2$$

$$\text{Ref. loss} = \cancel{\tfrac{1}{2}}\, 10\log\left(\frac{1}{1-K^2}\right)$$

$$= 10\log\left(\frac{1}{\left(\frac{2\sqrt{Z_RZ_0}}{Z_R+Z_0}\right)^2}\right)$$

$$= 20\log\frac{Z_R+Z_0}{2\sqrt{Z_RZ_0}}$$

$$\boxed{\text{Reflection loss} = 20\log(1/K)\ dB}$$

2m

$$K = \frac{2\sqrt{Z_RZ_0}}{Z_R+Z_0} \Rightarrow \text{Reflected factor}$$

$$\text{Reflection loss} \propto \frac{1}{\text{Reflected factor}}$$

Reflection factor:

2m Ratio which indicates the change in ~~con~~ current in the load under mismatched condition in the load.

Return loss:

2m. Ratio of power at recieving end due to incident wave to the power reflected by load is called return loss.

- Return loss $= 10 \log \left(\frac{P_1}{P_3} \right)$

$$= 10 \log \left(\frac{\cancel{P_1}}{k^2 \cancel{P_1}} \right)$$

$$= 10 \log \left(\frac{1}{k^2} \right)$$

Return loss $= 10 \log \left(\frac{1}{k} \right)^2$

(or)

$$= 20 \log \left(\frac{1}{k} \right)$$

$$\boxed{\text{Return loss} = 20 \log \left(\frac{Z_R + Z_0}{Z_R - Z_0} \right) \text{dB}.}$$

PROBLEMS :

1. The transmission line has Z_0 and it is terminated with $Z_R = 100\Omega$, $Z_0 = 745 \angle -12° \ \Omega$. Calculate Reflection loss and Return loss.

$$k = \frac{2\sqrt{Z_R Z_0}}{Z_R + Z_0}$$

$$k = \frac{2\sqrt{100 \times 745 \angle -12°}}{100 + 745 \angle -12°}$$

$$k = \frac{2\sqrt{100 \times 745 \angle -12°}}{100 + 628.67 + j\,399.74}$$

$$k = \frac{2 \,(272.94 \angle -6)}{728.67 + j\,399.74}$$

$$k = \frac{2 \,(272.94 \angle -6)}{831.11 \angle 0.5}$$

$$K = \frac{545.88 \angle -6}{831.11 \angle 0.5}$$

$$K = 0.65 \angle -6.5$$

$$|K| = 0.65$$

$$\text{Reflection loss} = 20 \log \left(\frac{1}{K}\right)$$

$$= 20 \log \left(\frac{1}{0.65}\right)$$

$$\text{Reflection loss} = 3.74 \text{ dB}$$

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} \quad (\text{Ref. coeff})$$

$$K = \frac{100 - 745 \angle -12^\circ}{100 + 745 \angle -12^\circ}$$

$$K = \frac{100 - 628.67 + j\,399.74}{100 + 628.67 + j\,399.74}$$

$$K = \frac{-528.67 + j\,399.74}{728.67 + j\,399.74}$$

$$K = \frac{662.78 \angle 2.49}{831.11 \angle 0.5}$$

$$K = 0.79 \angle 1.99$$

$$|K| = 0.79$$

$$\text{Return loss} = 20 \log (1/K)$$

$$= 20 \log (1/0.79)$$

$$\text{Return loss} = 2 \text{ dB}$$

UNIT - 2

HIGH FREQUENCY TRANSMISSION LINE

(30 MHz - 3000 MHz)

Dissipation less / Zero Dissipation

NOTE :

1. $G = 0$, $L = 0$ → Telephone line (audio freq)

2. $\frac{R}{G_1} = \frac{L}{C}$ → Transmission (distortionless line)

3. $R = 0$, $G = 0$ → Tx^{-n} line with high freq (dissipation less line) (or) High frequency.

The Radio frequency of 30MHz - 300MHz has made following assumption

i) current & voltages are flowing on conductor surface and internal inductance become zero.

ii) Due to skin effect the resistance decreases with $\sqrt{f}$ and inductance increases with f. ($\omega L >> R$)

iii) Leakage conductance $G = 0$, therefore $\omega_C >> G_1$

Transmission / Dissipation Less Line : (Definition)

Transmission line operates at Radio frequency ($R = G = 0$). Hence there is no dissapation. Such line called dissipation less line.

Skin Effect :

(*) 2m Current flows in outer surface of conductor at radio frequency is called as skin effect.

$f \downarrow \rightarrow \delta \uparrow es \rightarrow G = \uparrow es$

$f \uparrow \rightarrow \delta \downarrow es \rightarrow G = \downarrow es = 0.$

Skin Depth :

(*) 2m

$$\boxed{\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}}$$

Also called depth of penetration due to skin effect.

If frequency ↑es - depth of penetration decreases

If frequency ↓es - depth in...

μ = permiability

σ = conductivity

f = frequency.

Since the depth of penetration decreases, the ransmission line flows outside

Thus high freq, depth of penetration is very .mall, the current flows at outer surface f tr-n line, therefore no leakage of current (G = 0). Therefore for high frequency

$\boxed{R = G = 0}$. (*)

LINE CONSTANT OF TRANSMISSION LINE AT RADIO FREQUENCY (2° constants) :

$\alpha, \beta, v, \lambda, \gamma$

i. $$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{\cancel{j\omega} L}{\cancel{j\omega} C}}$$

$(R = G = 0)$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}} = R_0.}$$

ii. $$\gamma = \sqrt{ZY}$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$(R = G = 0)$

$$\gamma = \sqrt{(j\omega L)\,(j\omega C)}$$

$$\gamma = \sqrt{j^2\omega^2 LC}$$

$$\boxed{\gamma = j\omega\sqrt{LC}}$$

iii. $\gamma = \alpha + j\beta \Rightarrow$ when α=0 $\rightarrow$ then $\boxed{\gamma = j\beta}$

$$\boxed{\begin{aligned}\alpha &= 0\\ \beta &= \omega\sqrt{LC}\end{aligned}}$$

iv. $$v = \frac{\omega}{\beta} = \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{LC}}$$

$$\boxed{v = \frac{1}{\sqrt{LC}}}$$

$(\omega = 2\pi f)$

v. $$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}} = \frac{\cancel{2\pi}}{\cancel{2\pi} f\sqrt{LC}}$$

$$\boxed{\lambda = \frac{1}{f\sqrt{LC}}}$$

2

Voltage and Current Equation of Radio Frequency Transmission Line :

Generally transmission line,

$$V = V_R \cosh \gamma x + I_R Z_0 \sinh \gamma x$$
$$I = I_R \cosh \gamma x + \frac{V_R}{Z_0} \sinh \gamma x$$
Interms of cos, sin

$$V = \frac{V_R}{2Z_R} (Z_R + Z_0)[e^{\gamma x} + k e^{-\gamma x}]$$
$$I = \frac{I_R}{2Z_0} (Z_0 + Z_R)[e^{\gamma x} - k e^{-\gamma x}]$$
Interms of K

In high frequency tr^{-n} line, $R = G = 0$.

$0, 2^0$ const $\boxed{\begin{aligned} Z_0 &= R_0 \\ \gamma &= j\beta \end{aligned}}$

$$V = V_R \cosh j\beta x + I_R R_0 \sinh j\beta x.$$

$$I = I_R \cosh j\beta x + \frac{V_R}{R_0} \sinh j\beta x$$

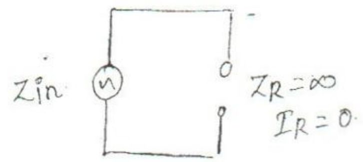
$$\cosh j\theta = \cos\theta$$
$$\sinh j\theta = j\sin\theta$$

$$\boxed{\begin{aligned} V &= V_R \cos\beta x + j I_R R_0 \sin\beta x \\ I &= I_R \cos\beta x + j \frac{V_R}{R_0} \sin\beta x \end{aligned}}$$

$$\boxed{\begin{aligned} V &= \frac{V_R}{2Z_R} (Z_R + R_0) [e^{j\beta x} + k e^{-j\beta x}] \\ I &= \frac{I_R}{2R_0} (Z_R + R_0) [e^{j\beta x} - k e^{-j\beta x}] \end{aligned}}$$

These are V and I equ for dissipation transmission line.

Case (i) Open circuit :



$$Z_{oc} = \frac{V_{oc}}{I_{oc}}$$

(*) $V \Rightarrow V_{oc}$ when $Z_R = \infty$, $I_R = 0$.

$$V_{oc} = V_R \cos\beta x + j\cancel{I_R}^{\,0} R_0 \sin\beta x$$

$$V_{oc} = V_R \cos\beta x$$

$$\cancel{V_{oc} = \frac{V_R}{2Z_R}(Z_R + R_0)\left[e^{j\beta x} + k e^{-j\beta x}\right]}$$

$$\left[\lambda = \frac{2\pi}{\beta} \Rightarrow \beta = 2\pi/\lambda\right]$$

$$\boxed{V_{oc} = V_R \cos\frac{2\pi x}{\lambda}}$$

) $I \Rightarrow I_{oc}$ when $Z_R = \infty$, $I_R = 0$.

$$I_{oc} = j\frac{V_R}{R_0}\sin\beta x$$

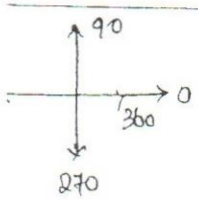
$$\boxed{I_{oc} = j\frac{V_R}{R_0}\sin\frac{2\pi x}{\lambda}}$$

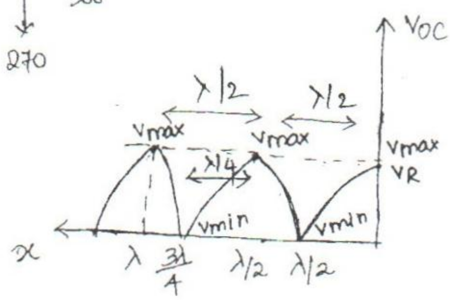
$$\Rightarrow Z_{oc} = \frac{V_{oc}}{I_{oc}} = \frac{\cancel{V_R}\cos 2\pi x/\lambda}{j\frac{\cancel{V_R}}{R_0}\sin\frac{2\pi x}{\lambda}}$$

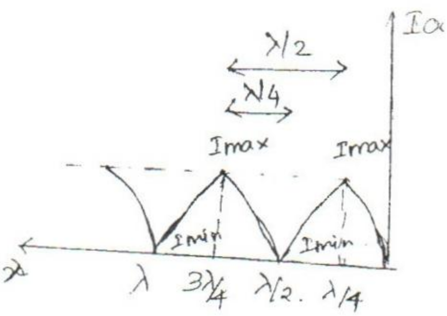
$$\boxed{Z_{oc} = -j\tan\frac{2\pi x}{\lambda} R_0}$$ $\left[\frac{\sin\theta}{\cos\theta} = \tan\theta\right]$

3

	distance x	$\beta x = \frac{2\pi}{\lambda} \times x$	$V_{oc} = V_R \cos \frac{2\pi x}{\lambda}$	$I_{oc} = \frac{jV_R}{R_0} \sin \frac{2\pi x}{\lambda}$
0	0	$\frac{2\pi}{\lambda} \times 0 = 0$ (0°)	V_R	0
$\lambda/4$	$\lambda/4$	$\frac{2\pi}{\lambda} \times \lambda/4 = \pi/2$ (90°)	0	jV_R/R_0
$2\lambda/\cancel{4}2$	$\lambda/2$	$\frac{2\pi}{\lambda} \times \lambda/2 = \pi$ (180°)	$-V_R$	0
$3\lambda/4$	$3\lambda/4$	$\frac{2\pi}{\lambda} \times 3\lambda/4 = \frac{3\pi}{2}$ (270°)	0	$-jV_R/R_0$
$4\lambda/4$	λ	$\frac{2\pi}{\lambda} \times \lambda = 2\pi$ (360°)	V_R	0





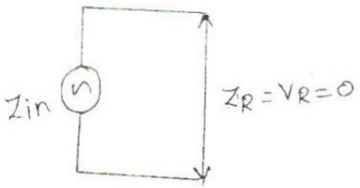


∴ Voltage and current equations in open ckt
in lines are out of phase by $\pi/2$ (ie) 90°

The distance between V_{max} and V_{min} is $\lambda/4$ and
also I_{max} and I_{min} is $\lambda/4$

Successive distance btwn V_{max} and I_{max} is $\lambda/2$
Transmission line properties are repeated
for every $\lambda/2$ distance..

Case (ii) Short circuit line :



↳ $V = V_{sc}$; $Z_R = 0$; $V_R = 0$.

$$V = \cancel{V_R \cos\beta x}^{\,0} + jI_R R_0 \sin\beta x$$

$$V_{sc} = jI_R R_0 \sin\beta x$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\boxed{V_{sc} = jI_R R_0 \sin\frac{2\pi x}{\lambda}}$$

↳ $I = I_{sc}$; $Z_R = 0$; $V_R = 0$.

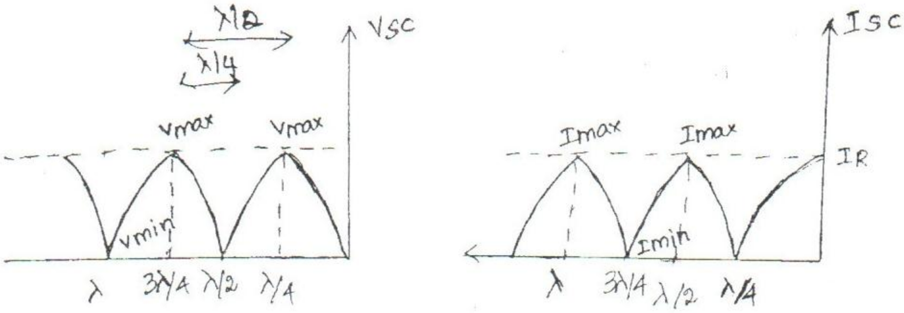
$$I_{sc} = I_R \cos\beta x$$

$$\beta = \frac{2\pi}{\lambda}$$

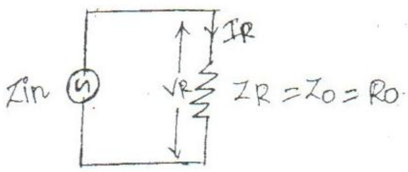
$$\boxed{I_{sc} = I_R \cos\frac{2\pi x}{\lambda}}$$

distance x	$\beta x = 2\pi/\lambda \times x$	V_{sc}	I_{sc}
0	$\frac{2\pi}{\lambda} \times 0 = 0\ (0^\circ)$	0	I_R
$\lambda/4$	$\frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \pi/2\ (90^\circ)$	$jI_R R_0$	0
$\lambda/2$	$\frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi\ (180^\circ)$	0	$-I_R$
$3\lambda/4$	$\frac{2\pi}{\lambda} \times \frac{3\lambda}{4} = \frac{3\pi}{2}\ (270^\circ)$	$-jI_R R_0$	0
λ	$\frac{2\pi}{\lambda} \times \lambda = 2\pi\ (360^\circ)$	0	I_R

4



ii) Matched load : $Z_R = Z_0$



$V_R = I_R R_0$

$V = V_R \cos\beta x + j I_R R_0 \sin\beta x$

$= V_R \cos\beta x + j V_R \sin\beta x$

$= V_R (\cos\beta x + j \sin\beta x)$

$V = V_R (e^{j\beta x})$

$V = V_R \angle \beta x$

$\boxed{|V| = V_R}$

$I = I_R \cos\beta x + j \frac{V_R}{R_0} \sin\beta x$

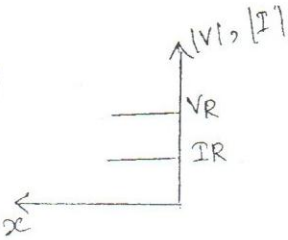
$I_R = V_R / R_0$

$I = I_R (\cos\beta x + j \sin\beta x)$

$= I_R (e^{j\beta x})$

$= I_R \angle \beta x$

$\boxed{|I| = I_R}$



Conclusion:-

At matched load condition ($Z_R = Z_0$) current & Vtg of txlon line maintained as constant as smooth line

Input Impedance of High Frequency Transmission line

$$Z_S \text{ (or) } Z_{in} = \frac{V_S}{I_S}$$

$$V = V_R \cos\beta x + j I_R R_0 \sin\beta x$$

$$I = I_R \cos\beta x + j \frac{V_R}{R_0} \sin\beta x$$

$$Z_S = \frac{V_S}{I_S} = \frac{V_R \cos\beta x + j I_R R_0 \sin\beta x}{I_R \cos\beta x + j \frac{V_R}{R_0} \sin\beta x}$$

$$Z_S = \frac{I_R Z_R \cos\beta x + j I_R R_0 \sin\beta x}{I_R \cos\beta x + j \frac{I_R Z_R}{R_0} \sin\beta x}$$

$$Z_S = \frac{\cancel{I_R} (Z_R \cos\beta x + j R_0 \sin\beta x)}{\cancel{I_R} \left(\cos\beta x + j \frac{Z_R}{R_0} \sin\beta x\right)}$$

$$\boxed{Z_S = \frac{R_0 (Z_R \cos\beta x + j R_0 \sin\beta x)}{(R_0 \cos\beta x + j Z_R \sin\beta x)}}$$

$$Z_S = \frac{V_S}{I_S}$$

$$V = \frac{V_R}{2Z_R} (Z_R + R_0) \left(e^{j\beta x} + K e^{-j\beta x}\right)$$

$$V = \frac{V_R}{2Z_R} (Z_R + R_0)\, e^{j\beta x} \left[1 + K \frac{e^{-j\beta x}}{e^{j\beta x}}\right]$$

$$V = \frac{V_R}{2Z_R} (Z_R + R_0)\, e^{j\beta x} \left[1 + K e^{-j2\beta x}\right]$$

$$e^{j\theta} = \angle\theta$$

$$e^{-2j\beta x} = \angle -2\beta x$$

$$K = |K|\angle\phi$$

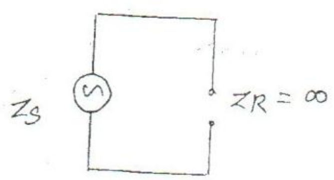
$$V = \frac{V_R}{2Z_R}(Z_R + R_0)\, e^{j\beta x}\left(1 + |K|\angle\phi \angle -2\beta x\right)$$

$$V = \frac{V_R}{2Z_R}(Z_R + R_0)\, e^{j\beta x}\left(1 + |K|\angle\phi - 2\beta x\right)$$

lly $$I = \frac{I_R}{2R_0}(Z_R + R_0)\, e^{j\beta x}\left(1 - |K|\angle\phi - 2\beta x\right)$$

$$\boxed{Z_{in} = \frac{V_S}{I_S} = R_0\left[\frac{1 + |K|\angle\phi - 2\beta x}{1 - |K|\angle\phi - 2\beta x}\right]}$$

Case (i) Open ckt input impedance of high frequency tx-n line :



$$Z_{oc} = \frac{R_0(Z_R\cos\beta x + jR_0\sin\beta x)}{(R_0\cos\beta x + jZ_R\sin\beta x)}$$

$$Z_{oc} = \frac{\cancel{Z_R}\left(R_0\cos\beta x + j\,R_0^2/Z_R\,\sin\beta x\right)}{\cancel{Z_R}\left(R_0/Z_R\,\cos\beta x + j\sin\beta x\right)}$$

$$Z_R = \infty$$

$$Z_{oc} = \frac{R_0\cos\beta x}{j\sin\beta x} \Rightarrow \boxed{Z_{oc} = -jR_0\cot\beta x}$$

$$\boxed{Z_{oc} = \frac{-jR_0}{\tan\beta x}}$$

Input Impedance of open ckt Dissipation less lines having only reactance (L (or) C)

→ If the reactance is +ve → inductance

→ If the reactance is −ve → capacitance

$-jR_0/\tan\beta x$ 0 → series

∞ → parallel.

Distance x	$\beta x = \frac{2\pi}{\lambda} \times x$	$\tan\beta x$	$Z_{oc} = -jR_0/\tan\beta x$	Z_{oc} polarity	Elements
0	0 (0°)	0	$+\infty$	∞	||el Resonance
$\lambda/6$	$\frac{2\pi}{\lambda} \times \frac{\lambda}{6} = \frac{\pi}{3}$ (60°)	1.732	$-j0.577R_0$	−ve	capacitance
$\lambda/4$	$\frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \pi/2$ (90°)	∞	0	0	~~Inductance~~ Series resonance
~~$2\lambda/6$~~ $\lambda/3$	$\frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3}$ (120°)	−1.732	$+j0.577R_0$	+ve	Inductance
$\lambda/2$	$\frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi$ (180°)	0	$-\infty$	∞	||el Resonance
~~$3\lambda/6$~~ $2\lambda/3$	$\frac{2\pi}{\lambda} \times \frac{2\lambda}{3} = \frac{4\pi}{3}$ (240°)	1.7	$-jR_0$ 0.5	−ve	Capacitance
$3\lambda/4$	$\frac{2\pi}{\lambda} \times \frac{3\lambda}{4} = \frac{3\pi}{2}$ (270°)	∞	0	0	Series resonance
~~$5\lambda/6$~~ $5\lambda/6$	$\frac{2\pi}{\lambda} \times \frac{5\lambda}{6} = \frac{5\pi}{3}$ (300)	−1.7	$+jR_0$ 0.5	+ve	Inductance
λ	$\frac{2\pi}{\lambda} \times \lambda = 2\pi$ (360)	0	$+\infty$	∞	||el resonance

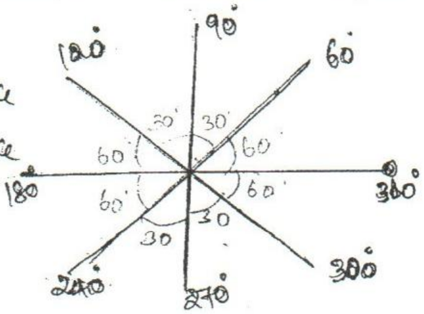
Polarity

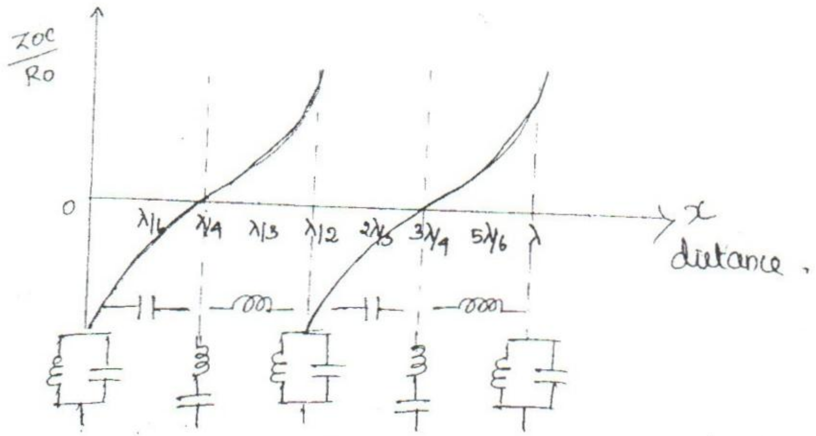
0 → Series Resonance

∞ → \|\|el Resonance

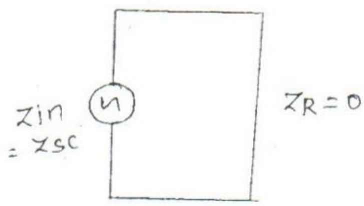
+ve → Inductance

−ve → Capacitance





case(ii) short ckt input impedance :



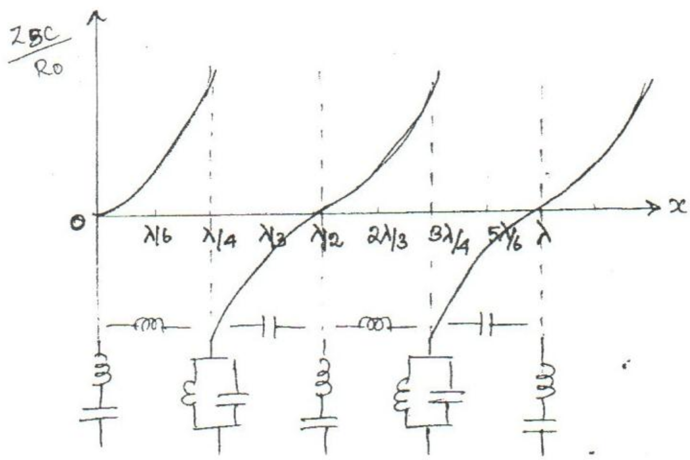
$$Z_{sc} = \frac{R_0 (Z_R \cos\beta x + j R_0 \sin\beta x)}{(R_0 \cos\beta x + j Z_R \sin\beta x)} \qquad Z_R = 0$$

$$Z_{sc} = \frac{\cancel{R_0} (j R_0 \sin\beta x)}{\cancel{R_0} \cos\beta x}$$

$$\boxed{Z_{sc} = j R_0 \tan\beta x}$$

(iii)

x	βx	$\tan\beta x$	$Z_{sc} = jR_o \tan\beta x$	Polarity	Nature of elements
0	0 (0°)	0	0	0	series resonance
$\lambda/6$	$\pi/3$ (60)	1.732	jR_o 1.732	+ve	Inductance
$\lambda/4$	$\pi/2$ (90)	∞	∞	∞	parallel Resonance
$\lambda/3$	$2\pi/3$ (120)	-1.732	$-jR_o$ 1.732	-ve	capacitance
$\lambda/2$	π (180°)	0	0	0	Series resonance
$2\lambda/3$	$\frac{4\pi}{3}$ (240°)	1.7	jR_o 1.7	+ve	Inductance
$3\lambda/4$	$\frac{3\pi}{2}$ (270)	∞	∞	∞	parallel resonance
$5\lambda/6$	$5\pi/3$ (300)	-1.7	$-jR_o$ 1.7	-ve	Capacitance
λ	2π (360°)	0	0.	0	Series resonance.



Applications of S.C and O.C input imp line :

1. Depends on distance x and wavelength λ open / short circuited line behaves like L, C, series and parallel resonance.

2. Depends on length x it is used for single and double stub matching.

3. Deppends on 'λ' it is used as bandstop and band pass filter (at μ-wave freq).

---

tanding waves :

When the line is not terminated with R_o. then ncident wave and reflected wave will have ame magnitude and travels in different irections. This type of wave is called as standing wave. Because of standing wave max and Vmin occurs

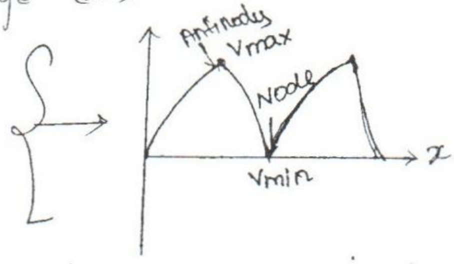
odes :

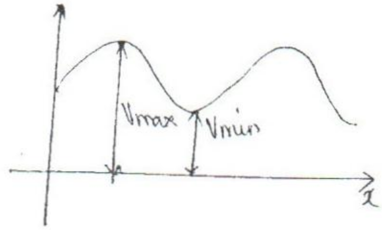
The point along the line, the magnitude of ltage (or) current is zero. called as nodes.

tinodes :

The point along the line, the magnitude tage (or) current is maximum.

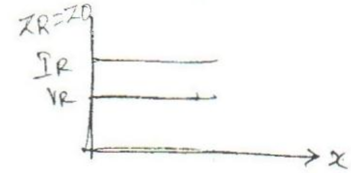
ing
es





. Smooth Line :

when the line is terminated with Z_0, then there is no standing waves called smooth line



Standing waves Ratio (SWR) :

Ratio of maximum voltage to minimum voltage

* Voltag SWR (S) = $\frac{V_{max}}{V_{min}}$
VSWR (S)

$ISWR = \frac{I_{max}}{I_{min}}$

* $V_{max} \rightarrow V_i$ and V_r inphase (0°)

$V_{max} = |V_i| + |V_r|$

* $V_{min} \rightarrow V_i$ and V_r out of phase (180° (or) π)

$V_{min} = |V_i| - |V_r|$

Standing wave Ratio is ratio of max vtg to min vtg

$$S = \frac{|V_i| + |V_r|}{|V_i| - |V_r|}$$

Relation btwn S and K :

$$S = \frac{\cancel{|V_i|}\left(1 + \frac{|V_r|}{|V_i|}\right)}{\cancel{|V_i|}\left(1 - \frac{|V_r|}{|V_i|}\right)}$$

Since $K = \frac{V_r}{V_i}$

$$\boxed{S = \frac{1+|K|}{1-|K|}}$$ $\rightarrow$ S interms of K

8

$$S(1-|K|) = 1+|K|$$

$$S - S|K| = 1+|K|$$

$$S-1 = S|K| + |K|$$

$$S-1 = (S+1)|K|$$

$$\boxed{|K| = \frac{S-1}{S+1}} \longrightarrow \text{K intorms}$$

$$|K| = \frac{\frac{V_{max}}{V_{min}} - 1}{\frac{V_{max}}{V_{min}} + 1} \Rightarrow \frac{\frac{V_{max} - V_{min}}{\cancel{V_{min}}}}{\frac{V_{max} + V_{min}}{\cancel{V_{min}}}}$$

$$\boxed{|K| = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}}$$

ase (i) :

matched load $(Z_0 = Z_R)$

$$K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$\boxed{|K| = 0}$$

$$S = \frac{1+|K|}{1-|K|} \Rightarrow \boxed{S = 1}$$

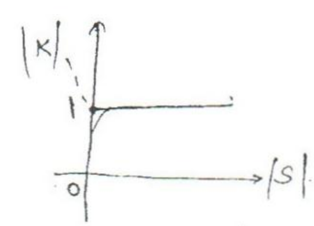
se (ii) :

mismatched load $(Z_R = \infty,\ Z_R = 0$ for OC & SC$)$

$$\boxed{K = 1}$$

$$S = \frac{1+|K|}{1-|K|} \Rightarrow \frac{1+1}{1-1}$$

$$\boxed{S = \infty}$$



ution:-

matched cond → K = 0 , S = 1 or   ② At mismatched → K=1 → S=↑

MEASUREMENT OF VSWR:—

W.K.T $Z_{in} = R_0 \left[\frac{1+|K| \angle \phi - 2\beta x}{1-|K| \angle \phi - 2\beta x} \right]$

Case(i) V_{max} occurs when V_r & V_i are inphase (0°)

$\angle \phi - 2\beta x = \angle 0^\circ$

$\angle \theta = e^{j\theta} = \cos\theta + j\sin\theta$

$= \cos 0 + j\sin 0 = 1$

$\boxed{\therefore \angle 0^\circ = 1}$

$Z_{in} = R_0 \left[\frac{1+|K|}{1-|K|} \right] \Rightarrow R_0 S$

$\boxed{Z_{in} = R_0 \times S}$

Case(ii) V_{min} occurs when V_r & V_i are out of phase (π or 180°)

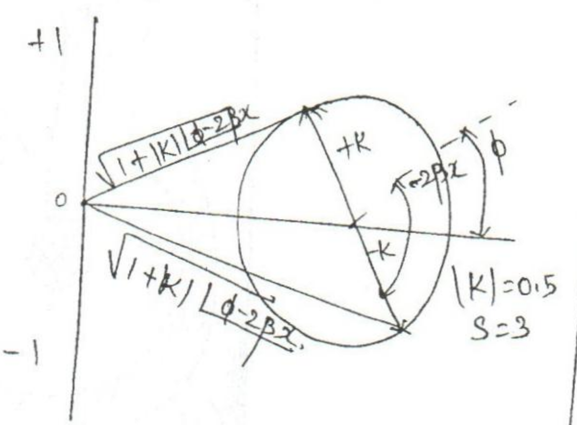
$\angle \phi - 2\beta x = \angle \pi$

$\cos\pi + j\sin\pi = -1$

$\boxed{\angle \pi = -1}$

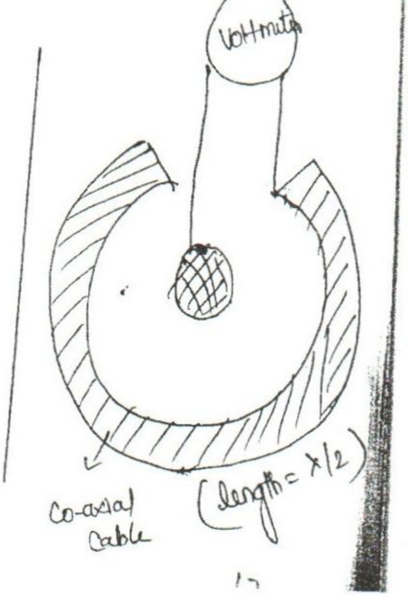
$Z_{in} = R_0 \left[\frac{1+|K)}{1+|K|} \right]$

$\boxed{Z_{in} = \frac{R_0}{S}}$



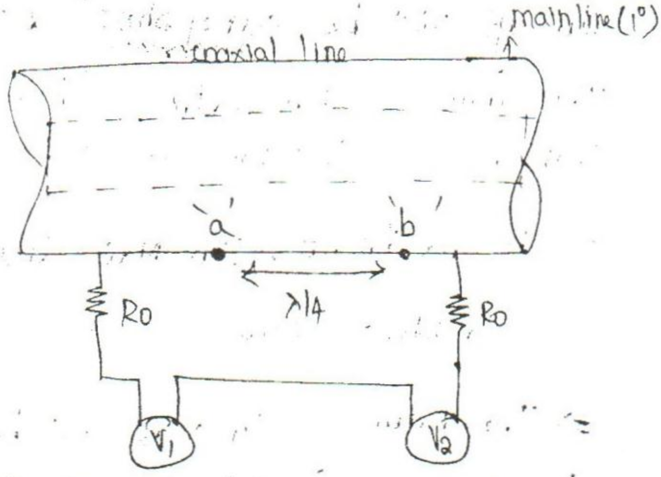
METHOD: 1

LECHER MEASUREMENT



(9)

DIRECTIONAL COUPLER :-



- Consider a coaxial txion line with two hole 'a' and 'b' in the outer sheath spaced by λ/4.
- To prevent reflection, R_0 is clamped over the holes. Some Energy will leak through the holes & travel as a wave from left to right in the Second line.

↳ 1° line txing wave to the right direction

case(i) :-

If in 2° line wave through hole 'a' travelling in right direction will be inphase with 1° line wave & it will re-inforce a wave entering at hole at 'b', which setting a wave travelling to right in the 2° line, which gives indication to V_2 **not** at V_1.

Case(ii):-

If in 2° line, wave through hole 'a' travelling to left will be out of phase & it will cancel the wave which enter at 'b' which gives Indication at V_1 not at V_2.

⇒ As a Result, In Main line wave to right is Incident wave & left is reflected wave.

⇒ The ratio of V_1 & V_2 will be the ratio of Incident & Reflected Wave.

METHOD-1 Lecher Measurement

Consider co-axial cable with $\lambda/2$ length, a u probe is inserted into air (dielectric of line. a voltmeter (or) detector connected between probe and ~~ta~~ transmission line.

If the meter measures linear value, then S is determined.

If the meter measures non-linear value, 'S' corrected and determined.

The same equipement and techniques are used to measure the wavelength, 'λ' of line

This measurement of open wire line wire system is called as **LECHER MEASUREMENT**

NOTE : $V(-x)$ Formulas

$$V = V_R \cos\beta x + j I_R R_0 \sin\beta x$$

$$I = I_R \cos\beta x + j \frac{V_R}{R_0} \sin\beta x$$

$$V = \frac{V_R}{2Z_R} (Z_0 + Z_R)\, e^{j\beta x} \left(1 + |K| \angle \phi - 2\beta x\right)$$

$$I = \frac{I_R}{2R_0} (Z_0 + Z_R)\, e^{j\beta x} \left(1 - |K| \angle \phi - 2\beta x\right)$$

$$Z_{in} = R_0 \left[\frac{Z_R \cos\beta x + j R_0 \sin\beta x}{R_0 \cos\beta x + j Z_R \sin\beta x} \right]$$

$$Z_{in} = R_0 \left[\frac{1 + |K| \angle \phi - 2\beta x}{1 - |K| \angle \phi - 2\beta x} \right]$$

Impedance and Power Measurement of High Frequency Transmission Line.

etermination of characteristic impedance ($Z_0 = R_0$):

$$V = \frac{V_R}{2Z_R}(Z_0 + Z_R)\, e^{j\beta x}\left(1 + |K| \angle \phi - 2\beta x\right)$$

$$I = \frac{I_R}{2R_0}(Z_R + Z_0)\, e^{j\beta x}\left(1 - |K| \angle \phi - 2\beta x\right)$$

$\Rightarrow V_{max} \rightarrow V_i$ & V_r are inphase

(ie) $0^\circ \Rightarrow \angle \phi - 2\beta x = 1$

$$V_{max} = \frac{V_R}{2Z_R}(Z_0 + Z_R)\, e^{j\beta x}(1 + |K|)$$

$I_{max} \rightarrow I_i$ & I_r are inphase

(ie) $0^\circ \Rightarrow \angle \phi - 2\beta x = 1$

$$\text{ax} = \frac{I_R}{2R_0}(Z_R + Z_0)\, e^{j\beta x}(1 - |K|)$$

$I_R Z_R$

$$= \frac{V_{max}}{I_{max}} = \frac{V_R / \cancel{2Z_R}\,(Z_0 + Z_R)\, e^{j\beta x}(1 + |K|)}{\cancel{I_R} / \cancel{2}R_0\,(Z_R + Z_0)\, e^{j\beta x}(1 - |K|)}$$

S

$$R_{max} = R_0 S$$

$$\boxed{R_0 = \frac{R_{max}}{S}}$$

Conclusion :

When incident wave and reflected wave are inphase then standing wave ratio decrea with respect to R_0

Similarly :

$V \rightarrow V_{min} \rightarrow V_i$ & V_r are out of phase

$$\underline{|\phi - 2\beta x} = \underline{|\pi} = -1$$

$$V_{min} = \frac{V_R}{2Z_R}(Z_0 + Z_R)\, e^{j\beta x}(1 - |K|)$$

$I \rightarrow I_{min} \rightarrow I_i$ & I_r are out of phase

$$\underline{|\phi - 2\beta x} = \underline{|\pi} = -1$$

$$I_{min} = \frac{I_R}{2R_0}(Z_R + Z_0)\, e^{j\beta x}(1 + |K|)$$

$$R_{min} = \frac{V_{min}}{I_{min}} = \frac{V_R/\cancel{2}\cancel{Z_R}\,(Z_0 + \cancel{Z_R})\, e^{\cancel{j\beta x}}(1 - |K|)}{\cancel{I_R}/\cancel{2}R_0\,(Z_0 + \cancel{Z_R})\, e^{\cancel{j\beta x}}(1 + |K|)}$$

$$R_{min} = \frac{R_0(1 - |K|)}{(1 + |K|)}$$

$$R_{min} = \frac{R_0}{S}$$

$$\boxed{R_0 = R_{min}\, S}$$

clusion :

When incident and reflected wave are phase, then standing wave ratio ncreases with respect to R_0.

Location of V_{max}, V_{min}, I_{max}, I_{min} :

* x_1 is the location where V_{min} and I_{max} will occur.

$$\phi - 2\beta x \Rightarrow \phi - 2\beta x_1 = 2n\pi$$

$$\boxed{x_1 = \frac{\phi - 2n\pi}{2\beta}} \qquad n = 0, 1, 2, \ldots$$

* x_2 is the location where V_{max} and I_{min} will occur.

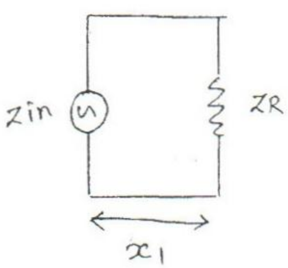
$$\phi - 2\beta x_2 = (2n+1)\pi$$

$$2\beta x_2 = \phi - (2n+1)\pi$$

$$\boxed{x_2 = \frac{\phi - (2n+1)\pi}{2\beta}}$$

Determination of load impedance (Z_R) :

(or) unknown load



$$Z_{in} = R_0 \left[\frac{Z_R \cos\beta x + j R_0 \sin\beta x}{R_0 \cos\beta x + j Z_R \sin\beta x} \right]$$

Taking $\cos\beta x$ outside.

$$Z_{in} = \frac{R_0 \cancel{\cos\beta x}(Z_R + jR_0 \tan\beta x)}{\cancel{\cos\beta x}(R_0 + jZ_R \tan\beta x)}$$ Since $\frac{\sin\beta x}{\cos\beta x} = \tan\beta x$

Replace $x = x_1$

$$(Z_{in})_1 = \frac{R_0 (Z_R + jR_0 \tan\beta x_1)}{(R_0 + jZ_R \tan\beta x_1)}$$

At x_1 V_{min} & I_{max} will occur.

$$\boxed{R_{min} = \frac{R_0}{S}}$$

$$(Z_{in})_1 = R_{min}$$

$$\frac{\cancel{R_0}}{S} = \cancel{R_0}\left[\frac{Z_R + jR_0 \tan\beta x_1}{R_0 + jZ_R \tan\beta x_1}\right]$$

$$R_0 + jZ_R \tan\beta x_1 = S(Z_R + jR_0 \tan\beta x_1)$$

$$jZ_R \tan\beta x_1 - SZ_R = SjR_0 \tan\beta x_1 - R_0$$

$$Z_R(j\tan\beta x_1 - S) = R_0(Sj\tan\beta x_1 - 1)$$

$$Z_R(S - j\tan\beta x_1) = R_0(1 - j\tan\beta x_1 S)$$

$$\boxed{Z_R = \frac{R_0(1 - jS\tan\beta x_1)}{(S - j\tan\beta x_1)}}$$

Determination of Power (P):

$$P = \frac{V_{max}^2}{R_{max}} \quad ; \quad P = \frac{V_{min}^2}{R_{min}}.$$

$$P \times P = \frac{V_{max}^2}{R_{max}} \times \frac{V_{min}^2}{R_{min}}$$

$$P^2 = \frac{V_{max}^2}{R_o \cancel{S}} \times \frac{V_{min}^2}{R_o/\cancel{S}} \qquad \left[\because R_{max} = R_o S, \; R_{min} = R_o/S\right]$$

$$P^2 = \frac{V_{max}^2 \, V_{min}^2}{R^2}$$

$$\boxed{P = \frac{V_{max} \, V_{min}}{R}}$$

Similarly,

$$P = I_{max}^2 \, R_{max} \quad ; \quad P = I_{min}^2 \, R_{min}$$

$$P \times P = (I_{max}^2 \, R_{max}) \, (I_{min}^2 \, R_{min})$$

$$P^2 = (I_{max}^2 \times R_o \cancel{S}) \, (I_{min}^2 \times \frac{R_o}{\cancel{S}})$$

$$P^2 = I_{max}^2 \, I_{min}^2 \, R_o^2$$

$$\boxed{P = I_{max} \, I_{min} \, R_o}$$

6

Formulas

1) $V = V_R \cos\beta x + j I_R R_0 \sin\beta x$

$I = I_R \cos\beta x + j \dfrac{V_R}{R_0} \sin\beta x$

$V = \dfrac{V_R}{2Z_R}(Z_R + R_0) e^{j\beta x}\left(1 + |K| \angle \phi - 2\beta x\right)$

$I = \dfrac{I_R}{2R_0}(Z_R + R_0) e^{j\beta x}\left(1 - |K| \angle \phi - 2\beta x\right)$

2) $Z_{in} = R_0\left[\dfrac{Z_R \cos\beta x + j R_0 \sin\beta x}{R_0 \cos\beta x + j Z_R \sin\beta x}\right]$

$Z_{in} = R_0\left[\dfrac{1 + |K| \angle \phi - 2\beta x}{1 - |K| \angle \phi - 2\beta x}\right]$

3) Rf. coeff (K):—

$K = \dfrac{Z_R - Z_0}{Z_R + Z_0}$

$|K| = \dfrac{S-1}{S+1}$

$|K| = \dfrac{|V_{max}| - |V_{min}|}{|V_{max}| + |V_{min}|}$

$|K| = \dfrac{V_r}{V_i}$

4) Standing wav[e] Ratio (SWR)(S

$S = \dfrac{1+|K|}{1-|K|}$

$S = \dfrac{|V_{max}|}{|V_{min}|}$

$S = \dfrac{|V_i| + |V_r|}{|V_i| - |V_r|}$

$V_{max} = |V_i| + |V_r|$

$V_{min} = |V_i| - |V_r|$

5) V_{max} occur when V_i & V_r inphase (0°)

$$\boxed{\phi - 2\beta x_1 = 0^\circ}$$ → (Answer is in -ve then add $\lambda/2$)

6) V_{min} occurs when V_i & V_r out of phase (π (or) 180°)

$$\boxed{\phi - 2\beta x_2 = \pi}$$ → (Answer is in -ve then add $\lambda/2$)

7) Location of V_{min} & I_{max}

$$\phi - 2\beta x_1 = 2n\pi$$

$$\boxed{x_1 = \frac{\phi - 2n\pi}{2\beta}}$$

8) Location of V_{max} & I_{min}.

$$\phi - 2\beta x_2 = (2n+1)\pi$$

$$\boxed{x_2 = \frac{\phi - (2n+1)\pi}{2\beta}}$$

(or)

$$\boxed{x_2 = x_1 + \frac{\lambda}{4}}$$

9) $\beta = \frac{2\pi}{\lambda}$

10) $\lambda = \frac{c}{f}$ (or) $\frac{v}{f}$ $\boxed{c = 3\times10^8 m}$

70 Ω line terminated by 115 − j80 Ω with the wavelength of 2.5 m. Calculate Reflection coefficient, standing wave ratio, maximum line impedance, minimum line impedance, location of Emax and Emin

Given data :

$Z_0 = 70\,\Omega$

$Z_R = 115 - j80\,\Omega$

$\lambda = 2.5\,m$

To find :

K, SWR (S), Rmax, Rmin

Sol :

(i)
$$\boxed{K = \frac{Z_R - Z_0}{Z_R + Z_0}}$$

$$\boxed{K = 0.362 - 0.275j}$$

$$\boxed{K = 0.457 \angle -0.645 \text{ rad}}$$

$K = |K| \angle \phi$

$|K| \rightarrow$ Magnitude, $\phi \rightarrow$ phase

$|K| = 0.455$

$\phi = -0.65$

(ii) $\boxed{S = \frac{1+|K|}{1-|K|}}$

$$= \frac{1+0.455}{1-0.455}$$

$\boxed{S = 2.66}$

(iii) $R_{max} = R_0 \times S \quad (\because R_0 = Z_0)$

$\boxed{R_{max} = 186.3}$

(iv) $R_{min} = \frac{R_0}{S}$

$$= \frac{70}{2.66}$$

$\boxed{R_{min} = 26.31}$

(v) location of $E_{max} \rightarrow \phi - 2\beta x_1 = 0°$

$\phi = 2\beta x_1$

$\boxed{x_1 = \frac{\phi}{2\beta}}$

$\phi = -0.65$

$\beta = \frac{2\pi}{\lambda}$

$$x_1 = \frac{-0.65}{2 \times \frac{2\pi}{\lambda}}$$

$$= \frac{-0.65\lambda}{4\pi}$$

$$x_1 = -0.05\lambda + \frac{\lambda}{2} \qquad (\lambda = 2.5m)$$

(Since the value is negative, for every $\lambda/2$ distance values are repeated).

$$\boxed{x_1 = 1.125} m.$$

vi) Location of E_{min}

$$\phi - 2\beta x_2 = \pi$$

$$x_2 = \frac{\phi - \pi}{2\beta}$$

$$x_2 = -0.3\lambda + \frac{\lambda}{2}$$

$$= (-0.3 \times 2.5) + \frac{2.5}{2}$$

$$\boxed{x_2 = 0.5m.}$$

(2) given that $Z_0 = 50\,\Omega$, $Z_R = 140\,\Omega$, $P_R = 75$ mW, $f = 50$ MHz, Calculate K, S, s_1 & s_2 position of V_{max}, V_{min}, I_{max} & I_{min}.

Sol :

(i) $\boxed{K = \dfrac{Z_R - Z_0}{Z_R + Z_0}}$

$$= \frac{140 - 50}{140 + 50}$$

$\boxed{K = 0.473}$ $|K| = 0.473$, $\phi = 0°$

(ii) $$S = \frac{1 + |K|}{1 - |K|}$$

$$= \frac{1 + 0.473}{1 - 0.473}$$

$\boxed{S = 2.795}$

(iii) (V_{min}, I_{max}) will occur at x_1

$$\phi - 2\beta x_1 = 2n\pi$$

$$x_1 = \frac{\phi - \cancel{2n}\pi}{2\beta} \quad (n = 0)$$

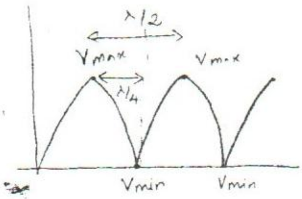
$\boxed{x_1 = 0}$ $(\because \phi = 0)$

(V_{max}, I_{min}) occurs at x_2

$$\phi - 2\beta x_2 = (2n+1)\pi$$

$$\boxed{x_2 = \frac{\phi - (2n+1)\pi}{2\beta}}$$ → (no need to use formula

$$x_2 = x_1 + \lambda/4$$



$$\boxed{x_2 = \lambda/4}$$

$$\lambda = c/f$$

$$= \frac{3 \times 10^8}{50}$$

$$\boxed{\lambda = 6000}$$

$$x_2 = \frac{6000}{4}$$

$$\boxed{x_2 = 1500\ m}$$

3) A 30 m long lossless Transmission line with Characteristic Impedance 50 Ω operating at 2 MHZ with terminating Impedance 60+j40 Ω Calculate K, SWR (S), i/p impedance if Velocity = 0.6 C .

Sol :

$$K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$= \frac{60 + j40 - 50}{60 + j40 + 50}$$

$$K = 0.197 + 0.291j$$

$$K = 0.3514 \angle 0.9756^\circ$$

$$\boxed{\begin{aligned} |K| &= 0.3514 \\ \phi &= 0.9756 \end{aligned}}$$

$$S = \frac{1 + |K|}{1 - |K|}$$

$$\boxed{S = 2.0835}$$

$$Z_{in} = \frac{R_0 \left[Z_R \cos\beta x + j R_0 \sin\beta x \right]}{\left[R_0 \cos\beta x + j Z_R \sin\beta x \right]}$$

$(R_0 = Z_0)$

$$\beta = \frac{2\pi}{\lambda}$$

$$\lambda = \frac{0.6c}{f} = \frac{0.6 \times 3 \times 10^8}{2 \times 10^6}$$

$$\boxed{\lambda = 90\,m}$$

$\beta = \frac{2\pi}{\lambda}$

Note: ($\cancel{\pi = 180^\circ}$ $\cancel{\text{calci degree mode}}$)

$\beta = \frac{2(\pi)}{90}$

$\boxed{\beta = 0.06}$ (rad mode)

$\beta x = \beta \times 30$

$= 4 \times 30$

$\boxed{\beta x = 120}$

$\cos \beta x = -0.5$

$\sin \beta x = 0.866$

$$Z_{in} = 50 \frac{[(60 + j40)(-0.5) + j\,50(0.866)]}{[50(-0.5) + j(60 + j40)(0.866)]}$$

$$= \frac{-1500 + 1165j}{-59.64 + 51.96j}$$

$$\boxed{\begin{aligned} Z_{in} &= 23.97 + 1.352j \\ Z_{in} &= 24.01 \angle 0.05 \end{aligned}}\ \text{rad}.$$

Note: (Calculations are in radian mode) and $\pi = 3.14$.

4. Characteristic Impedance of the line is 300 Ω, distance between successive voltage minimum is 15 cm and S = 3.33. calculate wavelength and load Impedance?

<u>given</u> :

$$Z_0 = 300\,\Omega$$

$$\lambda/2 = 15$$

$$S = 3.33$$

<u>Sol</u> :

(i) $\lambda/2 = 15$

$$\boxed{\lambda = 30 \text{ cm}}$$

(ii) $$K = \frac{S-1}{S+1}$$

$$\boxed{K = 0.538}$$

$$K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$(Z_R + Z_0)k = Z_R - Z_0$$

$$Z_R(K-1) = -Z_0 - Z_0 k.$$

$$Z_R = \frac{-Z_0(1+k)}{(K-1)}$$

$$Z_R = Z_0\left(\frac{1+k}{1-K}\right)$$

$$\boxed{Z_R = 998.76}\ \Omega.$$

UNIT-3

IMPEDANCE MATCHING IN HIGH FREQUENCY LINE

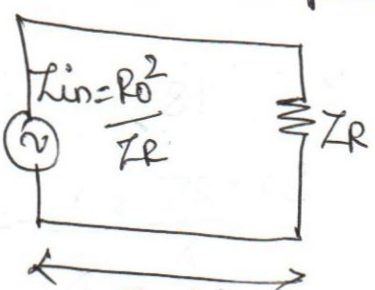
Impedance Matching:-

It is used to match the char. impedance of the txion line with load impedance ($Z_0 = Z_R$) in order to transfer max power to load and minimize the reflection loss.

Methods for impedance matching:

1) $\lambda/4$ (or) quarter wave line 2) $\lambda/2$ half wave line 3) $\lambda/8$ line
4) Single Stub matching 5) Double Stub matching.

1) $\lambda/4$ txion line (or) quater wave line:-



$x = \lambda/4$

i/p impedance $\boxed{Z_{in} = \dfrac{R_0(Z_R\cos\beta x + jR_0\sin\beta x)}{R_0\cos\beta x + jZ_R\sin\beta x}}$

let $x = \lambda/4$

$$\beta x = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$$

$$\cos\frac{\pi}{2} = 0 \; ; \; \sin\frac{\pi}{2} = 1$$

$$Z_{in} = \frac{R_0(0 + jR_0)}{(0 + jR_0)} = \frac{\cancel{j}R_0^2}{\cancel{j}Z_R}$$

$$\boxed{(Z_{in})_{\lambda/4} = \frac{R_0^2}{Z_R}}$$

$$\boxed{R_0 = \sqrt{Z_{in} \cdot Z_R}}$$

Applications:- 1) $\lambda/4$ line used as transformer for impedance matching

$$R_0 = \sqrt{Z_{in} Z_R}.$$

2) quater wave line can transform low impedance into high impedance and vice versa. So it can be used as Impedance Inverter.

3) used as an Insulators.

2) $\frac{\lambda}{2}$ line (or) Half wave line :-

$$Z_{in} = R_0 \left[\frac{Z_R \cos\beta x + j R_0 \sin\beta x}{R_0 \cos\beta x + j Z_R \sin\beta x} \right]$$

$x = \lambda/2$

$\beta x = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi$

$\cos\pi = -1$

$\sin\pi = 0$

$$Z_{in} = \cancel{R_0} \left[\frac{-Z_R}{-\cancel{R_0}} \right]$$

$$\boxed{Z_{in} = Z_R.}$$

Application:

1:1 matching

3) $\frac{\lambda}{8}$ line :-

$x = \lambda/8$, $\beta = \frac{2\pi}{\lambda}$

$\beta x = \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = \frac{\pi}{4}$

$\cos\pi/4 = 0.707$

$\sin\pi/4 = 0.707$

$$Z_{in} = R_0 \left[\frac{Z_R \cos\beta x + j R_0 \sin\beta x}{R_0 \cos\beta x + j Z_R \sin\beta x} \right]$$

$$= R_0 \left[\frac{Z_R(0.707) + j0.707 R_0}{0.707 R_0 + j0.707 Z_R} \right]$$

$$= R_0 \frac{\cancel{0.707} (Z_R + jR_0)}{\cancel{0.707} (R_0 + jZ_R)}$$

$$|Z_{in}| = R_0 \frac{\sqrt{Z_R^2 + \cancel{R_0^2}}}{\sqrt{R_0^2 + \cancel{Z_R^2}}}$$

$$\boxed{|Z_{in}| = R_0}$$

Applications → magnitude matches between Z_{in} & Z_0

STUB MATCHING :-

Txion line of finite length with either open or Short circuited at one end is called stub.

To deliver max power to load

↓

$\boxed{Z_R = Z_0}$ ie) No reflection on smooth line.

↓

Practically not possible to provide a loads with such antennas equal to R_0.

↓

It is necessary to add some impedance matching section b/n line & load.

Two ways are possible.

1) Quater wave line

2) Short/open ckted Stub with Suitable length

ie) STUB MATCHING.

Single Stub

Double Stub.

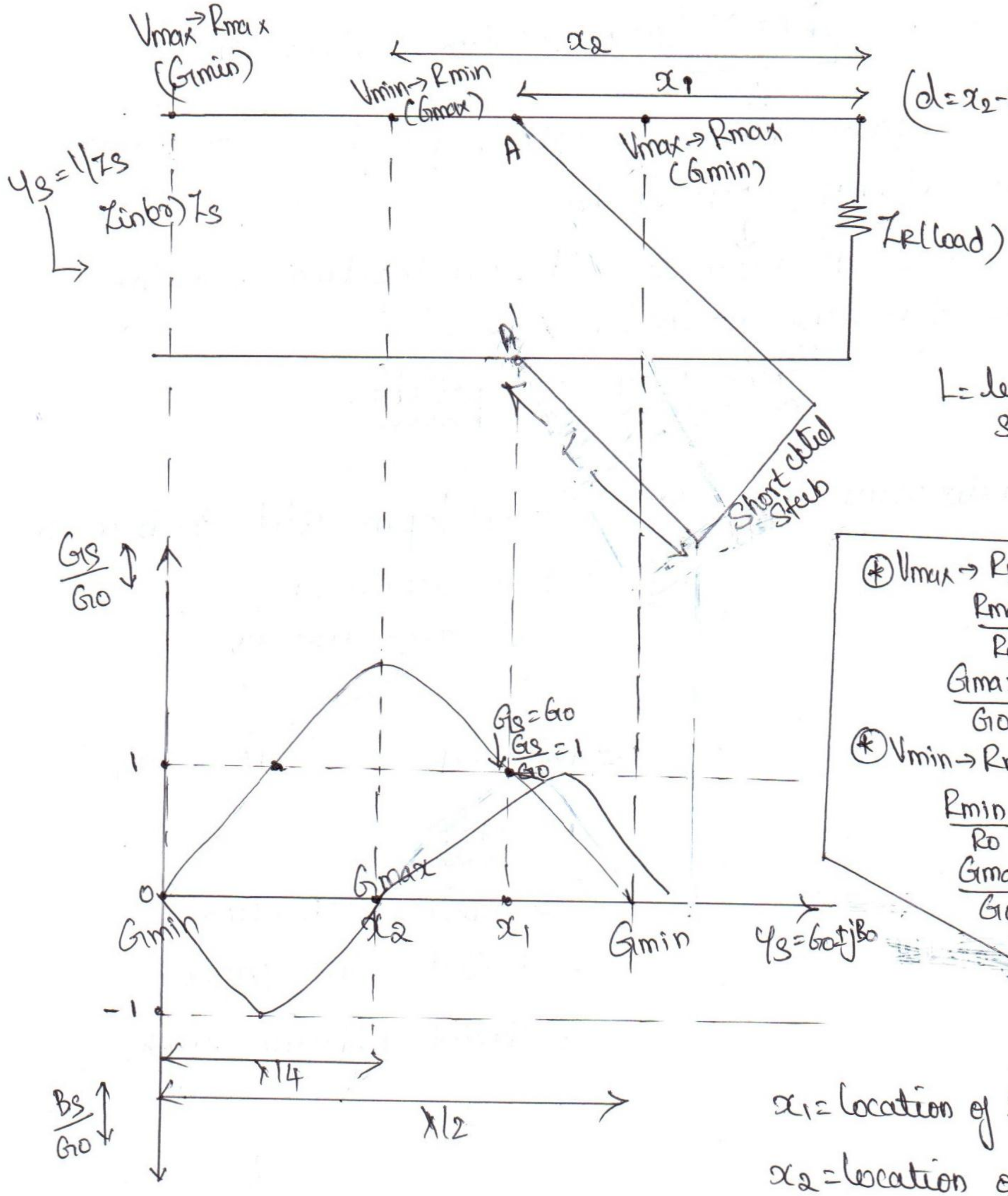
Advantages :-

1) Simpler construction
2) Radiation less power
3) Varied Effective length.

16m

SINGLE STUB IMPEDANCE MATCHING :—

Single Short (or) open ckted txion line with suitable length llel with txion line at particular distance from load.



* $V_{max} \to R_{max} = R_0 S$

$\frac{R_{max}}{R_0} = S$ (or)

$\frac{G_{max}}{G_0} = \frac{1}{S}$

* $V_{min} \to R_{min} = \frac{R_0}{S}$

$\frac{R_{min}}{R_0} = \frac{1}{S}$ (or)

$\frac{G_{max}}{G_0} = S$

x_1 = Location of Stub

x_2 = Location of V_{min}

∴ Stub is connected in ||el with txion line, it is convenient to use admittance (Y_s).

↳ Admittance at point 'A' before stub connected

$$\boxed{Y_s = G_0 \pm jB_0}$$

G = Conductance

B = Succeptance

↳ at point 'A' stub connected, so i/p impedance of stub is $\boxed{\mp jB_0}$

∴ Total admittance $Y_s = G_0 \pm j\cancel{B_0} \mp j\cancel{B_0}$

$$\boxed{\begin{array}{c} Y_s = G_0 \\ \text{(or)} \\ Z_s = R_0 = Z_0. \end{array}}$$

Conclusion:

1) R_0 is terminated from source to point 'A'
2) Reflection & SWR occurs in b/n 'A' and Z_R (load).
3) losses can be reduced by tring distance b/n 'A' and load(Z_R).

It is necessary to know,

↳ Exact point where stub is to be connect

↳ length of stub (L).

Two Measurement needed for this,

↳ 1) SWR

↳ 2) Vmin nearest to load.

I/p impedance is Z_{in} (or) $Z_s = R_0 \left[\dfrac{1+|K|\angle\phi-2\beta x}{1-|K|\angle\phi-2\beta x}\right]$

$$Y_s = \frac{1}{Z_s} = \frac{1}{R_0}\left[\frac{1-|K|\angle\phi-2\beta x}{1+|K|\angle\phi-2\beta x}\right]$$

$$Y_s = G_0\left[\frac{1-|K|\angle\phi-2\beta x}{1+|K|\angle\phi-2\beta x}\right]$$

For Normalization $\dfrac{Y_s}{G_0} = \dfrac{1-|K|\angle\phi-2\beta x}{1+|K|\angle\phi-2\beta x}$

Sub $y_s = G_s \pm jB_s$ $\angle \phi - 2\beta x = \angle \theta = e^{j\theta} = \cos\theta + j\sin\theta$

$$\frac{G_s + jB_s}{G_o} = \frac{1 - |K|(\cos\theta + j\sin\theta)}{1 + |K|(\cos\theta + j\sin\theta)}$$

$$= \frac{1 - |K|\cos\theta - j|K|\sin\theta}{1 + |K|\cos\theta + j|K|\sin\theta}$$

Take complex conjugate

$$= \frac{(1 - |K|\cos\theta) - (j|K|\sin\theta)}{(1 + |K|\cos\theta) + (j|K|\sin\theta)} \times \frac{(1 + |K|\cos\theta) - (j|K|\sin\theta)}{(1 + |K|\cos\theta) - (j|K|\sin\theta)}$$

$$= \frac{(1 - |K|\cos\theta)(1 + |K|\cos\theta) - (j|K|\sin\theta)(1 - |K|\cos\theta) \\ (-j|K|\sin\theta)(1 + |K|\cos\theta) + (j|K|\sin\theta)^2}{(1 + |K|\cos\theta)^2 - (j|K|\sin\theta)^2}$$

$$= \frac{1^2 - \underbrace{|K|^2\cos^2\theta} - j|K|\sin\theta + \cancel{j|K|^2\sin\theta\cos\theta} - j|K|\sin\theta \\ - \cancel{j|K|^2\sin\theta\cos\theta} - \underbrace{|K|^2\sin^2\theta}.}{1 + \underbrace{|K|^2\cos^2\theta} + 2|K|\cos\theta + \underbrace{|K|^2\sin^2\theta}}$$

$$\frac{G_s + jB_s}{G_o} = \frac{1 - |K|^2 - 2j|K|\sin\theta}{1 + |K|^2 + 2|K|\cos\theta}$$

$$\boxed{\frac{G_s}{G_o} + \frac{jB_s}{G_o} = \frac{1 - |K|^2}{1 + |K|^2 + 2|K|\cos\theta} - \frac{2j|K|\sin\theta}{1 + |K|^2 + 2|K|\cos\theta}} \longrightarrow (A)$$

Equate real & imag.

$$\frac{G_s}{G_o} = \frac{1 - |K|^2}{1 + |K|^2 + 2|K|\cos\theta} \longrightarrow (1) \qquad \frac{B_s}{G_o} = \frac{-2|K|\sin\theta}{1 + |K|^2 + 2|K|\cos\theta}. \longrightarrow (2)$$

To find x_2:— (Location of V_{min})

$$\theta = \phi - 2\beta x_2 = -\pi$$

$$\boxed{x_2 = \frac{\phi + \pi}{2\beta}}$$

[-Ve sign because $\cos\theta = -ve$
$\cos(-\pi) = -1$]

Eqn ①

$$\left.\frac{G_S}{G_O}\right|_{at\ x = x_2} = \frac{1-|K|^2}{1+|K|^2+2|K|(-1)}$$

$$= \frac{1-|K|^2}{1+|K|^2-2|K|} = \frac{1-|K|^2}{(1-|K|)^2} = \frac{(1+|K|)\cancel{(1-|K|)}}{(1-|K|)^{\cancel{2}}}$$

$$= \frac{1+|K|}{1-|K|} = S$$

$$\frac{G_S}{G_O} = S$$

$$G_S = G_O \times S = \frac{S}{R_O} \Rightarrow \frac{1}{R_{min}}$$

$$\boxed{\therefore R_{min} = \frac{R_O}{S}} \Rightarrow \text{(Location of } V_{min}) \rightarrow ③$$

To find x_1:—(location of stub)

W.K.T $Y_S = G_O \pm jB_O$

$$\boxed{\left.\frac{Y_S}{G_O}\right|_{x=x_1} = 1 \pm jB_o} \rightarrow ④ \quad \left(\frac{B_O}{G_O} = B_o \rightarrow \text{Normalized}\right)$$

$$\frac{Y_S}{G_O} = 1 \Rightarrow \boxed{Y_S = G_O} \rightarrow$$ This is the point at which stub is to be connected.

Equate ① & ④

$$\frac{1-|K|^2}{1+|K|^2+2|K|\cos\theta} = 1$$

$$\cancel{1} - |K|^2 = \cancel{1} + |K|^2 + 2|K|\cos\theta$$

$$\cos\theta = \frac{-|K|^2 - |K|^2}{2|K|} = \frac{-\cancel{2}|K|^{\cancel{2}}}{\cancel{2}\cancel{K}}$$

$$\boxed{\cos\theta = -|K|}$$

$$\theta = \cos^{-1}(-|K|)$$

$$\theta = -\pi \pm \cos^{-1}|K|$$

$$\phi - 2\beta x_1 = -\pi \pm \cos^{-1}|K| \qquad \text{Sub } \theta = \phi - 2\beta x_1$$

$$2\beta x_1 = \phi + \pi \mp \cos^{-1}|K|$$

$$\boxed{x_1 = \frac{\phi + \pi \mp \cos^{-1}|K|}{2\beta}}$$

→ x_1 is the location where Stub connected. For better performance Stub is placed on load on load side, so that syl. occurs at min. distance.

To find Susceptance of line and Stub :-

Compare ① & ③ (imag. parts)

$$\left.\frac{B_s}{G_0}\right|_{\text{at } x = x_1} = \frac{-2|K|\sin\theta}{1+|K|^2 + 2|K|\cos\theta}$$

W.K.T $\cos\theta = -|K|$

$\cos^2\theta + \sin^2\theta = 1$

$\sin\theta = \pm\sqrt{1-\cos^2\theta}$

$\sin\theta = \pm\sqrt{1-|K|^2}$

$$= \frac{-2|K|\left(\pm\sqrt{1-|K|^2}\right)}{1+|K|^2 + 2|K|(-|K|)}$$

$$= \frac{\mp 2|K|\sqrt{1-|K|^2}}{1+|K|^2 - 2|K|^2}$$

$$= \frac{\mp 2|K|\sqrt{1-|K|^2}}{1-|K|^2}$$

$$\frac{B_s}{G_0} = \frac{\mp 2|K|}{\sqrt{1-|K|^2}}$$

$$\boxed{B_s = G_0\left[\frac{\mp 2|K|}{\sqrt{1-|K|^2}}\right]}$$ → Susceptance of line at a distance x_1, where Stub is connected. To cancel this Susceptance of line, Susceptance of Stub should be

$$\boxed{B_s = G_0\left[\frac{\pm 2|K|}{\sqrt{1-|K|^2}}\right]}$$ → Susceptance of Stub.

<u>To find length of stub (L):-</u>

i/p impedance of Short ckted line $Z_{sc} = jR_0 \tan\beta x$

$(x=L)$ $$Y_{sc} = \frac{1}{Z_{sc}} = \frac{1}{jR_0\tan\beta L} = \frac{-jG_0}{\tan\beta L}$$

$$\boxed{Y_{sc} = \frac{-jG_0}{\tan\beta L}}$$ → Stub i/p imp (Short ckted line)

$$B_s = G_0\left[\frac{\pm 2|K|}{\sqrt{1-|K|^2}}\right]$$ → Susceptance of stub → (This eqn is imag part of admittance)

→ equate with eqn (A)

$$Y_{sc} = \frac{G_0}{\tan\beta L}$$

$$\cancel{G_0}\left[\frac{\pm 2|K|}{\sqrt{1-|K|^2}}\right] = \frac{\cancel{G_0}}{\tan\beta L}$$

$$\tan\beta L = \pm\frac{\sqrt{1-|K|^2}}{2|K|}$$

$$\beta L = \tan^{-1}\left[\frac{\pm\sqrt{1-|K|^2}}{2|K|}\right]$$

$$\boxed{L = \frac{1}{\beta}\tan^{-1}\left[\frac{\pm\sqrt{1-|K|^2}}{2|K|}\right]}$$

Conclusion :- $x_1 = \dfrac{\phi + \pi \mp \cos^{-1}|K|}{2\beta}$, $x_2 = \dfrac{\phi + \pi}{2\beta}$

$L = \dfrac{1}{\beta}\tan^{-1}\left(\dfrac{\pm\sqrt{1-|K|^2}}{2|K|}\right)$

At Position 1 :-

$$x_1' = \frac{\phi + \pi - \cos^{-1}|K|}{2\beta}$$

$$L' = \frac{1}{\beta}\tan^{-1}\left(\frac{+\sqrt{1-|K|^2}}{2|K|}\right)$$

At position 2 :-

$$x_1'' = \phi + \pi + \cos^{-1}|K|$$

$$L'' = \frac{1}{\beta}\tan^{-1}\left(\frac{-\sqrt{1-|K|^2}}{2|K|}\right)$$

In terms of Z_R & Z_O

$$x_1 = \frac{1}{\beta}\tan^{-1}\sqrt{\frac{Z_R}{Z_O}}$$

$$L = \frac{1}{\beta}\tan^{-1}\left[\frac{\sqrt{Z_R Z_O}}{Z_R - Z_O}\right]$$

Dis-advantages :-

↳ It is applicable only for single frequency.

↳ possible only for open wire line, not applicable for co-axial cables & waveguides.

Problems :-

① Determine length & location of single S.C line stub to produce impedance match on a trxion line with $R_0 = 600\,\Omega$ and terminated in $1800\,\Omega$.

$R_0 = Z_0 = 600\,\Omega$, $Z_R = 1800\,\Omega$.

1) $K = \dfrac{Z_R - Z_0}{Z_R + Z_0} = \dfrac{1800 - 600}{1800 + 600} = 0.5$

$K = |K| \angle \phi$

$|K| = 0.5, \ \phi = 0$

1) $$x_1 = \frac{\phi + \pi - \cos^{-1}|K|}{2\beta} \qquad \beta = \frac{2\pi}{\lambda}$$

$$x_1 = \frac{0 + \pi - \cos^{-1}|0.5|}{\frac{2 \times 2\pi}{\lambda}} \qquad \cos^{-1}(0.5) = 1.047$$

$$\boxed{x_1 = 0.1666\lambda}$$

2) $$x_2 = \frac{\phi + \pi + \cos^{-1}|K|}{2\beta} = \frac{0 + \pi + 1.047}{\frac{2 \times 2\pi}{\lambda}}$$

$$\boxed{x_2 = 0.3333\lambda}$$

3) $$L_1 = \frac{1}{\beta} \tan^{-1}\left(+\frac{\sqrt{1-|K|^2}}{2|K|}\right)$$

$$= \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{1 - 0.5^2}}{2(0.5)}$$

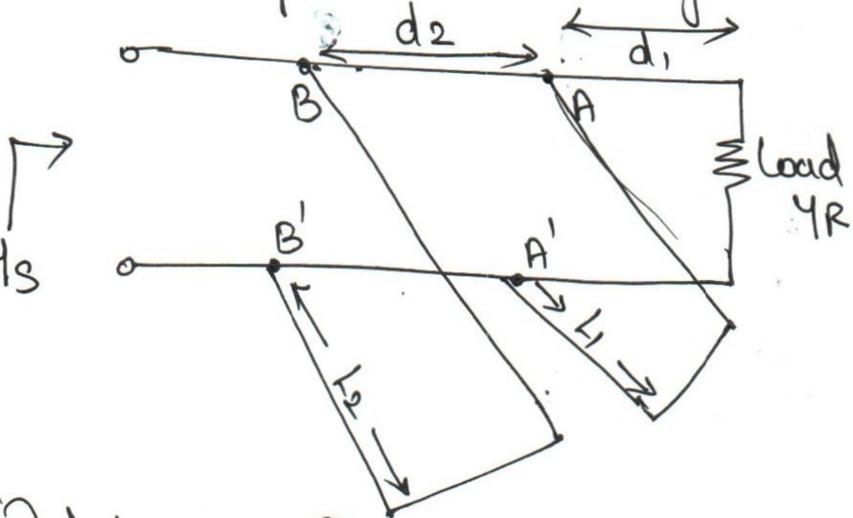
$$\boxed{L_1 = 0.1135\lambda}$$

4) $$L_2 = \frac{1}{\beta} \tan^{-1}\left(-\frac{\sqrt{1-|K|^2}}{2|K|}\right)$$

$$\boxed{L_2 = 0.386\lambda}$$

DOUBLE STUB MATCHING :—

Two different short ckted stub of length L_1 and L_2 are used for impedance matching.



* Let stub-① be located at point A-A' distance 'd₁' from the load & length of the stub is L_1.

* Let stub-② be located at point B-B' at distand 'd₂' from the stub ① and length is L_2.

↳ i/p impedance at any point is,

$$Z_{in} = Z_0\left[\frac{Z_R\cos\beta x + jR_0\sin\beta x}{R_0\cos\beta x + jZ_R\sin\beta x}\right]$$

$$Z_{in} = Z_0\left[\frac{Z_R + jR_0\tan\beta x}{R_0 + jZ_R\tan\beta x}\right]$$

$$\frac{1}{Y_{in}} = \frac{1}{Y_0}\left[\frac{\frac{1}{Y_R} + j\frac{1}{Y_0}\tan\beta x}{\frac{1}{Y_0} + j\frac{1}{Y_R}\tan\beta x}\right]$$

$$\frac{\frac{1}{Y_{in}}}{\frac{1}{Y_0}} = \frac{\cancel{\frac{1}{Y_R}}\left[1 + j\frac{Y_R}{Y_0}\tan\beta x\right]}{\cancel{\frac{1}{Y_R}}\left[\frac{Y_R}{Y_0} + j\tan\beta x\right]}$$

$$\frac{Y_0}{Y_{in}} = \left[\frac{1 + j\frac{Y_R}{Y_0}\tan\beta x}{\frac{Y_R}{Y_0} + j\tan\beta x}\right]$$

$$\frac{Y_{in}}{Y_0} = \left[\frac{\frac{Y_R}{Y_0} + j\tan\beta x}{1 + j\frac{Y_R}{Y_0}\tan\beta x}\right]$$

Let

$y_{in} = \frac{Y_{in}}{Y_0}$ = Normalized i/p admittance

$y_R = \frac{Y_R}{Y_0}$ = Normalized load admittance.

$$\boxed{y_{in} = \frac{y_R + j\tan\beta x}{1 + j y_R \tan\beta x}} \longrightarrow (1)$$

Take complex conjugate

$$y_{in} = \frac{(y_R + j\tan\beta x)}{(1 + j y_R\tan\beta x)} \times \frac{(1 - j y_R\tan\beta x)}{(1 - j y_R\tan\beta x)}$$

$$y_{in} = \frac{y_R - j y_R^2\tan\beta x + j\tan\beta x + y_R\tan^2\beta x}{1 + y_R^2\tan^2\beta x}.$$

$$y_{in} = \frac{y_R + y_R\tan^2\beta x + j(\tan\beta x - y_R^2\tan\beta x)}{1 + y_R^2\tan^2\beta x}.$$

$$\boxed{y_{in} = \frac{y_R(1 + \tan^2\beta x)}{1 + y_R^2\tan^2\beta x} + j\frac{(\tan\beta x)(1 - y_R^2)}{1 + y_R^2\tan^2\beta x}}$$

To find i/p admittance at Stub '1' (A-A'):-

Stub '1' is located at a pt A-A' at a distance $x = d_1$ from the load. i/p admittance at A-A' before stub '1' connected

$$y_A\Big|_{\text{when } x = d_1} = \frac{y_R(1 + \tan^2\beta d_1)}{1 + y_R^2\tan^2\beta d_1} + \frac{j(1 - y_R)\tan\beta d_1}{(1 + y_R^2\tan^2\beta d_1)} = g_i \pm j b_i$$

when Stub '1' Susceptance $\pm jb_1$ is added at a pt A-A', The new admittance will be.

$$y_A' = (g_i \pm jb_i) \pm jb_1$$

$$y_A' = g_i \pm j(b_i \pm b_1) \text{ where } b_i' = b_i \pm b_1$$

$$\boxed{y_A' = g_i \pm j\, b_i'}$$

∵ i/p admittance of Short ckted Stub is purely imagninary. So the real part ie) Conductance part of new admittance y_A' will remains the Same.

To find i/p admittance at Stub ② B-B' :-

The i/p admittance of line at pt B-B' Should be equal to G_0. So that line appears to be terminated into its char. impedance.

The Normalized admittance at this point is given by,

$$y_B' = g_2 \pm jb_2$$

$$\frac{y_B'}{g_2} = 1 \pm jb_2. \quad (b_2 \rightarrow \text{Normalized})$$

So that $\boxed{y_B' = G_0}$

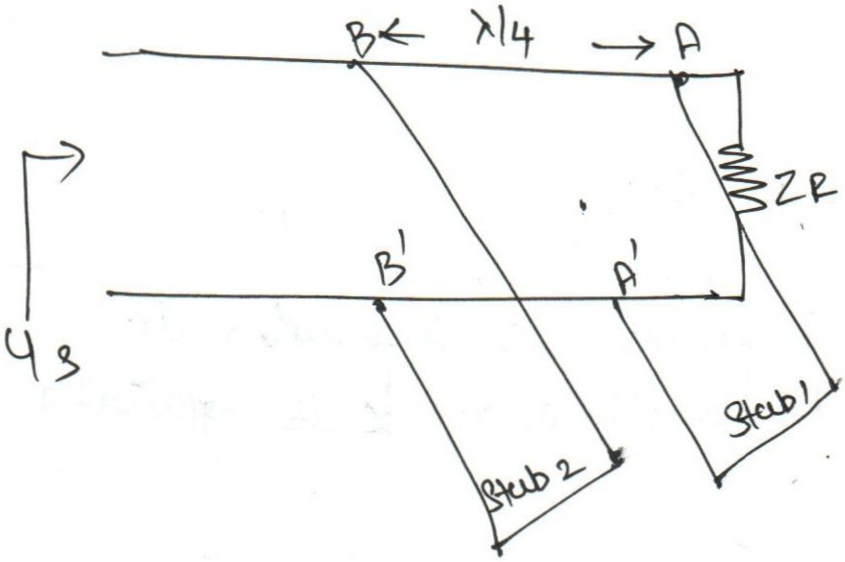
finally length of the Stub '2' is adjusted Such that Susceptance of the Stub 2 is $\mp jb_2$ which cancells the value of $\pm jb_2$ at point B-B' & desired admittance is Y.

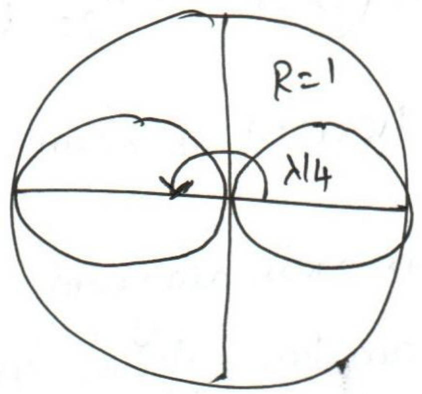
(*) Two stubs are seperated by fixed distance $\lambda/4$ (or) $3\lambda/8$.

(*) Matching takes places b/w point B-B' and Generator (Source).

(*) Reflection loss between BB' and load.

(*) To minimize these losses the stubs are located very close to the load & hence stub (1) is located at the load.





SMITH CHART :-

Introduction :-

→ Smith chart is basically a plot of polar factor reflection coefficient 'K' expressed in terms of Normalized impedance.

→ It is a graphical tool used for solving high frequency txion line and waveguides problems.

* It based on two set of orthogonal circles ie) constant resistance (R) circles and constant reactance circle (jx).

$$Z_R = R + jX$$

Normalized form $\frac{Z_R}{Z_0} = \frac{R+jX}{Z_0}$

* In circle diagram, the impedance is represented in rectangular form, while in Smith chart z is represented in circle form.

Construction of Smith chart :-

txion line i/p impedance

$$Z_{in} = R_0 \left(\frac{1+|K| \angle \phi - 2\beta x}{1-|K| \angle \phi - 2\beta x} \right)$$

Normalized i/p impedance

$$\frac{Z_{in}}{R_0} = \frac{1+|K| \angle \phi - 2\beta x}{1-|K| \angle \phi - 2\beta x}$$

$\frac{Z_{in}}{Z_0} = R+jX$ put $K = |K| \angle \phi - 2\beta x = U + iV$.

$\therefore (R+jX) = \frac{1+U+iV}{1-U-iV}$

Seporate real & imag.

$R+jX = \frac{1+U+iV}{1-U-iV} \times \frac{(1-U-iV)}{(1-U-iV)}$

$R+jX = \frac{1-U^2+iV(1-U)+iV(1+U)-V^2}{(1-U)^2+V^2} = \frac{1-U^2-V^2}{(1-U)^2+V^2} + i\frac{2V}{(1-U)^2+V^2}$

(real) $R = \frac{1-U^2-V^2}{(1-U)^2+V^2} \longrightarrow ①$ and $X = \frac{2V}{(1-U)^2+V^2} \longrightarrow ②$

Constant (R) circle:

circular form $x^2+y^2 = r^2$

$\left(U - \frac{R}{R+1}\right)^2 + (V-0)^2 = \left(\frac{1}{R+1}\right)^2$

Center $\left(\frac{R}{R+1}, 0\right)$ and radius $\left(\frac{1}{R+1}\right)$

when $R=0$, $c(0,0) \longrightarrow$ radius 1

$R=0.5$, $c(0.3,0) \longrightarrow$ radius 0.6

$R=1$, $c(0.5,0) \longrightarrow$ radius 0.5

$R=1.5$, $c(0.6,0) \longrightarrow$ radius 0.4

If $V=0$ circle becomes

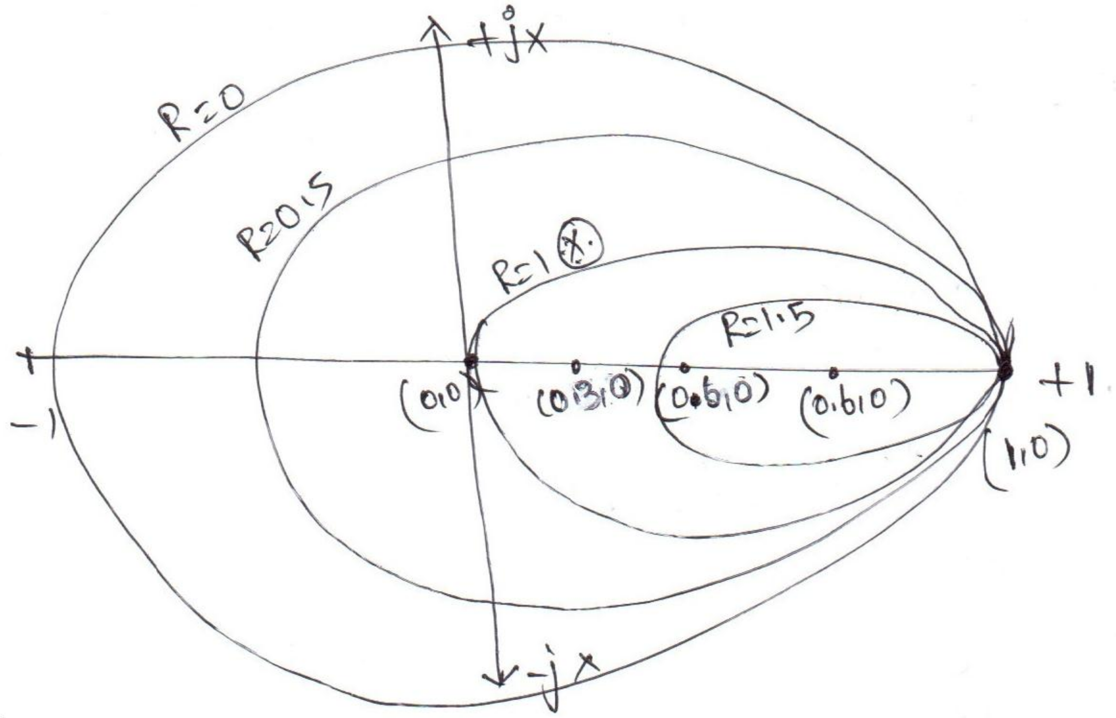
$$\left(U-\frac{R}{R+1}\right)^2 = \left(\frac{1}{R+1}\right)^2$$

$$U-\frac{R}{R+1} = \frac{1}{R+1}$$

$$U = \frac{1}{R+1} + \frac{R}{R+1} = \frac{R+1}{R+1} = 1$$

$(U,V) = (1,0) \longrightarrow$ All circle passes thro (1,0).

$0 \leq |K| \leq 1$



Constant X-circle :—

$$X = \frac{2V}{(1-U)^2+V^2}$$

re-arranging,

$$(U-1)^2 + \left(V-\frac{1}{X}\right)^2 = \left(\frac{1}{X}\right)^2$$

Center $\left(1, \frac{1}{X}\right)$ Radius $(1/X)$

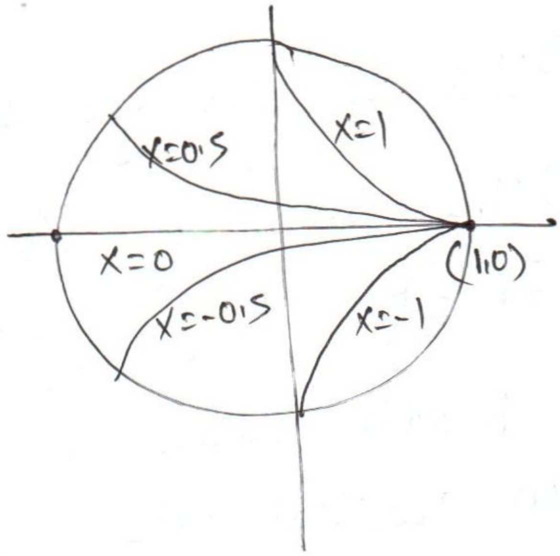
when $X = 0$ $c(1, \infty)$ radius (∞)

$X = 0.5$ $c(1, 2)$ radius (2)

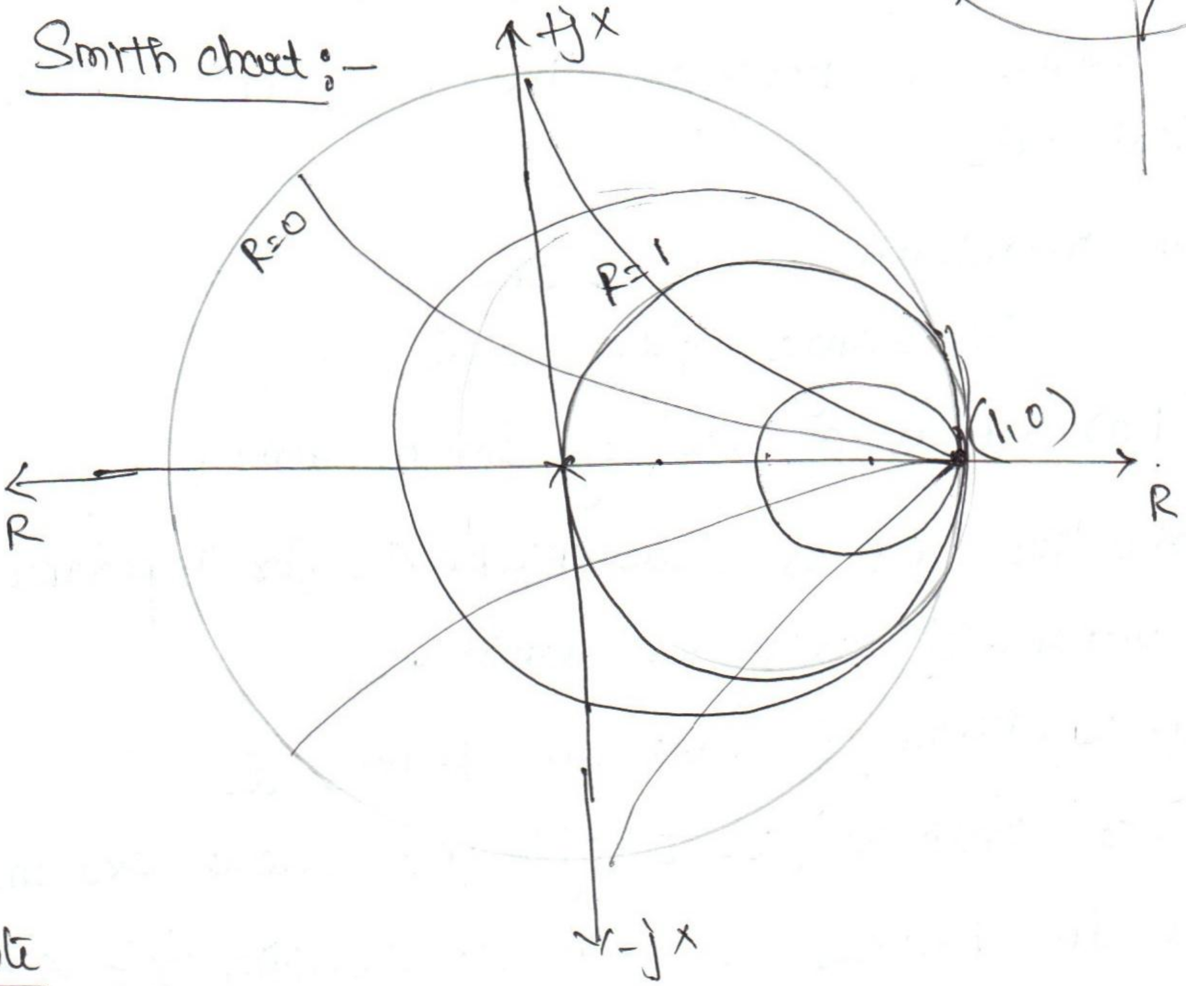
$X = 1$ $c(1, 1)$ radius (1)

$X = -0.5$ $c(1, -2)$ radius (-2)

$X = -1$ $c(1, -1)$ radius (-1)



Smith chart :-



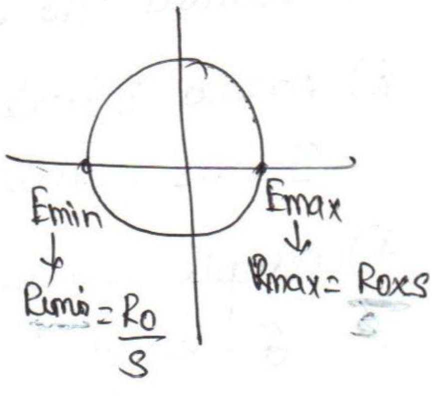
Note

1) center (R, X)
(1, 0)

2) R values (X-axis) → always +ve (0 .... 1 ..... 0)

3) Y values (Y-axis) → +ve & -ve (+0 ..... +30) up
(+0 ...... -30) down.

4) |K| → ranges from 0 to 1



Properties of Smith chart :—

1) It is used for both impedance & admittance

2) The values of Z_R and Y_R are Normalized values
$Z_r = \frac{Z_R}{Z_0}$ and $y_r = \frac{Y_R}{Y_0}$. $Z_R = R + jX$

R circle centers on horizontal line & Y circle centers on Vertical axis.

3) Maximum magnitude of K is unity
|K| ranges from 0 to 1.

4) point (1,0) act as a center of Smith chart.

5) Horizontal line act as resistance(R) axis for impedance and conductance (G) axis for admittance.

6) The left extreme of chart in R-axis is Short ckt condition ($Z = 0 + 0j$) and right extreme of chart in R-axis is open ckt condition ($Z = \infty + j\infty$) for impedance chart.

7) Outer rim of the chart is $\beta x = \frac{2\pi}{\lambda} x$ which indicates the electrical length of the line.

8) Arrow indicates the direction of travel along the line. It is scaled in degree (or) wavelength.

9) Comple revolution of 360° around chart = $\lambda/2$ distance.
* clockwise movement indicates travel towards Generator from load.

(*) anticlockwise movement $\xrightarrow{\text{indicates}}$ travel towards load from Generator.

$$\frac{\lambda}{2} = 360^\circ$$

$$\boxed{\lambda = 720^\circ}$$

10) outermost line scale = calculation of distance in wavelength from Generator

Next inner scale = calculation of distance in wavelength from load

Inner most scale = angle of refl. co-eff (K) in degree.

11) V_{max} occur at $R_{max} = R_0 \times S$

V_{min} occur at $R_{min} = \frac{R_0}{S}$

$V_{max} \rightarrow$ right side of the chart

$V_{min} \rightarrow$ left side of the chart.

—— x —— x ——

Parameters of open wire line at high frequency

* At high frequency, current flowing on the surface of the conductor in a skin of small depth.
* The internal flux and internal inductance are reduced to zero.

1) Inductance of the line:

$$L = \frac{\mu_0}{2\pi} \ln\left(\frac{d}{a}\right) = 4 \times 10^{-7} \ln\left(\frac{d}{a}\right) H/m$$

$$L = 9.21 \times 10^{-7} \log\left(\frac{d}{a}\right) H/m.$$

2) Capacitance of the line:

$$C = \frac{\pi \varepsilon}{\ln\left(\frac{d}{a}\right)} = \frac{12.07}{\ln\left(\frac{d}{a}\right)} \mu F/m.$$

d = distance (or) space b/n conductors

a = radius of the conductor.

3) Skin depth

$$\boxed{\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \text{ meter}}$$

f = frequency

μ = permeability $= 4\pi \times 10^{-7}$

σ = conductivity $= 5.75 \times 10^{7}$ ℧/m

$$\delta = \frac{0.0664}{\sqrt{f}}.$$

4) Resistance of the line:

For DC $\rightarrow R_{dc} = \frac{K}{\pi a^2}$ $\rightarrow$ $\frac{R_{ac}}{R_{dc}} = \frac{a}{2\delta} = \frac{a}{2}\sqrt{\pi f \mu \sigma}$

For AC $\rightarrow R_{ac} = \frac{K}{2\pi a \delta}$ $\longrightarrow$ $\frac{R_{ac}}{R_{dc}} = 7.53\, a\sqrt{f}$

Properties of coaxial line at higher frequency:-

* **Inductance per unit length**

$$L = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right) \ H/m$$

$$L = 4.6 \times 10^{-7} \log\left(\frac{b}{a}\right) \ H/m.$$

* **capacitance**

$$C = \frac{2\pi\epsilon}{\ln(b/a)} = \frac{24.14}{\log(b/a)} \ \mu F/m$$

* **Resistance** for dc, $R_{dc} = \frac{1}{\pi\sigma}\left[\frac{1}{a^2} + \frac{1}{c^2 - a^2}\right]$

for ac, $R_{ac} = 4.17 \times 10^{-8} \sqrt{f} \left(\frac{1}{b} + \frac{1}{a}\right) \Omega/m$

$a, b \rightarrow$ inner radius of two conductors

$c \rightarrow$ radius of outer Conductor.

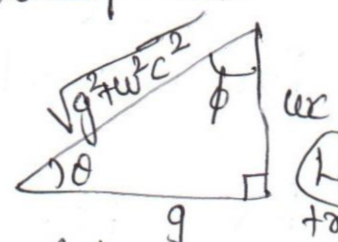
* Shunt Susceptance $y = g + j\omega c$

The quality of insulating material Expressed by power factor.

$$\cos\theta = \text{power factor} = \frac{g}{\sqrt{g^2 + \omega^2 c^2}} \approx \frac{g}{\omega c} \quad \because g << \omega c.$$

* The quality of dielectric Expressed by Dissipation

Factor $= \frac{\text{Energy dissipated}}{\text{Energy Stored.}}$



(Loss triangle of dielectric)

Good dielectric power factor = dissipation factor.

PANIMALAR INSTITUTE OF TECHNOLOGY
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DEPARTMENT OF ECE

UNIT-4 - PASSIVE FILTERS

Sub Code: EC 6503

Sub Name: Transmission Lines and Wave Guides

Class: III Year ECE/V Semester

Faculty: **HOD/ECE**

1. Dr.S.Sathya Priya

2. Mrs.D.Kalaiarasi

UNIT - 4 : PASSIVE FILTERS

The neper - the decibel - Characteristic Impedance of Symmetrical Networks - Current and Voltage ratios - Propagation Constant - Properties of Symmetrical Networks - Filter Fundamentals - Pass and stop bands - Behaviour of the characteristic Impedance - Constant K Filters - Low Pass, High Pass, Band Pass, Band Elimination Filters - M-derived sections - filter Circuit Design - Filter Performance - Crystal Filters.

Network:

An electrical network is a 2-port or four terminal system where a number of components or elements are connected together to perform a particular function.

Classification of Network.

(i) Lumped or Distributed Network.

A network which is formed by lumped components or discrete components like R, L, C is called a lumped network

A network which is formed by using sections of transmission lines is called a distributed network. In a distributed network, one cannot recognize the presence of R, L, C.

(ii) Linear or Non-linear Network

In a linear network, the output of the network is a linear function of input. A non-linear network will not have a linear relationship between output current and input voltage.

(iii) Active or Passive Network.

A network becomes active if it consists of an active element like a battery, transistor etc. within the two ports. A passive network is characterized by the presence of passive elements in the circuit.

(iv) Reciprocal or Non-reciprocal Network.

If the ratio of output voltage (i.e. response) to the input current in a network is a constant even when the position of input and output are interchanged, then the network is said to be reciprocal. If the ratio of output voltage to input current is not constant when the input and output terminals are interchanged, the network is said to be a non-reciprocal network.

(v) Symmetrical or Asymmetrical Network.

A network is symmetrical if the electrical properties of the network is not affected when the input and output are interchanged. If this is not so, then network is said to be asymmetrical.

(vi) Recurrent or Non-recurrent Network

When a large circuit consists of similar network sections connected one after another, the network is called recurrent network. A single isolated network is called a Non-recurrent network

(vii) Balanced or Unbalanced Network.

A balanced network is a network in which the corresponding series impedance elements are identical and these elements are symmetrical with respect to ground potential. A network which doesnot satisfy this is an unbalanced network.

Properties of Symmetrical Networks. U.Q – NOV 2011 (8 marks) U.Q – MAY 2012 (8 marks) U.Q – NOV 2012 (6 marks)

In a symmetrical network, the parameters to be characterised are

(a) Characteristic Impedance Z_0

(b) Open circuit Impedance Z_{oc}

(c) Short circuit Impedance Z_{sc}

(d) Propagation Constant γ

(a) Characteristic Impedance. U.Q – ARIL 2011 (2 marks)

Consider a symmetrical network with a characteristic impedance Z_0. If such a network is terminated at the output with an impedance equal to its characteristic impedance Z_0, then its impedance at the input terminal will have a value equal to its characteristic impedance Z_0.

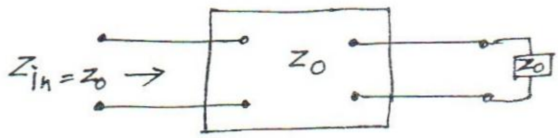
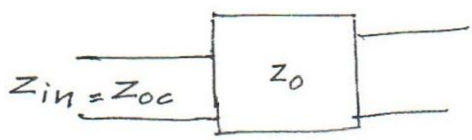


Fig. Symmetrical Network.

(b) Open Circuit Impedance

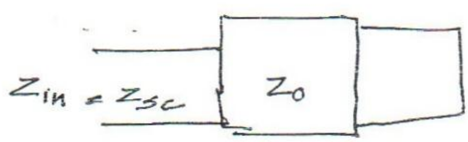
If the receiving end of a symmetrical network with characteristic impedance Z_0 is open circuited, then the input impedance will be $Z_{in} = Z_{oc}$.



$Z_L = \infty$ (or) Open circuit.

(c) Short Circuit Impedance

If the receiving end of a symmetrical network with characteristic impedance Z_0 is terminated in a short circuit, then $Z_{in} = Z_{sc}$.



$Z_L = 0$ (or) short circuit

$$Z_0 = \sqrt{Z_{oc} \times Z_{sc}}$$

(d) Propagation Constant. U.Q – APRIL 2011, NOV 2011, NOV 2012, MAY 2013 (2 marks)

Consider a symmetrical network with a characteristic impedance Z_0 terminated in a load equal to Z_0. If the current applied at the sending end is I_s, then the current I_R at the output of the network will suffer an exponential decay resulting in decrease in amplitude and a shift in phase angle. This property of a network is represented by a term propagation constant γ.

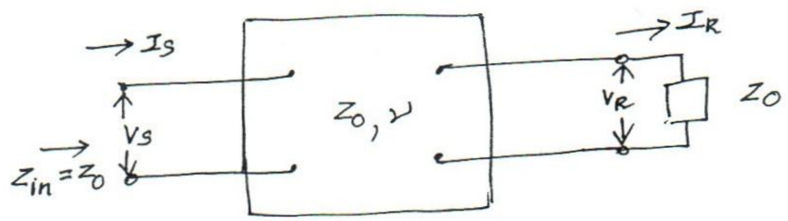


Fig. : Symmetrical Network

- The receiving end current is given by $I_R = I_S e^{-\gamma}$

$$\frac{I_S}{I_R} = e^{\gamma}$$

$$V_S = I_S Z_0 \quad \& \quad V_R = I_R Z_0$$

$$\frac{V_S}{V_R} = \frac{I_S Z_0}{I_R Z_0} = \frac{I_S}{I_R} = e^{\gamma}$$

$$\log_e \frac{I_S}{I_R} = \log_e \frac{V_S}{V_R} = \gamma$$

$$\boxed{\gamma = \ln\left(\frac{V_S}{V_R}\right) = \ln\left(\frac{I_S}{I_R}\right)}$$

$P_S = I_S^2 Z_0$ & $P_R = I_R^2 Z_0$

$$\frac{P_S}{P_R} = \frac{I_S^2}{I_R^2}$$

$$\frac{I_S}{I_R} = \sqrt{\frac{P_S}{P_R}} = e^{\gamma}$$

$$\gamma = \ln \left(\frac{P_S}{P_R}\right)^{1/2}$$

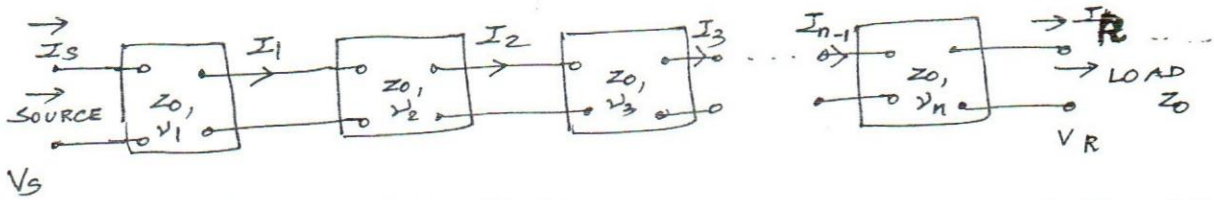
$$\boxed{\gamma = \frac{1}{2} \ln \left(\frac{P_S}{P_R}\right)}$$

$$\gamma = \alpha + j\beta$$

$\alpha \rightarrow$ Attenuation Constant in Nepers

$\beta \rightarrow$ Phase Constant in Radians.

Propagation Constant of Symmetrical Recurrent Network-



$$\gamma = \ln\left(\frac{I_S}{I_R}\right)$$

$$e^{\gamma} = \frac{I_S}{I_R}$$

$$e^{\gamma} = \frac{I_S}{I_R} = \frac{I_S}{I_1} \times \frac{I_1}{I_2} \times \frac{I_2}{I_3} \times \cdots \frac{I_{n-1}}{I_R}$$

$$e^{\gamma} = e^{\gamma_1} \cdot e^{\gamma_2} \cdot e^{\gamma_3} \cdots e^{\gamma_n}$$

$$e^{\gamma} = e^{\gamma_1 + \gamma_2 + \gamma_3 + \cdots + \gamma_n}$$

$$\gamma = \gamma_1 + \gamma_2 + \cdots \gamma_n$$

If $\gamma_1 = \gamma_2 = \cdots = \gamma_n$

$$\gamma = n\gamma_1$$

If $\gamma_1 = \alpha_1 + j\beta_1$

$$\gamma_n = \alpha_n + j\beta_n$$

for single stage network,

$$e^{\gamma} = e^{\alpha + j\beta} = e^{\alpha} \cdot e^{j\beta} = \frac{I_S}{I_R}$$

for multistage network and if all the network are identical,

$$\frac{I_s}{I_R} = e^{n\alpha} e^{jn\beta} = e^{n\alpha} \angle n\beta$$

$e^{n\alpha} \rightarrow$ Magnitude of $\frac{I_s}{I_R}$

$\angle n\beta \rightarrow$ Phase angle between I_s and I_R.

Taking modulus on both sides,

$$\left|\frac{I_S}{I_R}\right| = e^{n\alpha} = e^{\alpha_1 + \alpha_2 + \cdots + \alpha_n}$$

$$\ln\left|\frac{I_S}{I_R}\right| = \alpha_1 + \alpha_2 + \cdots + \alpha_n$$

$$\boxed{\ln\left|\frac{I_S}{I_R}\right| = n\alpha \quad \text{nepers}}$$

Neper.

Neper is defined as the natural logarithm of ratio of input voltage or current to the output voltage or current in a network which is properly terminated in its characteristic impedance.

$$\left|\frac{I_S}{I_R}\right| = \left|\frac{V_S}{V_R}\right| = e^{N}$$

$$\log_e\left|\frac{V_S}{V_R}\right| = \log_e\left|\frac{I_S}{I_R}\right| = N$$

$$\boxed{N\,(\text{No. of nepers}) = \ln\left|\frac{V_S}{V_R}\right| = \ln\left|\frac{I_S}{I_R}\right|}$$

$P_S = I_S^2 Z_0$ & $P_R = I_R^2 Z_0$

$$e^{N} = \frac{I_S}{I_R} = \left(\frac{P_S}{P_R}\right)^{1/2}$$

$$e^{2N} = \left(\frac{P_S}{P_R}\right)$$

$$2N = \log_e\left(\frac{P_S}{P_R}\right)$$

$$2N = \ln\left(\frac{P_S}{P_R}\right)$$

$$\boxed{N = \frac{1}{2}\ln\left|\frac{P_S}{P_R}\right|}$$

The term neper was selected to honour John Napier, a mathematician who first proposed the use of logarithm.

Decibel.

Decibel is another logarithmic unit abbreviated as dB.

$$bel = \log_{10} \left|\frac{P_S}{P_R}\right|$$

$$\text{Decibel in dB, } D = 10 \log_{10} \left|\frac{P_S}{P_R}\right|$$

$$\log_{10}\left|\frac{P_S}{P_R}\right| = \frac{D}{10}$$

$$P_S = \frac{V_S^2}{Z_0} \quad \& \quad P_R = \frac{V_R^2}{Z_0} \quad \text{and} \quad P_S = I_S^2 Z_0 \quad \& \quad P_R = I_R^2 Z_0$$

for current ratio, $D = 10 \log_{10} \dfrac{I_S^2}{I_R^2}$

$$D = 10 \log_{10} \left|\frac{I_S}{I_R}\right|^2$$

$$D = 20 \log_{10} \left|\frac{I_S}{I_R}\right|$$

For voltage ratio, $D = 20 \log_{10} \left|\dfrac{V_S}{V_R}\right|$

$$\boxed{D = 20 \log_{10}\left|\frac{V_S}{V_R}\right| \text{ dB} = 20 \log_{10}\left|\frac{I_S}{I_R}\right| \text{dB}}$$

<u>Conversion of Neper to dB.</u> U.Q - APRIL 2012 (2 marks
U.Q - APRIL 2013 (4 marks

$$\log_e x = \log_{10} x \times \log_e 10$$

$$\log_e\left(\frac{P_S}{P_R}\right) = \log_{10}\left(\frac{P_S}{P_R}\right) \times 2.3026$$

$$2N = \frac{D}{10} \times 2.3026.$$

$$D = db = \frac{10 \times 2N}{2.3026} = 8.686\ N$$

Attenuation in db = 8.686 × Attenuation in Nepers.

$$\boxed{\begin{aligned} 1 \ dB &= 8.686 \ np \\ 1 \ np &= 0.115 \ dB. \end{aligned}}$$

Problems.

1) find α in dB. Given α = 3 nepers

Sol:

α in dB = $8.686 \times \alpha$ in nepers

$\alpha = 26.05$ dB //

2) Find the received power of symmetrical network whose input power is 20 m watts. The attenuation is 3 dB.

Sol:

Gn. $P_S = 20$ mW

$\alpha = 3$ dB

$$\alpha_{\text{in dB}} = 10 \log_{10} \left| \frac{P_S}{P_R} \right|$$

$$\frac{\alpha}{10} = \log_{10} \left| \frac{P_S}{P_R} \right|$$

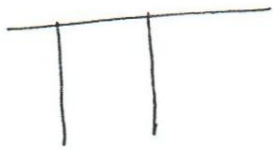
$$10^{\alpha/10} = \frac{P_S}{P_R} = \frac{20 \times 10^{-3}}{P_R}$$

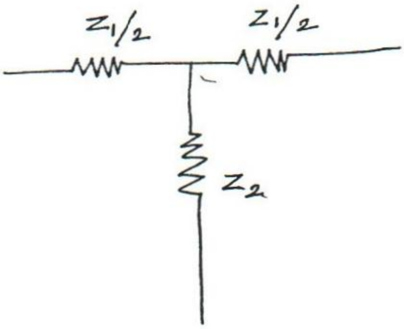
$$P_R = \frac{20 \times 10^{-3}}{10^{3/10}} = 0.01 \text{ watts}$$

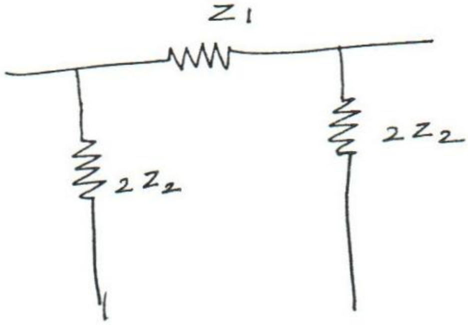
$P_R = 10$ m watts //

Construction of Symmetrical T and π Networks.

Symmetrical T and π Networks are the most important networks in filter theory as they form the building blocks of electrical filters, equalizers and attenuators.

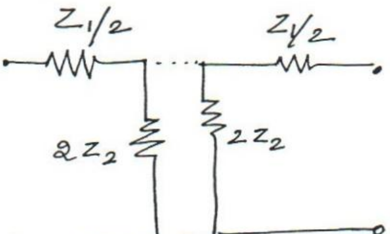




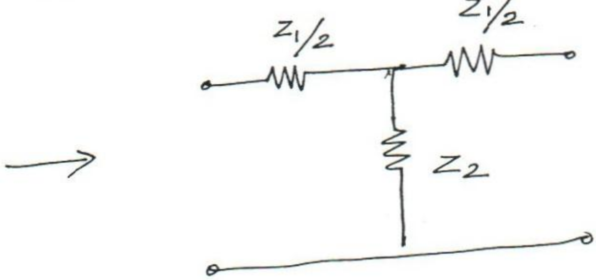


* In a T network, the impedances in series arm is $\frac{Z_1}{2}$ and in shunt arm it is Z_2. In a π network, the impedance in the series arm is Z_1 and in the shunt arm, it is $2Z_2$.

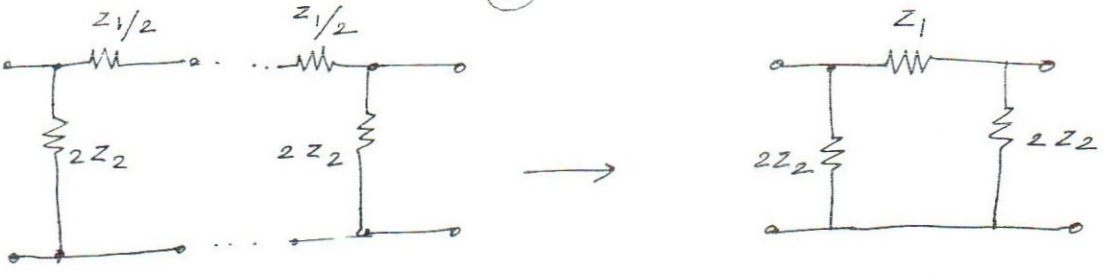
* Both T and π networks are built using two asymmetrical L-half sections.



Two L-sections



Fig_T section

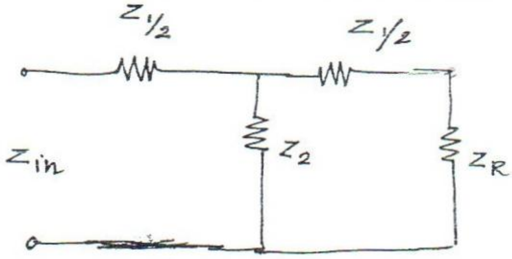


Two L-sections

π- section

fig → π section.

Symmetrical T-Network.



$$Z_{in} = \left(Z_R + Z_1/_2\right) \parallel Z_2 \quad + Z_1/_2$$

$$Z_{in} = \frac{\left(Z_R + Z_1/_2\right) Z_2}{Z_R + Z_2 + \frac{Z_1}{2}} + \frac{Z_1}{2}$$

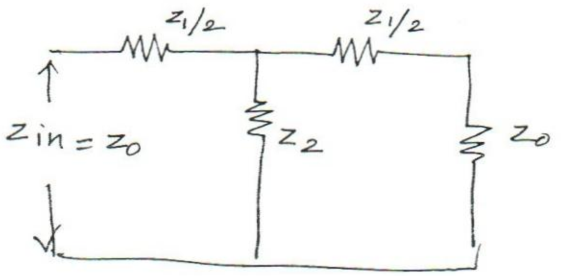
$$Z_{in} = \frac{\frac{Z_1 Z_2}{2} + Z_R Z_2 + \frac{Z_1}{2}\left(Z_R + Z_2 + Z_1/_2\right)}{Z_R + Z_2 + \frac{Z_1}{2}}$$

$$Z_{in} = \frac{\frac{Z_1 Z_2}{2} + Z_R Z_2 + \frac{Z_1 Z_R}{2} + \frac{Z_1 Z_2}{2} + \frac{Z_1^2}{4}}{Z_R + Z_2 + \frac{Z_1}{2}}$$

$$Z_{in} = \frac{Z_1 Z_2 + \frac{Z_1^2}{4} + Z_R\left(Z_2 + \frac{Z_1}{2}\right)}{Z_R + Z_2 + \frac{Z_1}{2}} \quad \text{—— (1)}$$

Characteristic Impedance (Z_0)

U.Q - NOV 2011 (4 marks)
U.Q - MAY 2013 (6 marks)



Sub. $Z_{in} = Z_0 = Z_R$ in ①

$$Z_0 = \frac{Z_1 Z_2 + \frac{Z_1^2}{4} + Z_0\left(\frac{Z_1}{2} + Z_2\right)}{Z_0 + Z_2 + \frac{Z_1}{2}}$$

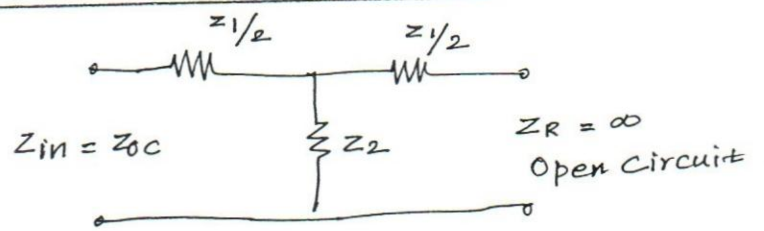
$$Z_0\left(Z_0 + \frac{Z_1}{2} + Z_2\right) - Z_0\left(Z_2 + \frac{Z_1}{2}\right) = Z_1 Z_2 + \frac{Z_1^2}{4}$$

$$Z_0^2 + \frac{Z_0 Z_1}{2} + Z_0 Z_2 - Z_0 Z_2 - \frac{Z_1 Z_0}{2} = Z_1 Z_2 + \frac{Z_1^2}{4}$$

$$Z_0^2 = Z_1 Z_2 + \frac{Z_1^2}{4}$$

$$\boxed{Z_0 = \sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}} = \sqrt{Z_1 Z_2\left(1 + \frac{Z_1}{4 Z_2}\right)}}$$

Open Circuit Impedance



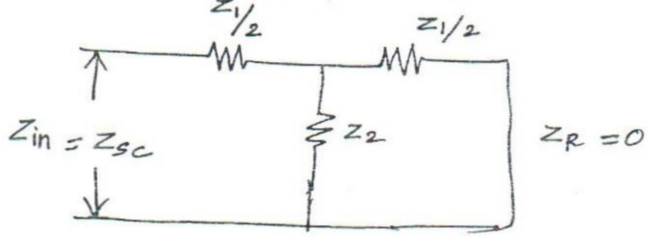
① ⇒

$$Z_{in} = \frac{Z_1 Z_2 + \frac{Z_1^2}{4} + Z_R Z_2 + \frac{Z_1 Z_R}{2}}{Z_R + Z_2 + \frac{Z_1}{2}} = \frac{\cancel{Z_R}\left[\frac{Z_1 Z_2}{Z_R} + \frac{Z_1^2}{4 Z_R} + Z_2 + \frac{Z_1}{2}\right]}{\cancel{Z_R}\left[1 + \frac{Z_2}{Z_R} + \frac{Z_1}{2 Z_R}\right]}$$

Sub. $Z_{in} = Z_{oc}$ & $Z_R = \infty$

$$\boxed{Z_{oc} = \frac{Z_1}{2} + Z_2 = \frac{Z_1 + 2Z_2}{2}}$$

<u>Short Circuit Impedance.</u>



$$Z_{sc} = \left(\frac{Z_1}{2} \parallel Z_2\right) + \frac{Z_1}{2}$$

$$Z_{sc} = \frac{\frac{Z_1}{2} \cdot Z_2}{\frac{Z_1}{2} + Z_2} + \frac{Z_1}{2} = \frac{\frac{Z_1 Z_2}{2} + \frac{Z_1}{2}\left(\frac{Z_1}{2} + Z_2\right)}{\frac{Z_1}{2} + Z_2}$$

$$Z_{sc} = \frac{\frac{Z_1 Z_2}{2} + \frac{Z_1^2}{4} + \frac{Z_1 Z_2}{2}}{\frac{Z_1}{2} + Z_2}$$

$$\boxed{Z_{sc} = \frac{Z_1 Z_2 + \frac{Z_1^2}{4}}{\frac{Z_1}{2} + Z_2}}$$

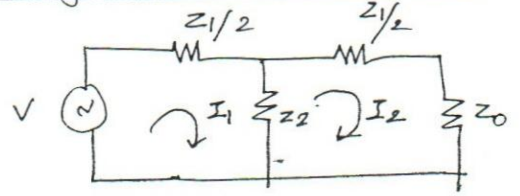
<u>Product of Z_{oc}, Z_{sc}:</u>

$$Z_{oc} \cdot Z_{sc} = \cancel{\left(\frac{Z_1}{2} + Z_2\right)} \left(\frac{Z_1 Z_2 + \frac{Z_1^2}{4}}{\cancel{\frac{Z_1}{2} + Z_2}}\right)$$

$$Z_{oc} Z_{sc} = Z_0^2$$

$$\boxed{Z_0 = \sqrt{Z_{oc} Z_{sc}}}$$

Propagation Constant.



$$\gamma = \log_e \frac{I_1}{I_2}$$

$$e^{\gamma} = \frac{I_1}{I_2}$$

Apply KVL to second loop,

$$I_2 \frac{Z_1}{2} + I_2 Z_0 + Z_2 (I_2 - I_1) = 0$$

$$I_2 \left[\frac{Z_1}{2} + Z_0 + Z_2 \right] = I_1 Z_2$$

$$\frac{I_1}{I_2} = \frac{\frac{Z_1}{2} + Z_0 + Z_2}{Z_2} = 1 + \frac{Z_0}{Z_2} + \frac{Z_1}{2Z_2}$$

$$\gamma = \ln \left(\frac{I_1}{I_2} \right)$$

$$\boxed{\gamma = \ln \left[1 + \frac{Z_0}{Z_2} + \frac{Z_1}{2Z_2} \right]}$$

Sub. $Z_0 = \sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}}$

$$\gamma = \ln \left[1 + \frac{Z_1}{2Z_2} + \frac{\sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}}}{Z_2} \right]$$

$$\gamma = \ln \left[1 + \frac{Z_1}{2Z_2} + \sqrt{\frac{Z_1}{Z_2} + \left(\frac{Z_1}{2Z_2} \right)^2} \right]$$

$$\boxed{\gamma = \ln \left[1 + \frac{Z_1}{2Z_2} + \sqrt{\frac{Z_1}{Z_2} + \left(\frac{Z_1}{2Z_2} \right)^2} \right]}$$

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Relation between Characteristic Impedance, Propagation Constant Z_1 & Z_2

Taking KVL to second loop,

$$I_2 \frac{Z_1}{2} + I_2 Z_0 + Z_2 (I_2 - I_1) = 0$$

$$I_1 Z_2 = I_2 \left[\frac{Z_1}{2} + Z_0 + Z_2 \right]$$

$$\frac{I_1}{I_2} = \frac{\frac{Z_1}{2} + Z_0 + Z_2}{Z_2} = e^{\gamma}$$

$$Z_0 = Z_2 e^{\gamma} - Z_2 - \frac{Z_1}{2}$$

$$Z_0 = Z_2 \left[e^{\gamma} - 1 \right] - \frac{Z_1}{2} \quad \text{—(1)}$$

$$Z_0 = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} \quad \text{—(2)}$$

Sub. (2) in (1)

$$\sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = Z_2 \left[e^{\gamma} - 1 \right] - \frac{Z_1}{2}$$

Squaring both sides,

$$\frac{Z_1^2}{4} + Z_1 Z_2 = \left[Z_2 (e^{\gamma} - 1) - \frac{Z_1}{2} \right]^2$$

$$\frac{Z_1^2}{4} + Z_1 Z_2 = Z_2^2 (e^{\gamma} - 1)^2 - 2 \frac{Z_1 Z_2}{2} (e^{\gamma} - 1) + \frac{Z_1^2}{4}$$

$$\cancel{\frac{Z_1^2}{4}} + \cancel{Z_1 Z_2} = Z_2^2 (e^{2\gamma} - 2e^{\gamma} + 1) - e^{\gamma} Z_1 Z_2 + \cancel{Z_1 Z_2} + \cancel{\frac{Z_1^2}{4}}$$

$$Z_2^2 (e^{2\gamma} - 2e^{\gamma} + 1) = e^{\gamma} Z_1 Z_2$$

Dividing by Z_2^2

$$(e^{2\gamma} - 2e^{\gamma} + 1) = \frac{Z_1 e^{\gamma}}{Z_2}$$

Dividing by e^{γ},

$$e^{\gamma} - 2 + e^{-\gamma} = \frac{Z_1}{Z_2}$$

$$e^{\gamma} + e^{-\gamma} = 2 + \frac{Z_1}{Z_2}$$

Dividing by 2,

$$\frac{e^{\gamma} + e^{-\gamma}}{2} = 1 + \frac{Z_1}{2Z_2}$$

$$\boxed{\cosh\gamma = 1 + \frac{Z_1}{2Z_2}} \quad \text{—(3)}$$

Using the identity, $\cosh^2\gamma - \sinh^2\gamma = 1$

$$\sinh^2\gamma = \cosh^2\gamma - 1 = \left[1 + \frac{Z_1}{2Z_2}\right]^2 - 1$$

$$\sinh^2\gamma = 1 + \frac{Z_1}{Z_2} + \frac{Z_1^2}{4Z_2^2} - 1$$

$$\sinh^2\gamma = \frac{Z_1}{Z_2} + \frac{Z_1^2}{4Z_2^2} = \frac{1}{Z_2^2}\left[\frac{Z_1^2}{4} + Z_1 Z_2\right]$$

$$\sinh^2\gamma = \frac{Z_0^2}{Z_2^2}$$

$$\boxed{\sinh\gamma = \frac{Z_0}{Z_2}} \quad \text{—(4)}$$

$$\tanh\gamma = \frac{\sinh\gamma}{\cosh\gamma} = \frac{Z_0/Z_2}{1 + \frac{Z_1}{2Z_2}} = \frac{\frac{Z_0}{Z_2}}{\frac{2Z_2 + Z_1}{2Z_2}}$$

$$\tanh\gamma = \frac{2Z_0}{2Z_2 + Z_1} = \frac{Z_0}{Z_2 + Z_1/2}$$

$$\boxed{\tanh\gamma = \frac{Z_0}{Z_{oc}}}$$

But, $Z_{oc} = \frac{Z_1}{2} + Z_2$.

$$Z_0 = \sqrt{Z_{OC} \cdot Z_{SC}}$$

$$\tan h\nu = \frac{\sqrt{Z_{OC} Z_{SC}}}{Z_{OC}}$$

$$\boxed{\tan h\nu = \sqrt{\frac{Z_{SC}}{Z_{OC}}}}$$

$$\cos h^2\nu - \sin h^2\nu = 1$$

$$\cos h\, 2\theta = \cos h^2\theta + \sin h^2\theta$$

$$\cos h\nu = \cos h^2\frac{\nu}{2} + \sin h^2\frac{\nu}{2}$$

$$= 1 + \sin h^2\left(\frac{\nu}{2}\right) + \sin h^2\left(\frac{\nu}{2}\right)$$

$$\cos h\nu = 1 + 2\sin h^2\left(\frac{\nu}{2}\right)$$

$$2\sin h^2\left(\frac{\nu}{2}\right) = \cos h\nu - 1$$

$$2\sin h^2\left(\frac{\nu}{2}\right) = 1 + \frac{Z_1}{2Z_2} - 1 = \frac{Z_1}{2Z_2}$$

$$\sin h^2\left(\frac{\nu}{2}\right) = \frac{Z_1}{4Z_2}$$

$$\boxed{\sin h\left(\frac{\nu}{2}\right) = \sqrt{\frac{Z_1}{4Z_2}}}$$

$$\cos h\nu = 1 + \frac{Z_1}{2Z_2} \quad \text{—— (5)}$$

$$\sin h\nu = \frac{Z_0}{Z_2} \quad \Rightarrow \quad Z_2 = \frac{Z_0}{\sin h\nu} \quad \text{—— (6)}$$

Sub. (6) in (5)

$$\cosh \nu = 1 + \frac{Z_1}{\dfrac{2Z_0}{\sinh \nu}}$$

$$\cosh \nu = 1 + \frac{Z_1}{2} \cdot \frac{\sinh \nu}{Z_0}$$

$$\cosh \nu - 1 = \frac{Z_1}{2} \cdot \frac{\sinh \nu}{Z_0}$$

$$2 \sinh^2\left(\frac{\nu}{2}\right) = \frac{Z_1}{2} \cdot \frac{\sinh \nu}{Z_0}$$

$$\frac{Z_1}{2} = 2Z_0 \cdot \frac{\sinh^2\left(\frac{\nu}{2}\right)}{\sinh \nu}$$

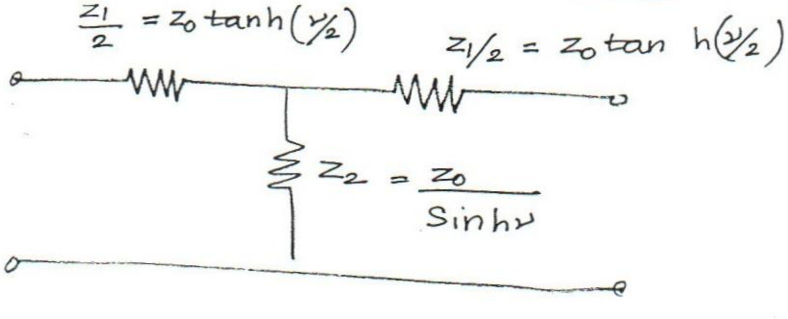
$$\sin 2\theta = 2 \sin\theta \cos\theta$$

$$\sin\theta = 2 \sin\frac{\theta}{2} \cos\frac{\theta}{2}$$

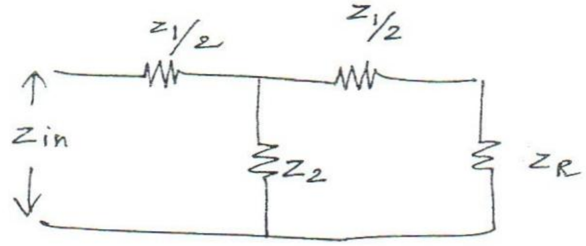
$$\frac{Z_1}{2} = 2Z_0 \cdot \frac{\sinh^2\left(\frac{\nu}{2}\right)}{2 \sinh\frac{\nu}{2} \cosh\frac{\nu}{2}}$$

$$\boxed{\frac{Z_1}{2} = Z_0 \tanh\left(\nu/2\right)}$$

$$\sinh \nu = \frac{Z_0}{Z_2} \qquad \boxed{Z_2 = \frac{Z_0}{\sinh \nu}}$$



Input Impedance of Symmetrical T-Network for generalized load Z_R.



$$Z_{in} = \frac{Z_1 Z_2 + \frac{Z_1^2}{4} + Z_R\left(Z_2 + \frac{Z_1}{2}\right)}{Z_R + Z_2 + \frac{Z_1}{2}}$$

$$Z_{in} = \frac{Z_0^2 + Z_R\left(\frac{Z_1}{2} + Z_2\right)}{Z_R + Z_2 + \frac{Z_1}{2}}$$

Sub. $Z_{OC} = \frac{Z_1}{2} + Z_2 = \frac{Z_0}{\tanh \gamma}$

$$Z_{in} = \frac{Z_0^2 + Z_R \cdot \frac{Z_0}{\tanh \gamma}}{\frac{Z_0}{\tanh \gamma} + Z_R} \Rightarrow \frac{Z_0^2 \tanh \gamma + Z_R Z_0}{Z_0 + Z_R \tanh \gamma}$$

$$\boxed{Z_{in} = \frac{Z_0\left(Z_0 \tanh \gamma + Z_R\right)}{Z_0 + Z_R \tanh \gamma}}$$

$$Z_{in} = \frac{Z_0\left[Z_0 \frac{\sinh \gamma}{\cosh \gamma} + Z_R\right]}{Z_0 + Z_R \frac{\sinh \gamma}{\cosh \gamma}}$$

$$\boxed{Z_{in} = \frac{Z_0\left(Z_0 \sinh \gamma + Z_R \cosh \gamma\right)}{Z_0 \cosh \gamma + Z_R \sinh \gamma}}$$

Case 1 : Matched condition

$Z_R = Z_0$

$$Z_{in} = \frac{Z_0 (Z_0 \sinh \gamma + Z_0 \cosh \gamma)}{Z_0 \cosh \gamma + Z_0 \sinh \gamma}$$

$$\boxed{Z_{in} = Z_0}$$

Case 2 : Short circuit , $Z_R = 0$

$$Z_{in} = Z_0 \cdot \frac{Z_0 \sinh \gamma}{Z_0 \cosh \gamma}$$

$$\boxed{Z_{in} = Z_0 \tanh \gamma}$$

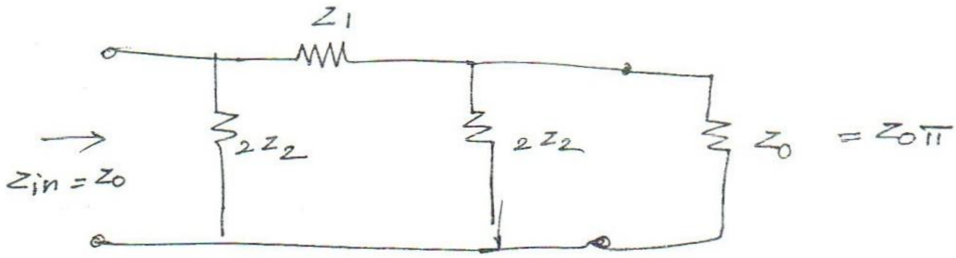
Case 3 : Open Circuit , $Z_R = \infty$

$$Z_{in} = \frac{Z_0 \cancel{Z_R} \left(\frac{Z_0}{Z_R} \sinh \gamma + \cosh \gamma \right)}{\cancel{Z_R} \left(\sinh \gamma + \frac{Z_0}{Z_R} \cosh \gamma \right)}$$

$$Z_{in} = Z_0 \frac{\cosh \gamma}{\sinh \gamma} = \frac{Z_0}{\tanh \gamma}$$

$$\boxed{Z_{in} = Z_0 \coth \gamma}$$

Symmetrical π Network.



Characteristic Impedance; U.Q – NOV 2011 (4 marks)
U.Q – MAY 2013 (6 marks)

$$Z_{in} = \frac{2Z_2 \cdot \left[\dfrac{2Z_2 Z_{0\pi}}{2Z_2 + Z_{0\pi}} + Z_1\right]}{2Z_2 + Z_1 + \dfrac{2Z_2 Z_{0\pi}}{2Z_2 + Z_{0\pi}}}$$

$$Z_{in} = \frac{2Z_2\left[2Z_2 Z_{0\pi} + Z_1(2Z_2 + Z_{0\pi})\right]}{2Z_2(2Z_2 + Z_{0\pi}) + Z_1(2Z_2 + Z_{0\pi}) + 2Z_2 Z_{0\pi}}$$

$$Z_{in} = \frac{4Z_2^2 Z_{0\pi} + 4Z_1 Z_2^2 + 2Z_1 Z_2 Z_{0\pi}}{4Z_2^2 + 2Z_2 Z_{0\pi} + 2Z_1 Z_2 + 2Z_2 Z_{0\pi} + Z_1 Z_{0\pi}}$$

Sub, $Z_{in} = Z_{0\pi}$

$$Z_{0\pi}(4Z_2^2 + 4Z_2 Z_{0\pi} + 2Z_1 Z_2 + Z_1 Z_{0\pi}) = 4Z_2^2 Z_{0\pi} + 4Z_1 Z_2^2 + 2Z_1 Z_2 Z_{0\pi}$$

$$\cancel{4Z_2^2 Z_{0\pi}} + 4Z_2 Z_{0\pi}^2 + \cancel{2Z_1 Z_2 Z_{0\pi}} + Z_1 Z_{0\pi}^2 = \cancel{4Z_2^2 Z_{0\pi}} + 4Z_1 Z_2^2 + \cancel{2Z_1 Z_2 Z_{0\pi}}$$

$$4Z_2 Z_{0\pi}^2 + Z_1 Z_{0\pi}^2 = 4Z_1 Z_2^2$$

$$Z_{0\pi}^2\left[4Z_2 + Z_1\right] = 4Z_1 Z_2^2$$

$$Z_{0\pi}^2 = \frac{4 Z_1 Z_2^2}{4 Z_2 + Z_1} = \frac{Z_1 Z_2^2}{\frac{Z_1}{4} + Z_2}$$

Multiply numerator & denominator by Z_1,

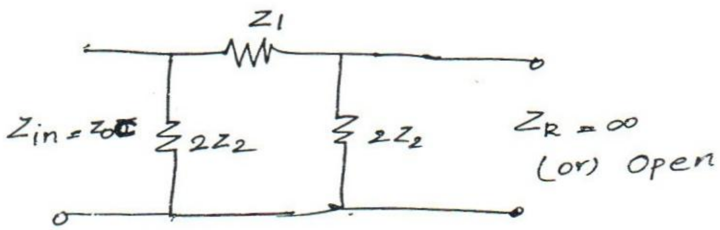
$$Z_{0\pi}^2 = \frac{Z_1^2 Z_2^2}{Z_1 Z_2 + \frac{Z_1^2}{4}}$$

$$Z_{0T} = \sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}}$$

$$Z_{0\pi}^2 = \frac{Z_1^2 Z_2^2}{Z_{0T}^2}$$

$$\boxed{Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}}}$$

Open Circuit Impedance



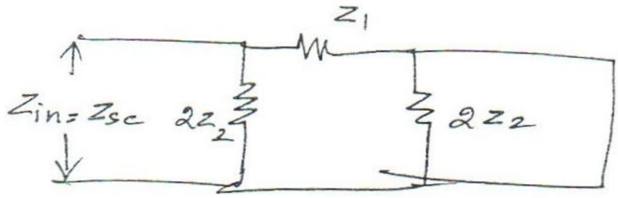
$$Z_{in} = (Z_1 + 2 Z_2) \parallel 2 Z_2$$

$$Z_{in} = \frac{(Z_1 + 2 Z_2)\, 2 Z_2}{Z_1 + 2 Z_2 + 2 Z_2} = \frac{2 Z_1 Z_2 + 4 Z_2^2}{Z_1 + 4 Z_2}$$

$$Z_{in} = Z_{oc}$$

$$\boxed{Z_{oc} = \frac{2 Z_1 Z_2 + 4 Z_2^2}{Z_1 + 4 Z_2}}$$

Short Circuit Impedance



When a resistor is short circuited, its parallel resistor is removed.

$$Z_{sc} = Z_1 \parallel 2Z_2$$

$$\boxed{Z_{sc} = \frac{2Z_1 Z_2}{Z_1 + 2Z_2}}$$

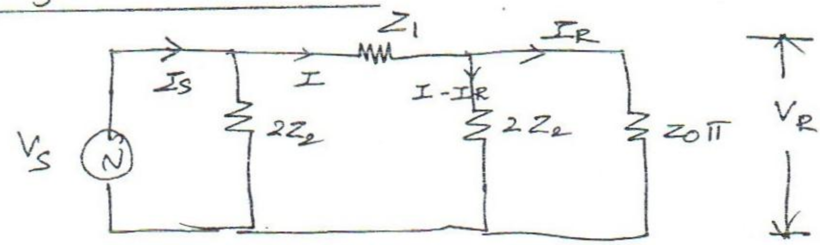
$$Z_{oc} \cdot Z_{sc} = \frac{2Z_1Z_2 + 4Z_2^2}{Z_1 + 4Z_2} \cdot \frac{2Z_1Z_2}{Z_1 + 2Z_2}$$

$$= \frac{2Z_2\cancel{(Z_1 + 2Z_2)}}{Z_1 + 4Z_2} \cdot \frac{2Z_1Z_2}{\cancel{Z_1 + 2Z_2}}$$

$$Z_{oc} Z_{sc} = \frac{4Z_1 Z_2^2}{Z_1 + 4Z_2} = Z_{0\pi}^2$$

$$\boxed{Z_{0\pi} = \sqrt{Z_{oc} Z_{sc}}}$$

Propagation Constant.



$$I_R = \frac{I \cdot 2Z_2}{2Z_2 + Z_0}$$

But $$I = \frac{I_S \cdot 2Z_2}{2Z_2 + Z_1 + \frac{2Z_2 Z_0}{2Z_2 + Z_0}}$$

$$I_S = \frac{I\left[2Z_2 + Z_1 + \frac{2Z_2 Z_0}{2Z_2 + Z_0}\right]}{2Z_2}$$

$$\frac{I_S}{I_R} = \frac{\dfrac{\cancel{I}\left[2Z_2 + Z_1 + \frac{2Z_2 Z_0}{2Z_2 + Z_0}\right]}{2Z_2}}{\cancel{I} \cdot \dfrac{2Z_2}{2Z_2 + Z_0}}$$

$$\frac{I_S}{I_R} = \frac{(2Z_2 + Z_0)\left(2Z_2 + Z_1 + \frac{2Z_2 Z_0}{2Z_2 + Z_0}\right)}{4Z_2^2}$$

$$= \frac{4Z_2^2 + 2Z_2 Z_1 + 2Z_2 Z_0 + 2Z_2 Z_0 + Z_1 Z_0}{4Z_2^2}$$

$$\frac{I_S}{I_R} = \frac{4Z_2^2 + 2Z_1 Z_2}{4Z_2^2} + \frac{4Z_2 Z_0 + Z_1 Z_0}{4Z_2^2}$$

$$\frac{I_S}{I_R} = \frac{4Z_2^2 + 2Z_1 Z_2}{4Z_2^2} + \frac{Z_0(4Z_2 + Z_1)}{4Z_2^2}$$

$$\frac{I_S}{I_R} = \left(1 + \frac{Z_1}{2Z_2}\right) + \frac{Z_0}{Z_2}\left(1 + \frac{Z_1}{4Z_2}\right) \quad \text{--- (1)}$$

$$\text{Sub. } Z_0 = Z_{0\pi} = \sqrt{\frac{4Z_1 Z_2^2}{4\cancel{Z}_2 + Z_1}}$$

$$Z_0 = \sqrt{\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}} \quad \text{--- (2)}$$

Sub. (2) in (1)

$$\frac{I_S}{I_R} = \left(1 + \frac{Z_1}{2Z_2}\right) + \sqrt{\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}} \cdot \frac{1}{Z_2}\left(1 + \frac{Z_1}{4Z_2}\right)$$

$$\frac{I_S}{I_R} = \left(1 + \frac{Z_1}{2Z_2}\right) + \sqrt{\frac{Z_1}{Z_2}} \cdot \left(1 + \frac{Z_1}{4Z_2}\right)^{1/2}$$

$$\frac{I_S}{I_R} = \left(1 + \frac{Z_1}{2Z_2}\right) + \sqrt{\frac{Z_1}{Z_2} + \frac{Z_1^2}{4Z_2^2}}$$

$$\frac{I_S}{I_R} = 1 + \frac{Z_1}{2Z_2} + \frac{\sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}}{Z_2} = e^{\gamma}$$

$$e^{\gamma} = 1 + \frac{Z_1}{2Z_2} + Z_1 \cdot \frac{\sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}}{Z_1 Z_2}$$

$$\frac{Z_1 Z_2}{Z_{0T}} = Z_{0\pi}$$

$$\frac{1}{Z_{0\pi}} = \frac{Z_{0T}}{Z_1 Z_2}$$

$$e^{\gamma} = 1 + \frac{Z_1}{2Z_2} + \frac{Z_1}{Z_{0\pi}}$$

$$\text{Sub. } Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}}$$

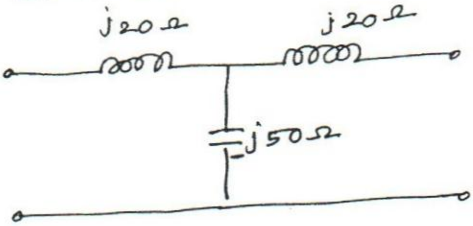
$$e^{\gamma} = 1 + \frac{Z_1}{2Z_2} + \frac{Z_{0T}}{Z_2}$$

$$\boxed{\gamma = \ln\left(1 + \frac{Z_1}{2Z_2} + \frac{Z_{OT}}{Z_2}\right)}$$

Hence, propagation constant for T and π networks are same because total series arm resistance and total shunt arm resistance are same for T and π networks

Problems.

1) Find the characteristics of the given symmetrical network.



Sol :

Given is a T network.

$\frac{Z_1}{2} = j20\ \Omega$; $Z_2 = -j50\ \Omega$

$Z_1 = j40\ \Omega$.

$$Z_0 = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

$$Z_{OC} = \frac{Z_1}{2} + Z_2$$

$$Z_{SC} = \frac{Z_0^2}{Z_{OC}}$$

$$\gamma = \ln\left[1 + \frac{Z_1}{2Z_2} + \frac{Z_0}{Z_2}\right)$$

$$Z_0 = \sqrt{(j20)^2 + (j40)(-j50)} = \sqrt{-400 + 2000}$$

$$Z_0 = \sqrt{1600} \qquad \boxed{Z_0 = 40\,\Omega}$$

$$Z_{oc} = \frac{Z_1}{2} + Z_2 = j20 - j50 = -j30\,\Omega$$

$$\boxed{Z_{oc} = -j30\,\Omega}$$

$$Z_{sc} = \frac{Z_0^2}{Z_{oc}} = \frac{1600}{-j30} = \frac{j1600}{30}$$

$$\boxed{Z_{sc} = j53.33\,\Omega}$$

$$\gamma = \ln\left[1 + \frac{Z_1}{2Z_2} + \frac{Z_0}{Z_2}\right]$$

$$\gamma = \ln\left[1 + \frac{j40}{2(-j50)} + \frac{40}{-j50}\right]$$

$$\gamma = \ln\left[1 - 0.4 + j0.8\right]$$

$$\gamma = \ln\left[0.6 + j0.8\right]$$

$$e^{\gamma} = 0.6 + j0.8 = r\angle\theta$$

$$r = \sqrt{0.6^2 + 0.8^2} = 1$$

$$\theta = \tan^{-1}\left(\frac{0.8}{0.6}\right) = 53.13^\circ = 0.93 \text{ radians}$$

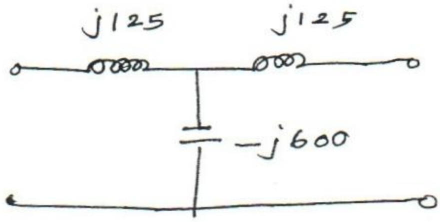
$$e^{\gamma} = e^{\alpha + j\beta} = e^{\alpha} . e^{j\beta} = e^{\alpha} \angle \beta = 1 \angle 0.93 \text{ rad}.$$

$$e^{\alpha} = 1$$

$$\alpha = \ln 1 = 0$$

$\beta = 0.9$ radians (or) 53.13 degrees.

2) Find the parameters Z_o, Z_{oc}, Z_{sc} and γ for the given circuit.



Sol :

$$\frac{Z_1}{2} = j125 \Rightarrow Z_1 = j250 \quad \& \quad Z_2 = -j600$$

$$Z_o = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = \sqrt{\frac{(j250)^2}{4} + (j250)(-j600)}$$

$$Z_o = \sqrt{-15625 + 150000} \qquad \boxed{Z_o = 366.6\,\Omega}$$

$$Z_{oc} = \frac{Z_1}{2} + Z_2 = j125 - j600 \qquad \boxed{Z_{oc} = -j475\,\Omega}$$

$$Z_o^2 = Z_{oc} Z_{sc}$$

$$Z_{sc} = \frac{Z_o^2}{Z_{oc}} \qquad Z_{sc} = \frac{(366.6)^2}{-j475} \qquad \boxed{Z_{sc} = j282.9\,\Omega}$$

$$\gamma = \ln \left[1 + \frac{Z_1}{2Z_2} + \frac{Z_o}{Z_2} \right]$$

$$\gamma = \ln\left[1 + \frac{j250}{2(-j600)} + \frac{366.6}{-j600}\right]$$

$$\gamma = \ln\left[1 - 0.208 + j\,0.611\right]$$

$$\gamma = \ln\left[0.792 + j0.611\right]$$

$$e^{\gamma} = 0.792 + j0.611 = r\angle\theta$$

$$r = \sqrt{0.792^2 + 0.611^2} = 1$$

$$\theta = \tan^{-1}\left(\frac{0.611}{0.792}\right) = 37.65 \text{ degrees} = 0.66 \text{ radians}$$

$$e^{\gamma} = e^{\alpha + j\beta} = e^{\alpha}e^{j\beta} = e^{\alpha}\angle\beta = 1\angle 0.66 \text{ radians}$$

$$e^{\alpha} = 1, \quad \text{so} \quad \alpha = \ln(1) = 0$$

$$\beta = 37.65 \text{ degress (or) } 0.66 \text{ radians}$$

3) ~~A~~ symmetrical T-network has the following parameters, $Z_{oc} = 700\,\Omega$, $Z_{sc} = 100\,\Omega$. Find the value of network elements, characteristic impedance Z_0 and propagation constant γ.

Sol:

$$Z_0 = \sqrt{Z_{oc}Z_{sc}} = \sqrt{700 \times 100}$$

$$\boxed{Z_0 = 264.57\,\Omega}$$

$$\tan h\gamma = \frac{Z_0}{Z_{oc}} = \frac{264.57}{700} = 0.378$$

$$\gamma = \tan h^{-1}\left(\frac{Z_0}{Z_{oc}}\right) = \tan h^{-1}\, 0.378 = 0.398 \text{ np}$$

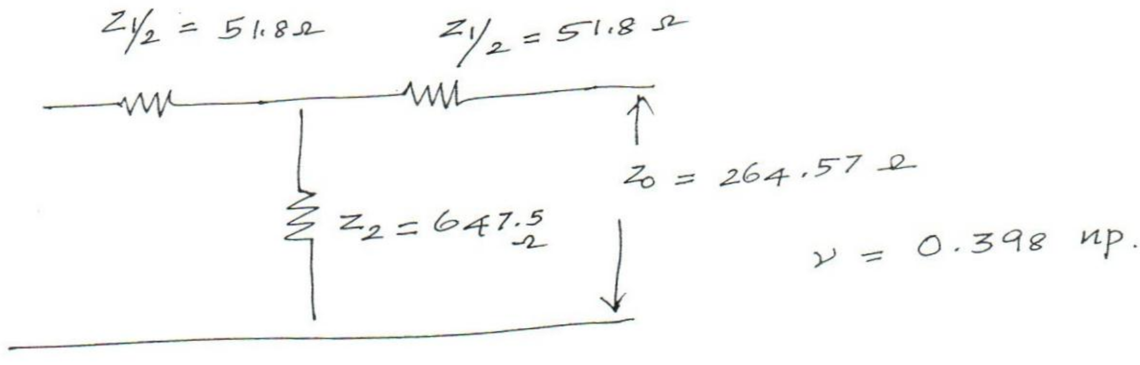
$$\boxed{\gamma = 0.398 \text{ np}}$$

$$\frac{Z_1}{2} = Z_0 \tan h\left(\frac{\gamma}{2}\right) = 264.57 \tan h(0.198)$$

$$\boxed{\frac{Z_1}{2} = 51.8\ \Omega}$$

$$Z_2 = \frac{Z_0}{\sin h\gamma} = \frac{264.57}{\sin h\, 0.398} = \frac{264.57}{0.408}$$

$$\boxed{Z_2 = 647.5\ \Omega}$$



FILTERS

An electric filter is a network with the following properties.

(i) It can transmit signals within a specified frequency band. This frequency band is called pass band.

(ii) It suppresses or attenuates signals outside the pass band. The frequency over which the signal is attenuated is called the stop band or attenuation band.

CUT-OFF FREQUENCY:

The frequency that separates the stop band and pass band (ie) transition point is called Cut-off frequency.

Characteristics of Ideal filter.

(1) Zero attenuation in Pass band.

(2) Infinite attenuation in stop band.

(3) Abrupt / Sudden transition occurs in pass band to stop band and stop band to pass band

(4) To avoid reflection loss in entire pass band, Z_0 should match characteristic impedance of the circuit

Uses of filters.

(1) In radio receivers.

(2) In TV receivers.

(3) In radio transmitters.

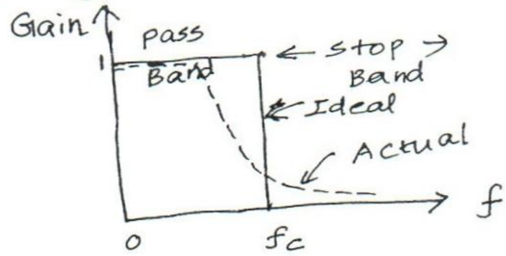
(4) In regulated d.c. power supplies.

(5) In electrical engineering, LPF & HPF are used in thyristor controlled circuits to eliminate undesired frequency components.

TYPES OF FILTERS.

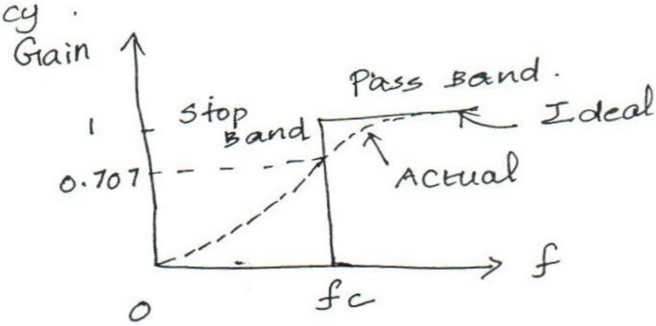
1. LOW PASS FILTER (LPF)

The filter transmits signals of all frequencies from zero to some designated frequency called cut off frequency. Beyond this, it attenuates all the signals.



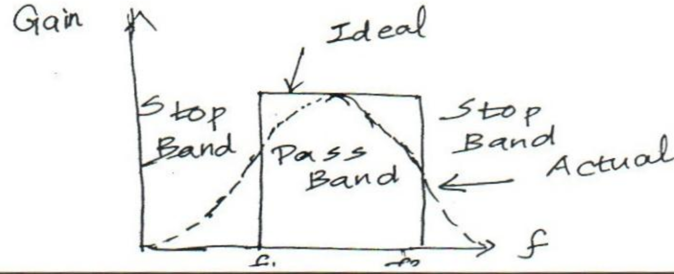
2. HIGH PASS FILTER (HPF)

this filter transmits frequencies only above a designated cutoff frequency and attenuates below this frequency.



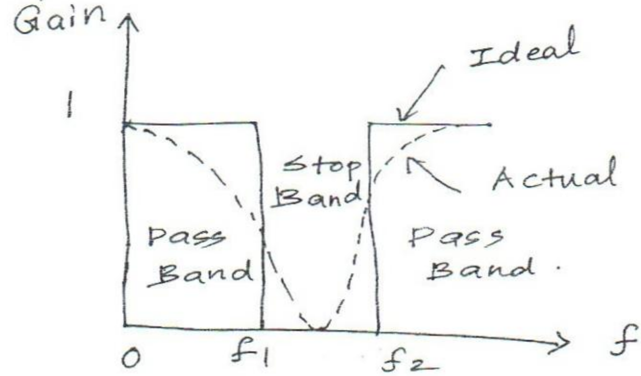
3. BAND PASS FILTER (BPF)

An ideal BPF passes all frequencies within a limited band between 2 designated cutoff frequencies f_1 and f_2 without attenuation. It fully attenuates signals below f_1 and above f_2.



4) BAND STOP (OR) BAND ELIMINATION FILTER (BSF)

It attenuates all frequencies between two designated frequencies f_1 & f_2 and passes all other frequencies.



FILTER FUNDAMENTALS.

Analysis and study of practical filters involves determination of the following.

① Attenuation Constant (α) and phase Constant (β) of the filter as a function of frequency in stop and pass bands.

② Characteristic Impedance of the filter network and its variation with frequency.

③ Determination of the filter cut-off frequency or frequencies.

$\frac{j\omega L_1}{4j\omega L_2}$, $\frac{j\omega L}{\frac{1}{j\omega C}} = j\omega L \times j\omega C$

$= j^2 \omega C$.

PASS AND STOP BANDS.

As the propagation constant γ is a function of frequency, the pass band, stop band and the cut-off point (ie) the point of separation between the two bands can be identified.

$$\sin h \frac{\gamma}{2} = \sqrt{\frac{Z_1}{4 Z_2}}$$

$$\cos h\gamma = 1 + \frac{Z_1}{2Z_2}$$
$$\cos h\gamma - 1 = \frac{Z_1}{2Z_2} \Rightarrow 2\sin h^2 \frac{\gamma}{2} = \frac{Z_1}{2Z_2}$$
$$\sin h^2 \frac{\gamma}{2} = \frac{Z_1}{4Z_2}$$
$$\sin h\left(\frac{\gamma}{2}\right) = \sqrt{\frac{Z_1}{4Z_2}}$$

$$\sin h \frac{\gamma}{2} = \sin h \left(\frac{\alpha + j\beta}{2}\right) = \sin h\left(\frac{\alpha}{2} + j\frac{\beta}{2}\right)$$

$$\sin h\left(\frac{\alpha}{2} + j\frac{\beta}{2}\right) = \sin h \frac{\alpha}{2} \cos h j\frac{\beta}{2} + \cos h\frac{\alpha}{2} \sin h j\frac{\beta}{2}$$

$$\sin h \frac{\gamma}{2} = \sin h\frac{\alpha}{2} \cos\frac{\beta}{2} + j\cos h\frac{\alpha}{2} \sin\frac{\beta}{2} \quad (1)$$

$\sin h(i\theta) = i\sin\theta$
$\cos h(i\theta) = \cos\theta$

If the network contains only pure reactances, $\frac{Z_1}{4Z_2}$ will be purely real and positive or negative

CASE 1 : Z_1 and Z_2 are of the same type.

If Z_1 & Z_2 are of same type of reactances, $\left|\frac{Z_1}{4Z_2}\right|$ is real and positive. $\left|\frac{Z_1}{4Z_2}\right| > 0$.

So imaginary term in (1) RHS will be zero.

(i) $\cos h\frac{\alpha}{2} \sin\frac{\beta}{2} = 0$

(ii) $\sin h\frac{\alpha}{2} \cos\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$

Both equations (i) & (ii) should be simultaneously satisfied.

In eqn (i) $\cosh\frac{\alpha}{2}\sin\frac{\beta}{2}=0$, $\cosh\frac{\alpha}{2}$ is not equal to zero, since its minimum value is 1.

So, $\sin\frac{\beta}{2}=0$

(ie) $\frac{\beta}{2}=\pm n\pi$ where $n=0,2,4$, etc. (iii)

$\cos\theta \to -1$ to 1

In eqn. (ii) $\sinh\frac{\alpha}{2}\cos\frac{\beta}{2}=\sqrt{\frac{Z_1}{4Z_2}}$

Based on (iii), $\cos\frac{\beta}{2}=1$. We cannot take $\cos\frac{\beta}{2}=-1$ because we cannot take RHS of (ii) to be negative. So $\cos\frac{\beta}{2}=1$.

So (ii) $\Rightarrow$ $\sinh\frac{\alpha}{2}=\sqrt{\frac{Z_1}{4Z_2}}$

$$\alpha = 2\sinh^{-1}\sqrt{\frac{Z_1}{4Z_2}}.$$

So, the condition $\left|\sqrt{\frac{Z_1}{4Z_2}}\right|>0$ implies a stop band or attenuation band of frequencies.

So, in stop band, we summarize

$$\boxed{\alpha\neq 0,\ \beta=\pi \ \ \&\ \ \alpha = 2\sinh^{-1}\sqrt{\frac{Z_1}{4Z_2}}}$$

<u>CASE 2</u> ; Z_1 and Z_2 are of opposite type.

If Z_1 and Z_2 are of opposite type,

$\frac{Z_1}{4Z_2}$ is negative. $\left|\frac{Z_1}{4Z_2}\right| < 0$ and it will be imaginary.

So ① $\Rightarrow$ $\sinh\frac{\alpha}{2}\cos\frac{\beta}{2} = 0$ — (iii)

$$\cosh\frac{\alpha}{2}\sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}} \quad \text{— (iv)}$$

Two conditions are possible from above eqUations,

(iii) $\sinh\frac{\alpha}{2} = 0$, so $\alpha = 0$; $\beta \neq 0$

$\cosh\frac{\alpha}{2} = \cosh 0 = 1$

$\sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}}$

(iv) $\cos\frac{\beta}{2} = 0$; so $\sin\frac{\beta}{2} = \pm 1$

$\alpha \neq 0$; $\beta = (2n-1)\pi$; $\cosh\frac{\alpha}{2} = \sqrt{\frac{Z_1}{4Z_2}}$

(iii) leads to pass band or region of zero attenuation, It is required that $-1 < \frac{Z_1}{4Z_2} < 0$

Here, $\beta = 2\sin^{-1}\sqrt{\frac{Z_1}{4Z_2}}$.

$\cosh\gamma = 1 + \frac{Z_1}{2Z_2} \Rightarrow \cosh(\alpha + j\beta) = 1 + \frac{Z_1}{2Z_2}$
$\cosh\alpha\cos\beta + j\sinh\alpha\sin\beta = 1 + \frac{Z_1}{2Z_2}$
In pass band, $\alpha = 0 \Rightarrow \cos\beta = 1 + \frac{Z_1}{2Z_2}$
$\cos\beta \Rightarrow -1$ to 1
$-1 < 1 + \frac{Z_1}{2Z_2} < 1 \Rightarrow -2 < \frac{Z_1}{2Z_2} < 0$

$\boxed{-1 < \frac{Z_1}{4Z_2} < 0}$

(iv) Leads to a stop band or attenuation band since $\alpha \neq 0$. The phase angle is π and attenuation is given by $\alpha = 2\cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}}$.

In this region, $\frac{Z_1}{4Z_2} < -1$.

$\beta = \pi$
$-\cosh\alpha = 1 + \frac{Z_1}{2Z_2}$
$\cosh\alpha = -1 - \frac{Z_1}{2Z_2}$
$\cosh\alpha > 1 \Rightarrow -1 - \frac{Z_1}{2Z_2} > 1$
$-1 - 1 > \frac{Z_1}{2Z_2} \Rightarrow -2 > \frac{Z_1}{2Z_2}$
$-1 > \frac{Z_1}{4Z_2}$ $\boxed{\frac{Z_1}{4Z_2} < -1}$

CONCLUSION:

If Z_1, Z_2 are same, we will get only stop band. If it is different, we will get both stop and pass bands.

Cut off frequency:

The frequencies at which the network changes from a pass network to a stop network or vice versa are called cut off frequencies.

At cut off frequency,

$$\frac{Z_1}{4Z_2} = -1$$

$$\boxed{Z_1 = -4Z_2}$$

Behaviour of Characteristic Impedance.

$$Z_0 = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = \sqrt{Z_1 Z_2\left(1 + \frac{Z_1^2}{4Z_1 Z_2}\right)}$$

$$Z_0 = \sqrt{Z_1 Z_2\left(1 + \frac{Z_1}{4Z_2}\right)}$$

When Z_1 & Z_2 are of same type

$$Z_1 = jX_1 \text{ (or) } -jX_1$$

$$Z_2 = jX_2 \text{ (or) } -jX_2$$

$$Z_0 = \sqrt{-X_1X_2\left(1+\frac{X_1}{4X_2}\right)} \Rightarrow Z_0 \text{ is Imaginary.}$$

When z_1 & Z_2 are of different type,

$$Z_1 = jX_1 \quad \& \quad Z_2 = -jX_2$$

$$Z_0 = \sqrt{\underset{+ve}{Z_1Z_2}\left(1+\underbrace{\frac{Z_1}{4Z_2}}_{-ve}\right)}$$

In Pass band, $-1 < \frac{Z_1}{4Z_2} < 0$

Eg → If $\frac{Z_1}{4Z_2} = -0.5$

$$Z_0 = \sqrt{\underset{+ve}{Z_1Z_2}\underbrace{\left(1+\frac{Z_1}{4Z_2}\right)}_{+ve}} = \sqrt{+ve} = \text{Real}$$

In the Pass band, Z_0 = Real.

In stop band, $\frac{Z_1}{4Z_2} < -1$

Eg → If $\frac{Z_1}{4Z_2} = -2$ $\quad 1+\frac{Z_1}{4Z_2} = 1-2 = -1$

$$Z_0 = \sqrt{\underset{+ve}{Z_1Z_2}\underset{-ve}{\left(1+\frac{Z_1}{4Z_2}\right)}} = \sqrt{-ve} \rightarrow \text{Imaginary.}$$

In stop band, Z_0 = Imaginary

CONSTANT K FILTERS.

Constant K filters are symmetrical T or π networks. They are called Constant K filters because their series and shut impedances Z_1 & Z_2 are related by the equation $Z_1 Z_2 = K^2$ Where K is a real number independent of frequency. It is also called prototype filter because the other more complex networks can be derived from it. Filters are classified according to the relationship between the arm impedances Z_1 & Z_2.

They are
(i) Constant K or Prototype filter.
(ii) m - derived filter.

$Z_1 Z_2 = R_K^2$

$R_K \rightarrow$ Design (or) Nominal Impedance /

U.Q - NOV 2011 (8 marks)

CONSTANT K - LPF

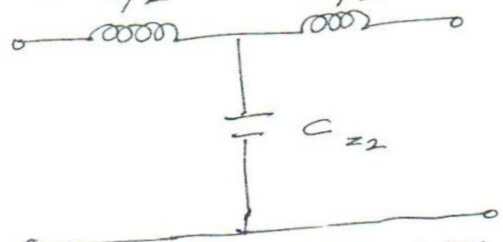


Fig → T - section LPF

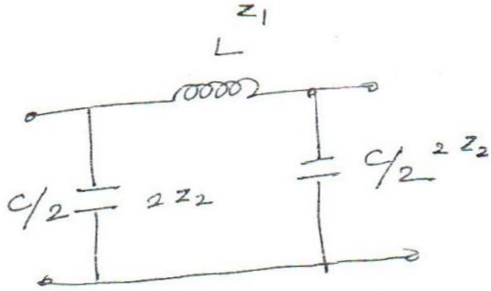


Fig → π - section LPF

The total series and shunt impedance of Symmetrical T and π networks are same. Hence, Cut off frequency f_c and design impedance R_K are same

(i) Design Impedance (R_K) :

Total series impedance is $Z_1 = j\omega L$

Total shunt impedance is $Z_2 = \frac{1}{j\omega C}$

$$Z_1 Z_2 = \frac{L}{C} = R_K^2$$

$$\boxed{R_K = \sqrt{\frac{L}{C}}} \quad \text{——} (1)$$

(ii) Cut off Frequency (f_c)

At $\omega = \omega_c$, $\quad Z_1 = -4 Z_2$

$$j\omega_c L = -\frac{4}{j\omega_c C}$$

$$\omega_c^2 = \frac{4}{LC} \quad \text{——} (A)$$

$$\omega_c = \frac{2}{\sqrt{LC}} \Rightarrow 2\pi f_c = \frac{2}{\sqrt{LC}}$$

$$\boxed{f_c = \frac{1}{\pi\sqrt{LC}}} \quad \text{——} (2)$$

(1) $\Rightarrow$ $R_K = \sqrt{\frac{L}{C}}$

(2) $\Rightarrow$ $\sqrt{L} = \frac{1}{\pi f_c \sqrt{C}}$

$$R_k = \frac{1}{\frac{\pi f_c \sqrt{C}}{\sqrt{C}}} = \frac{1}{\pi f_c \cdot C}$$

$$R_K = \frac{1}{\pi f_c\, C}$$

$$\boxed{C = \frac{1}{\pi R_k f_c}} \quad \text{—— (3)}$$

(2) $\Rightarrow \sqrt{C} = \dfrac{1}{\pi f_c \sqrt{L}}$

(1) $\Rightarrow R_k = \sqrt{\dfrac{L}{C}} = \dfrac{\sqrt{L}}{\frac{1}{\pi f_c \sqrt{L}}}$

$$R_K = \pi f_c L$$

$$\frac{R_K}{\pi f_c} = L$$

$$\boxed{L = \frac{R_K}{\pi f_c}} \quad \text{—— (4)}$$

(iii) <u>Characteristic Impedance.</u>

$$Z_{OT} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

$$Z_1 = j\omega L, \quad Z_2 = \frac{1}{j\omega C}$$

$$Z_{OT} = \sqrt{-\frac{\omega^2 L^2}{4} + \frac{L}{C}}$$

$$Z_{OT} = \sqrt{\frac{L}{C}\left[1 - \frac{\omega^2 LC}{4}\right]}$$

(A) $\Rightarrow \omega_c^2 = \dfrac{4}{LC}$

$$Z_{OT} = \sqrt{\frac{L}{C}\left(1 - \frac{\omega^2}{\omega_c^2}\right)} \Rightarrow Z_{OT} = \sqrt{\frac{L}{C}}\sqrt{1 - \frac{\omega^2}{\omega_c^2}} \quad \text{--- (5)}$$

$$Z_{OT} = R_K \sqrt{1 - \frac{f^2}{f_c^2}} \quad \text{--- (6)}$$

$$\boxed{Z_{OT} = R_K \sqrt{1 - \left(\frac{f}{f_c}\right)^2}}$$

$$Z_{O\pi} = \frac{Z_1 Z_2}{Z_{OT}} = \frac{j\omega L \cdot \frac{1}{j\omega C}}{Z_{OT}} = \frac{\frac{L}{C}}{R_K \sqrt{1 - \left(\frac{f}{f_c}\right)^2}}$$

$$Z_{O\pi} = \frac{R_K^2}{R_K \sqrt{1 - (f/f_c)^2}}$$

$$\boxed{Z_{O\pi} = \frac{R_K}{\sqrt{1 - \left(\frac{f}{f_c}\right)^2}}}$$

Graph :

	Z_{OT}	$Z_{O\pi}$
$f = 0$	R_K	R_K
$f = f_c$	0	∞
$f > f_c$	Imaginary	Imaginary

* When $\frac{f}{f_c} < 1$, Z_{OT} and $Z_{O\pi}$ are real & band is pass band.

* When $\frac{f}{f_c} > 1$, Z_{OT} & $Z_{O\pi}$ are imaginary.

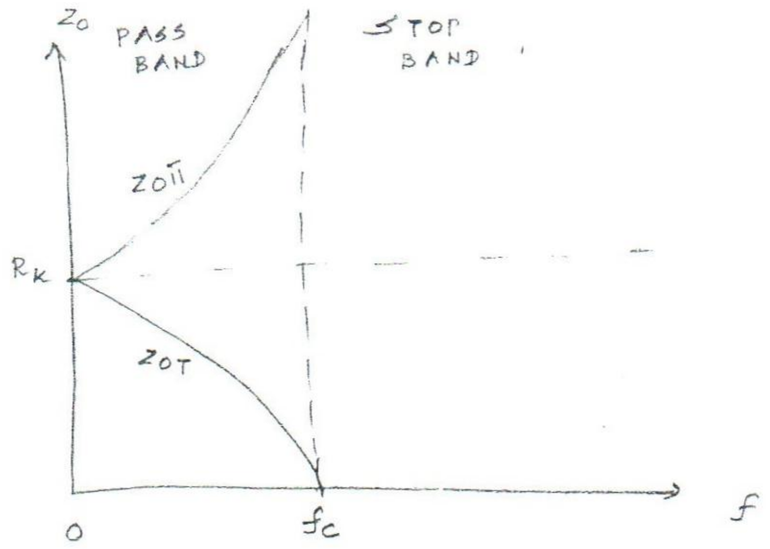


Fig → Characteristic Impedance Profile.

(iv) Attenuation and Phase Characteristics of LPF

For a two port T and π sections,

$$\cosh \gamma = 1 + \frac{Z_1}{2 Z_2} = 1 + \frac{j\omega L}{\frac{2}{j\omega C}} = 1 - \frac{\omega^2 LC}{2}$$

$$\omega_c^2 = \frac{4}{LC} \Rightarrow \frac{\omega_c^2}{2} = \frac{2}{LC}$$

$$\cosh \gamma = 1 - 2\frac{\omega^2}{\omega_c^2}$$

$$\cosh \gamma = 1 - 2\left(\frac{\omega}{\omega_c}\right)^2$$

$$\gamma = \alpha + j\beta$$

$$\cosh(\alpha + j\beta) = \cosh\alpha \cos\beta + j\sinh\alpha \sin\beta = 1 - 2\left(\frac{\omega}{\omega_c}\right)^2$$

Equating real and imaginary part,

$$\cosh\alpha \cos\beta = 1 - 2\left(\frac{\omega}{\omega_c}\right)^2$$

$$\sinh\alpha \sin\beta = 0$$

In Pass Band:

$$\alpha = 0$$

$$\cos\beta = 1 - 2\left(\frac{\omega}{\omega_c}\right)^2$$

$$\beta = n\pi$$

Since $\beta = n\pi$, $\cos\beta$ varies from $+1$ to -1,

When $\cos\beta = 1$ and $\alpha = 0 \Rightarrow$

$$1 - \frac{\omega^2 LC}{2} = 1$$

$$\omega^2 LC = 0$$

$$\omega = 0.$$

This gives the lower cut-off frequency in pass band of LPF. Here $\beta = 0$

When $\cos\beta = -1$

$$+1 - \frac{\omega^2 LC}{2} = -1$$

$$\omega^2 LC = 4$$

$$\omega^2 = \frac{4}{LC} \qquad f = \frac{1}{\pi\sqrt{LC}}$$

This corresponds to higher cutoff freq in pass band.

Here $\beta = \pi$.

$$\cos\beta = 1 - 2\left(\frac{\omega}{\omega_c}\right)^2$$

$$1 - \cos\beta = 2\left(\frac{\omega}{\omega_c}\right)^2$$

$$2\left(\frac{\omega}{\omega_c}\right)^2 = 2\sin^2\left(\frac{\beta}{2}\right)$$

$$\sin^2(\beta/2) = \left(\frac{\omega}{\omega_c}\right)^2 = \left(f/f_c\right)^2$$

$$\sin\frac{\beta}{2} = f/f_c$$

$$\frac{\beta}{2} = \sin^{-1}\left(f/f_c\right)$$

$$\boxed{\beta = 2\sin^{-1} f/f_c}$$

CONCLUSION : In pass band,

Attenuation, $\alpha_p = 0$

Phase Shift, $\beta_p = 2\sin^{-1}\left(f/f_c\right)$ radians.

In stop Band :

$\alpha \neq 0$; $\sin\beta = 0 \Rightarrow \beta = n\pi$.

When $n = 1$,

$$\cosh\alpha\cos\beta = 1 - 2\left(\frac{\omega}{\omega_c}\right)^2$$

$$-\cosh\alpha = 1 - 2\left(\frac{\omega}{\omega_c}\right)^2$$

$$1 + \cosh\alpha = 2\left(\frac{\omega}{\omega_c}\right)^2$$

$$2\cosh^2\left(\frac{\alpha}{2}\right) = 2\left(\frac{\omega}{\omega_c}\right)^2$$

$$\cosh^2\left(\frac{\alpha}{2}\right) = \left(\frac{\omega}{\omega_c}\right)^2$$

$$\cosh\left(\frac{\alpha}{2}\right) = \frac{f}{f_c}$$

$$\boxed{\alpha_s = 2 \cosh^{-1}(f/f_c)}$$

CONCLUSION:

In stop Band,

Attenuation, $\alpha_s = 2 \cosh^{-1}(f/f_c)$

Phase shift, $\beta = \pm \pi$

GRAPH:

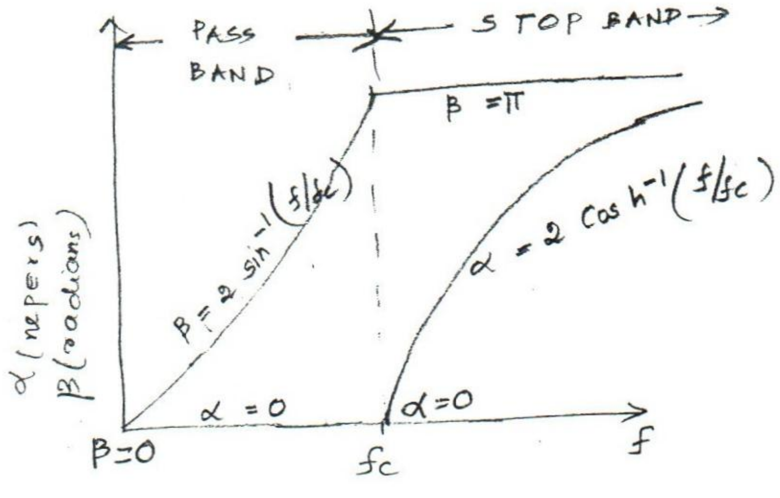


FIG: α and β characteristics of a T or π LPF.

DESIGN EQUATIONS FOR CONSTANT K-LPF (T & π SECTIONS)

(i) Design Impedance, $R_K = \sqrt{\frac{L}{C}}$

(ii) Cut off Frequency, $f_c = \frac{1}{\pi\sqrt{LC}}$

(iii) Capacitance, $C = \frac{1}{\pi R_K f_c}$

(iv) Inductance, $L = \frac{R_K}{\pi f_c}$

Problems.

1) Design a low pass filter for 600 Ω impedance having cut off frequency 3000 HZ.

Sol:

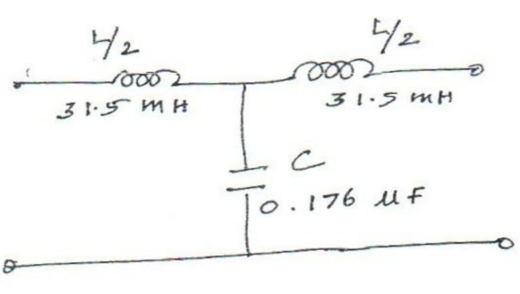
$f_c = 3000$ HZ, $R_K = 600\,\Omega$

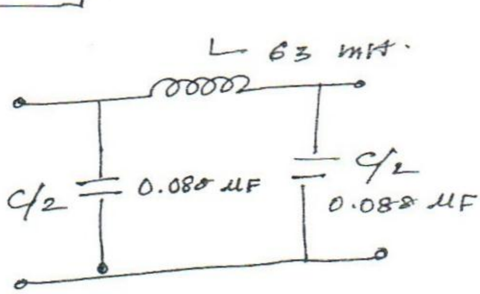
$$C = \frac{1}{\pi f_c R_K} = \frac{1}{3.14 \times 3000 \times 600}$$

$$\boxed{C = 0.1768\ \mu F}$$

$$L = \frac{R_K}{\pi f_c} = \frac{600}{3.14 \times 3000}$$

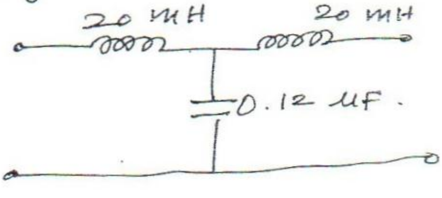
$$\boxed{L = 0.063\ H = 63\ mH}$$





2) For the constant k LPF, determine (i) f_c (ii) Design Impedance (iii) Phase Difference between I/p and O/p and the ratio of I/p voltage to O/p voltage at a frequency 1.5 KHZ and 5 KHZ.

The given network is



Sol :

$L/2 = 20 \text{ mH} \Rightarrow L = 40 \text{ mH}$

$C = 0.12 \ \mu F.$

(i) Cut off frequency, $f_c = \dfrac{1}{\pi\sqrt{LC}} = \dfrac{1}{\pi\sqrt{40\times10^{-3}\times0.12\times10^{-6}}}$

$$\boxed{f_C = 4.59 \text{ KHZ}}$$

(ii) $R_K = \sqrt{\dfrac{L}{C}} = \sqrt{\dfrac{40\times10^{-3}}{0.12\times10^{-6}}}$

Design Impedance, $\boxed{R_K = 577.35 \ \Omega}$

(iii) Phase difference between input, output.

$f_1 = 1.5 \text{ KHZ}, \ f_2 = 5 \text{ KHZ}.$

$f_1 < f_c$. So, f_1 lies in pass band.

In pass band, $\alpha = 0$

$$\beta = 2\sin^{-1}(f/f_c) = 2\sin^{-1}\left(\frac{1.5\times10^3}{4.59\times10^3}\right)$$

$$\boxed{\beta = 38.14^\circ \text{ (or) } 0.66 \text{ radians}}$$

Attenuation, Phase shift at 5 KHZ.

$f_2 = 5 \text{ KHZ}, \ f_c = 4.6 \text{ KHZ}.$

$f_2 > f_c$. So f_2 lies in stop band.

In stop band, $\beta = \pm\pi$

$$\alpha = 2\cosh^{-1}\left(\frac{f}{f_c}\right) = 2\cosh^{-1}\left(\frac{5\times10^3}{4.6\times10^3}\right)$$

$$\boxed{\alpha = 0.828 \text{ nepers}; \ \beta = \pm\pi}$$

U.Q. - NOV 2011 (8 marks)

CONSTANT K - HIGH PASS FILTER

HPF can be constructed by interchanging series and shunt arm capacitances of LPF (ie) Capacitor is the series component and inductor is the shunt component.

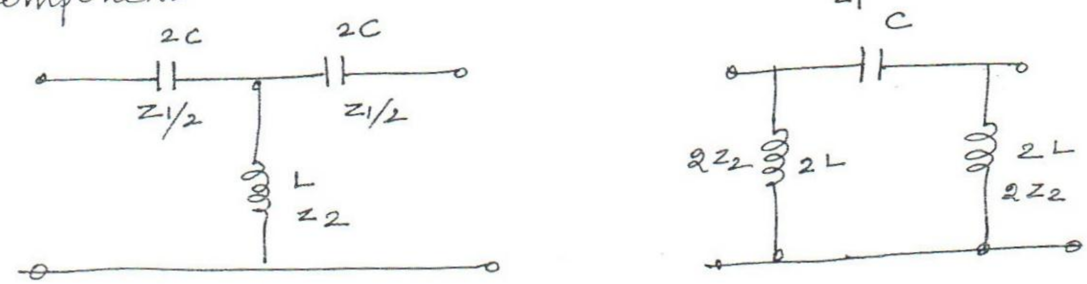


fig. T and π section HPF.

Series arm impedance, $Z_1 = \frac{1}{j\omega C}$

Shunt arm impedance, $Z_2 = j\omega L$.

As the series and shunt elements of T and π networks are same, f_c and R_K will be same for both T and π network.

(i) Design Impedance (R_K):

$$Z_1 Z_2 = R_K^2$$

$$\frac{1}{j\omega C} j\omega L = R_K^2$$

$$R_K^2 = \frac{L}{C}$$

$$\boxed{R_K = \sqrt{\frac{L}{C}}} \quad \text{—— (1)}$$

R_K → Design / Nominal Impedance, independent of frequency

(ii) Cut-off Frequency

$$Z_1 = -4 Z_2$$

$$\frac{1}{j\omega C} = -4 \cdot j\omega L$$

$$1 = 4\omega^2 LC$$

$$\boxed{\omega_c^2 = \frac{1}{4LC}} \longrightarrow (2)$$

$$(2\pi f_c)^2 = \frac{1}{4LC}$$

$$4\pi^2 f_c^2 = \frac{1}{4LC}$$

$$f_c^2 = \frac{1}{16\pi^2 LC}$$

$$\boxed{f_c = \frac{1}{4\pi\sqrt{LC}}} \longrightarrow (3)$$

(iii) Design Equations.

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

$$\sqrt{L} = \frac{1}{4\pi f_c \sqrt{C}}$$

Sub. $\sqrt{L}$ in (1)

$$R_K = \frac{1}{4\pi f_c C}$$

$$\boxed{C = \frac{1}{4\pi f_c R_K}} \longrightarrow (4)$$

$$R_K = \sqrt{\frac{L}{C}} = \frac{\sqrt{L}}{\frac{1}{4\pi f_c \sqrt{L}}}$$

$$R_K = 4\pi f_c L$$

$$\boxed{L = \frac{R_K}{4\pi f_c}} \quad \text{---} (5)$$

(iv) Characteristic Impedance

$$Z_{0T} = \sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}}$$

$$Z_1 = \frac{1}{j\omega C} ; \quad Z_2 = j\omega L$$

$$Z_{0T} = \sqrt{\frac{1}{j\omega C} j\omega L + \frac{(1/j\omega C)^2}{4}} = \sqrt{\frac{L}{C} - \frac{1}{4\omega^2 C^2}}$$

$$Z_{0T} = \sqrt{\frac{L}{C}\left(1 - \frac{1}{4\omega^2 LC}\right)}$$

$$\omega_c^2 = \frac{1}{4LC}$$

$$Z_{0T} = \sqrt{\frac{L}{C}\left(1 - \frac{\omega_c^2}{\omega^2}\right)}$$

$$Z_{0T} = \sqrt{\frac{L}{C}\left[1 - \left(\frac{f_c}{f}\right)^2\right]} = \sqrt{\frac{L}{C}} \sqrt{1 - \frac{f_c^2}{f^2}}$$

$$\boxed{Z_{0T} = R_K \sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \quad \text{---} (6)$$

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{\frac{1}{j\omega C}\cdot j\omega L}{Z_{0T}} = \frac{R_K^2}{R_K\sqrt{1-\frac{f_c^2}{f^2}}}$$

$$\boxed{Z_{0\pi} = \frac{R_K}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}} \quad \text{—— (7)}$$

	Z_{0T}	$Z_{0\pi}$	
$f=0$	Imaginary	Imaginary	Stop band
$f=f_c$	0	∞	
$f=\infty$	R_K	R_K	Pass band.

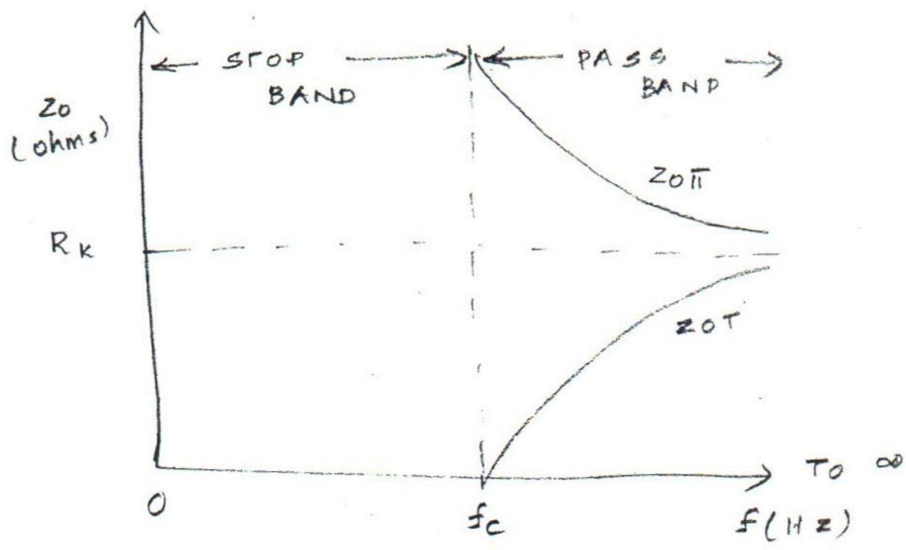


FIG : Z_0 profile for T and π HPF.

(V) <u>Attenuation and phase shift characteristics of HPF.</u>

$$\cosh \gamma = 1 + \frac{Z_1}{2Z_2}$$

$$\cosh \gamma = 1 + \frac{\frac{1}{j\omega C}}{2(j\omega L)} = 1 - \frac{1}{2\omega^2 LC}$$

$$\cosh\alpha \cos\beta = 1 - \frac{1}{2\omega^2 LC} \quad \text{—— (i)}$$

$$\sinh\alpha \sin\beta = 0 \quad \text{—— (ii)}$$

Pass Band.

If $\alpha = 0$, $\cosh\alpha = 1$, $\cos\beta = 1 - \frac{1}{2\omega^2 LC}$

$$1 - \cos\beta = \frac{1}{2\omega^2 LC}$$

$$2\sin^2\frac{\beta}{2} = \frac{1}{2\omega^2 LC}$$

$$\sin^2\frac{\beta}{2} = \frac{1}{4\omega^2 LC} \qquad \omega_c^2 = \frac{1}{4LC}$$

$$\sin^2\frac{\beta}{2} = \frac{\omega_c^2}{\omega^2} = \frac{f_c^2}{f^2}$$

$$\sin\frac{\beta}{2} = \frac{f_c}{f}$$

$$\boxed{\beta = 2\sin^{-1}\left(\frac{f_c}{f}\right)} \quad \text{—— (8)}$$

PASS BAND

$\alpha_p = 0$

$\beta_p = 2\sin^{-1}\left(\frac{f_c}{f}\right)$

Stop Band.

$$\beta = \pm\pi, \quad \alpha \neq 0$$

$\beta = \pi \Rightarrow_{(i)} \Rightarrow -\cosh\alpha = 1 - \frac{1}{2\omega^2 LC}$

$$\cosh\alpha = \frac{1}{2\omega^2 LC} - 1$$

$$\cosh\alpha = 2\left(\frac{\omega_c^2}{\omega^2}\right) - 1$$

55

$$1 + \cosh\alpha = 2\frac{f_c^2}{f^2}$$

$$2\cosh^2\frac{\alpha}{2} = 2\frac{f_c^2}{f^2}$$

$$\cosh\left(\frac{\alpha}{2}\right) = \frac{f_c}{f}$$

$$\boxed{\alpha = 2\cosh^{-1}\left(\frac{f_c}{f}\right)} \quad \text{—— (9)}$$

STOP BAND

$$\beta_s = \pm\pi$$

$$\alpha_s = 2\cosh^{-1}\left(\frac{f_c}{f}\right)$$

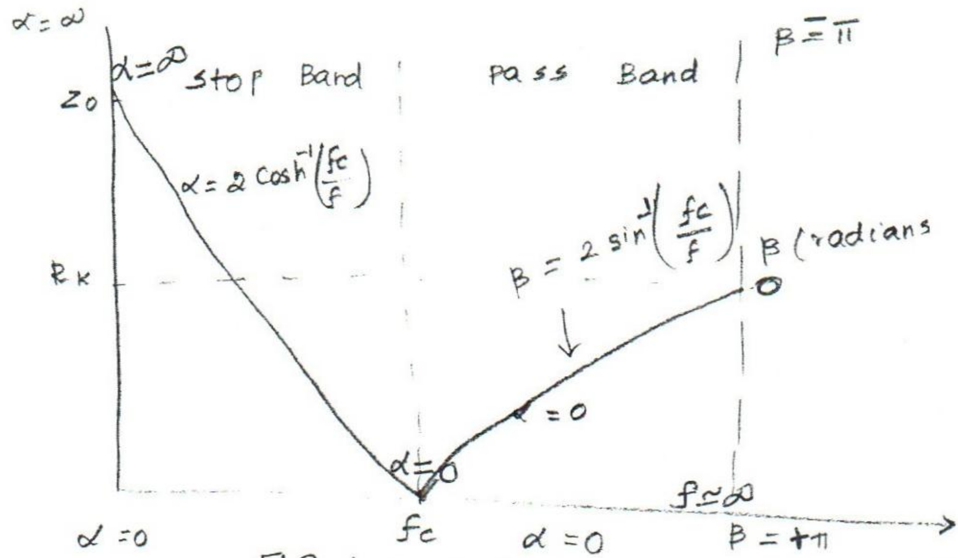


FIG. : α and β variation of HPF for T & π sections

DESIGN EQUATIONS

(i) Design Impedance, $R_K = \sqrt{\frac{L}{C}}$

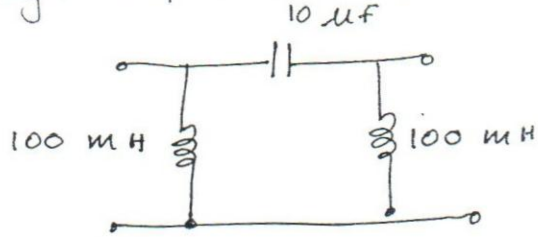
(ii) Cut off Frequency, $f_c = \frac{1}{4\pi\sqrt{LC}}$

(iii) Inductance, $L = \frac{R_K}{4\pi f_c}$

(iv) Capacitance, $C = \frac{1}{4\pi R_K f_c}$

Problems -

1) Determine the cut off frequency of HPF and design impedance of the given network.



Sol:

$Z_1 = 10\ \mu F$; $2Z_2 = 100\ mH$, $Z_2 = 50\ mH$

$$f_c = \frac{1}{4\pi\sqrt{LC}} = \frac{1}{4\pi\sqrt{10\times10^{-6}\times50\times10^{-3}}}$$

$$\boxed{f_c = 112.59\ Hz}$$

$$R_K = \sqrt{\frac{L}{C}} = \sqrt{\frac{50\times10^{-3}}{10\times10^{-6}}}$$

$$\boxed{R_K = 70.71\ \Omega}$$

2) Design a HPF whose cut-off frequency is 2000 Hz and design impedance is 600 Ω,

Sol:

$f_c = 2000\ Hz$

$R_K = 600\ \Omega$

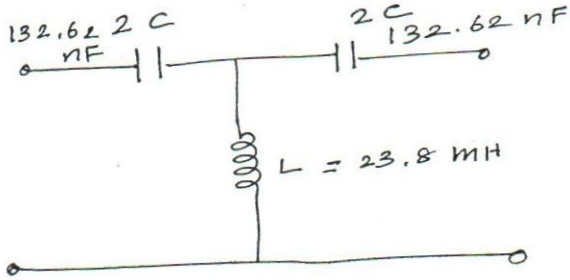
$$C = \frac{1}{4\pi R_K f_c} = \frac{1}{4\pi\times600\times2000}$$ $\boxed{C = 66.31\ nF}$

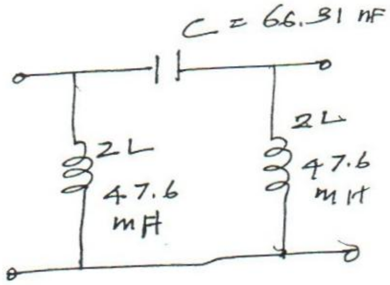
$$L = \frac{R_k}{4\pi f_c} = \frac{600}{4\pi \times 2000}$$

$$\boxed{L = 0.0238 \text{ H} = 23.8 \text{ mH}}$$

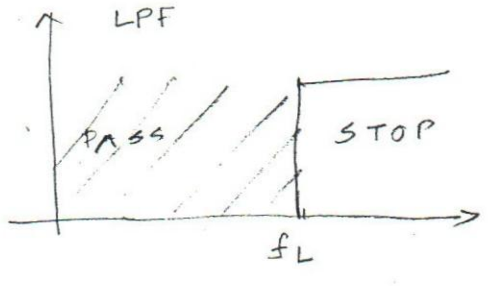
$2C = 132.62$ nF

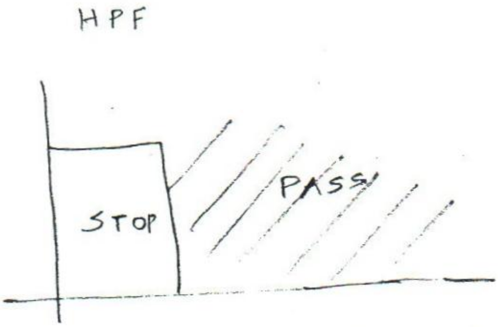
$2L = 47.6$ mH



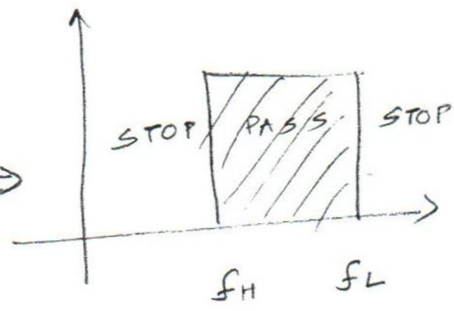


CONSTANT K – BAND PASS FILTER (BPF)





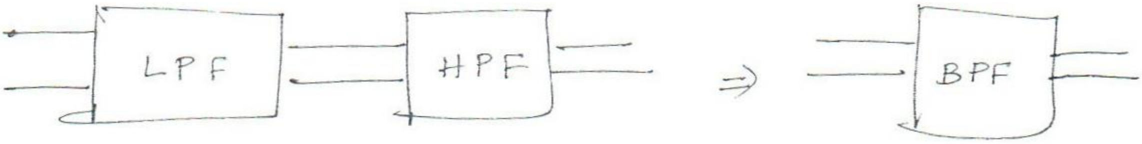
$\Rightarrow$



$f_L > f_H$

A BPF can be realized by a LPF followed by HPF and cut off frequency of LPF is higher than cut-off frequency of HPF

$$\boxed{f_{cL} > f_{cH}}$$



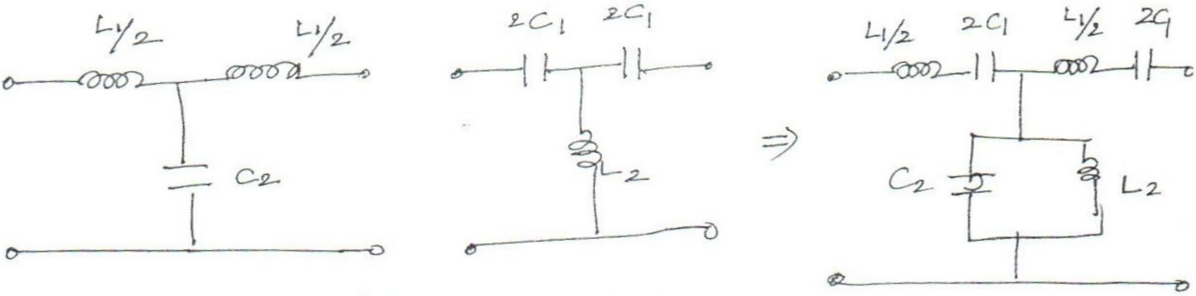


FIG → T - BPF

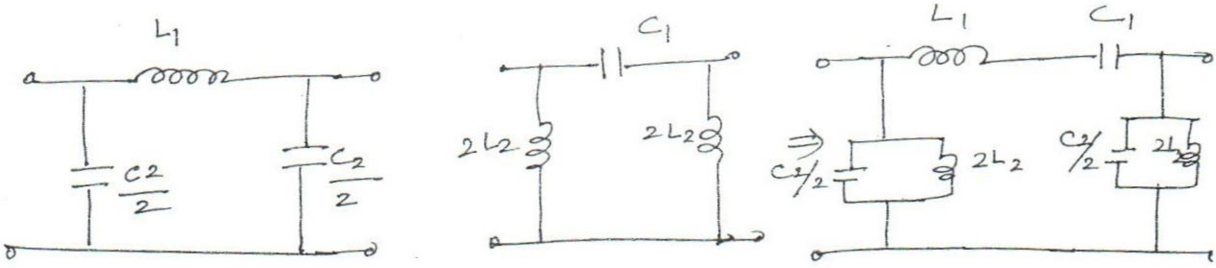


FIG. → π BPF

Series arm Impedance, $Z_1 = j\omega L_1 + \frac{1}{j\omega C_1}$ (L_1, C_1 in series)

$$Z_1 = \frac{1 - \omega^2 L_1 C_1}{j\omega C_1} \quad \text{——} (1)$$

Shunt arm Impedance, $Z_2 = L_2 \parallel C_2$

$$Z_2 = j\omega L_2 \parallel \frac{1}{j\omega C_2} = \frac{j\omega L_2 \cdot \frac{1}{j\omega C_2}}{j\omega L_2 + \frac{1}{j\omega C_2}}$$

$$Z_2 = \frac{L_2/C_2}{\frac{1 - \omega^2 L_2 C_2}{j\omega C_2}} \Rightarrow Z_2 = \frac{j\omega L_2}{1 - \omega^2 L_2 C_2} \quad \text{——} (2)$$

At resonance, $\omega = \omega_0$, $\omega L_1 = \frac{1}{\omega C_1}$

$$\omega_0^2 = \frac{1}{L_1 C_1}$$

$$\omega_0 L_2 = \frac{1}{\omega_0 C_2} \Rightarrow \omega_0^2 = \frac{1}{L_2 C_2}$$

Comparing, $\omega_0^2 = \frac{1}{L_1 C_1} = \frac{1}{L_2 C_2}$

$$L_1 C_1 = L_2 C_2 \quad \text{——— (3)}$$

(i) Design Impedance.

$$R_K^2 = Z_1 Z_2$$

$$R_K^2 = \left(\frac{1 - \omega^2 L_1 C_1}{j\omega C_1}\right)\left(\frac{j\omega L_2}{1 - \omega^2 L_2 C_2}\right)$$

(3) $\Rightarrow L_1 C_1 = L_2 C_2$

$$R_K^2 = \frac{L_2}{C_1}$$

$L_1 C_1 = L_2 C_2$

$\frac{L_2}{C_1} = \frac{L_1}{C_2}$

$$\boxed{R_K = \sqrt{\frac{L_2}{C_1}} = \sqrt{\frac{L_1}{C_2}}} \quad \text{——→ (4)}$$

(ii) Cut off Frequency.

$$Z_1 = -4 Z_2$$

Multiply by Z_1 on both sides, $Z_1^2 = -4 Z_1 Z_2$

$$Z_1^2 = -4 R_K^2$$

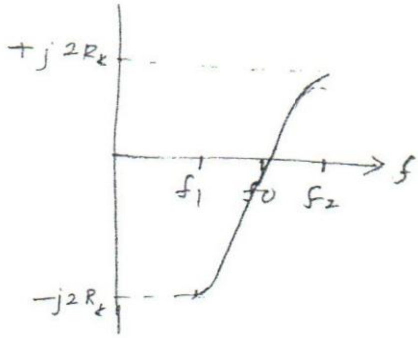
$$Z_1 = \pm 2j R_K$$

At lower cut off freq, $f = f_1$, $Z_1 = -2j R_K$

At upper cut off freq $f = f_2$, $Z_1 = +2jR_K$.

$$Z_1 = -2jR_K \Big|_{\omega = \omega_1}$$

$$Z_1 = 2jR_K \Big|_{\omega = \omega_2}$$



To find lower cut off frequency.

At $\omega = \omega_1$, $Z_1 = -2jR_K$

$$\frac{1 - \omega_1^2 L_1 C_1}{j\omega_1 C_1} = -2jR_K$$

$$1 - \omega_1^2 L_1 C_1 = 2R_K \omega_1 C_1$$

$$1 = 2R_K \omega_1 C_1 + \omega_1^2 L_1 C_1$$

$$\boxed{\omega_1^2 L_1 C_1 + 2R_K C_1 \omega_1 - 1 = 0}$$

$$\boxed{ax^2 + bx + c = 0}$$

$a = L_1 C_1$

$b = 2R_K C_1$

$c = -1$

$$\omega_1 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\omega_1 = \frac{-2R_K C_1 \pm \sqrt{4R_K^2 C_1^2 + 4L_1 C_1}}{2L_1 C_1}$$

$$\omega_1 = -\frac{R_K}{L_1} \pm \sqrt{\frac{4R_K^2 C_1^2}{4L_1^2 C_1^2} + \frac{4L_1 C_1}{4L_1^2 C_1^2}}$$

$$\omega_1 = -\frac{R_K}{L_1} \pm \sqrt{\frac{R_K^2}{L_1^2} + \frac{1}{L_1 C_1}}$$

$$\omega_1 = -\frac{R_K}{L_1} \pm \frac{1}{L_1}\sqrt{R_K^2 + \frac{L_1}{C_1}}$$

Taking positive term,

$$\omega_1 = -\frac{R_K}{L_1} + \frac{1}{L_1}\sqrt{R_K^2 + \frac{L_1}{C_1}}$$

$$2\pi f_1 = \frac{-R_K + \sqrt{R_K^2 + \frac{L_1}{C_1}}}{L_1}$$

$$\boxed{f_1 = \frac{-R_K + \sqrt{R_K^2 + \frac{L_1}{C_1}}}{2\pi L_1}} \tag{5}$$

Similarly,

$$\boxed{f_2 = \frac{R_K + \sqrt{R_K^2 + \frac{L_1}{C_1}}}{2\pi L_1}} \tag{6}$$

Bandwidth.

$$\Delta f = f_2 - f_1$$

$$\Delta f = \frac{R_K + \sqrt{R_K^2 + \frac{L_1}{C_1}}}{2\pi L_1} + \frac{R_K - \sqrt{R_K^2 + \frac{L_1}{C_1}}}{2\pi L_1}$$

$$\Delta f = \frac{2 R_K}{2\pi L_1}$$

$$\boxed{\Delta f = \frac{R_K}{\pi L_1}} \tag{7}$$

Resonant frequency.

$Z_1 = -j2 R_K$ at f_1

$Z_2 = +j2 R_K$ at f_2

Z_1 at $\omega_1 = -Z_1$ at ω_2

$$\omega_0^2 = \frac{1}{L_1C_1}$$

$$\omega_0^2 = \frac{1}{L_2C_2}$$

$$\frac{1-\omega_1^2L_1C_1}{j\omega_1C_1} = \frac{\omega_2^2L_1C_1-1}{j\omega_2C_1}$$

$$\frac{1-\frac{\omega_1^2}{\omega_0^2}}{\omega_1} = \frac{\frac{\omega_2^2}{\omega_0^2}-1}{\omega_2}$$

$$1-\frac{\omega_1^2}{\omega_0^2} = \frac{\omega_1}{\omega_2}\left[\frac{\omega_2^2}{\omega_0^2}-1\right]$$

$$\omega_0^2-\omega_1^2 = \frac{\omega_1}{\omega_2}\left[\omega_2^2-\omega_0^2\right]$$

$$\omega_2\omega_0^2-\omega_2\omega_1^2 = \omega_1\omega_2^2-\omega_1\omega_0^2$$

$$\omega_0^2(\omega_1+\omega_2) = \omega_1\omega_2(\omega_1+\omega_2)$$

$$\omega_0^2 = \omega_1\omega_2$$

$$\boxed{f_0 = \sqrt{f_1f_2}} \quad \text{— (8)}$$

(iii) Characteristic Impedance

$$Z_{0T} = \sqrt{Z_1Z_2+\frac{Z_1^2}{4}}$$

$$Z_{0T} = \sqrt{\underbrace{\frac{(1-\omega^2L_1C_1)}{j\omega C_1}\cdot\frac{j\omega L_2}{(1-\omega^2L_2C_2)}}_{R_K} + \frac{\left[\frac{1-\omega^2L_1C_1}{j\omega C_1}\right]^2}{4}}$$

$$Z_1Z_2 = R_K{}^2$$

$$Z_{OT} = \sqrt{R_K^2 - \frac{(1-\omega^2 L_1 C_1)^2}{4\omega^2 C_1^2}}$$

Resonant frequency, $\omega_0^2 L_1 C_1 = 1$

At resonance, $Z_{OT} = R_K$

At f_1 & f_2, $Z_{OT} = 0$

At $f = f_1 \Rightarrow Z_1 = -2j\,R_K$
At $f = f_2 \Rightarrow Z_2 = +2j\,R_K$
At $f = f_1$, $Z_{OT} = \sqrt{R_K^2 - 4\frac{R_K^2}{4}} = 0$
At $f = f_2$, $Z_{OT} = \sqrt{R_K^2 - 4\frac{R_K^2}{4}} = 0$

$$Z_{O\pi} = \frac{Z_1 Z_2}{Z_{OT}} = \frac{R_K^2}{\sqrt{R_K^2 - \frac{(1-\omega^2 L_1 C_1)^2}{4\omega^2 C_1^2}}}$$

At resonance, $Z_{O\pi} = \frac{R_K^2}{R_K}$, $Z_{O\pi} = R_K$

$Z_{O\pi}$ at f_1 & f_2 $\Rightarrow \frac{R_K^2}{Z_{OT}}$, $Z_{O\pi} = \infty$.

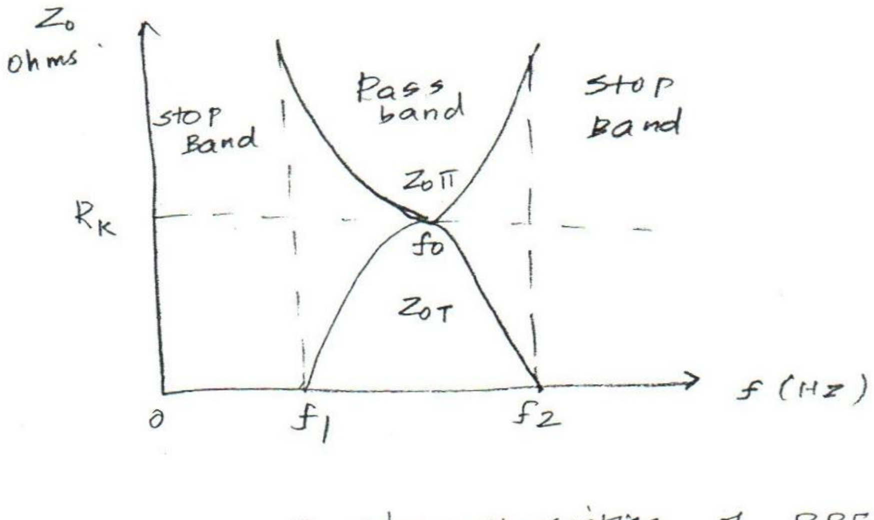


FIG. Z_0 characteristics of BPF.

(iv) Design Equations.

To find C_1, C_2, L_1 and L_2

$$\omega_1^2 L_1 C_1 + 2 R_K C_1 \omega_1 - 1 = 0$$

Sub, $\omega_0^2 = \frac{1}{L_1 C_1} \Rightarrow L_1 C_1 = \frac{1}{\omega_0^2}$

$$\frac{\omega_1^2}{\omega_0^2} + 2 R_K C_1 \omega_1 - 1 = 0$$

$$\omega_1^2 + 2 R_K C_1 \omega_1 \omega_0^2 - \omega_0^2 = 0$$

$$2 R_K C_1 \omega_1 \omega_0^2 = \omega_0^2 - \omega_1^2$$

Sub $\omega_0^2 = \omega_1 \omega_2$

$$2 R_K C_1 \omega_1 \omega_0^2 = \omega_1 \omega_2 - \omega_1^2$$

$$2 R_K C_1 \cancel{\omega_1} \omega_0^2 = \cancel{\omega_1}(\omega_2 - \omega_1)$$

$$2 R_K C_1 \omega_0^2 = \omega_2 - \omega_1$$

$$C_1 = \frac{\omega_2 - \omega_1}{2 R_K \omega_0^2} = \frac{2\pi(f_2 - f_1)}{2 R_K \cdot (2\pi f_0)^2}$$

$$\boxed{C_1 = \frac{f_2 - f_1}{4 \pi R_K f_0^2}} \quad \text{—— (9)}$$

$$\boxed{f_0^2 = f_1 f_2}$$

$$R_K^2 = \frac{L_2}{C_1}$$

$$L_2 = C_1 R_K^2$$

$$\boxed{L_2 = \frac{R_K (f_2 - f_1)}{4 \pi f_0^2}} \quad \text{—— (10)}$$

$$\omega_0^2 = \frac{1}{L_1 C_1} \Rightarrow L_1 = \frac{1}{\omega_0^2 C_1}$$

$$L_1 = \frac{1}{4\pi^2 f_0^2 \cdot \dfrac{(f_2 - f_1)}{4\pi R_K f_0^2}}$$

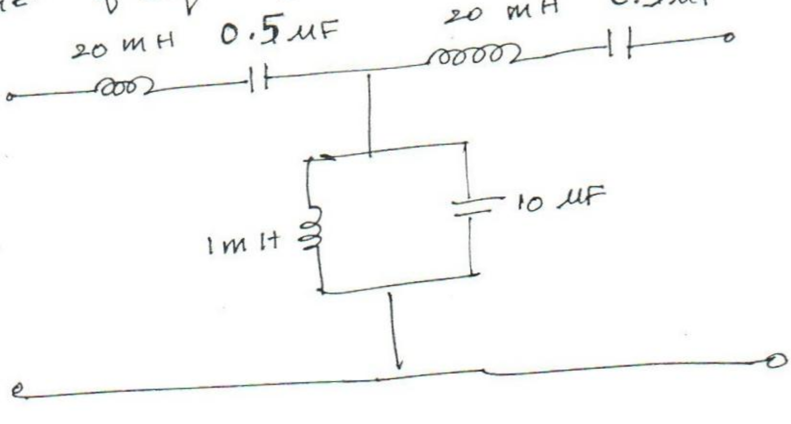
$$\boxed{L_1 = \frac{R_K}{\pi (f_2 - f_1)}} \quad \text{—— (11)}$$

$$R_K^2 = \frac{L_1}{C_2} \Rightarrow C_2 = \frac{L_1}{R_K^2}$$

$$\boxed{C_2 = \frac{1}{\pi R_K (f_2 - f_1)}} \quad \text{—— (12)}$$

Problems.

1) For the network shown below, calculate resonant frequency.



Sol:

$\frac{L_1}{2} = 20$ mH $\Rightarrow L_1 = 40$ mH

$2C_1 = 0.5\ \mu F \Rightarrow C_1 = 0.25\ \mu F$

$L_2 = 1$ mH & $C_2 = 10\ \mu F$.

$$f_1 = \frac{-R_K + \sqrt{R_K^2 + \frac{L_1}{C_1}}}{2\pi L_1}$$

$$R_K = \sqrt{\frac{L_1}{C_2}} = 63.24\ \Omega.$$

$$f_1 = 1.35\ \text{Hz}$$

$$f_2 = \frac{R_K + \sqrt{R_K^2 + \frac{L_1}{C_1}}}{2\pi L_2}$$

$$f_2 = 1.86\ \text{Hz}$$

$$f_0^2 = f_1 f_2 = 79.97 \times 10^3\ \text{Hz}$$

$$\boxed{f_0 = 1.58\ \text{Hz}}$$

2) Design a band pass filter to work into input and output resistances of 100 ohms and have a pass band from 4800 Hz to 5200 Hz.

Sol:

$R_K = 100\ \Omega$

$f_1 = 4800\ \text{Hz}$

$f_2 = 5200\ \text{Hz}$

$$L_1 = \frac{R_K}{\pi (f_2 - f_1)}$$

$$L_1 = 79.57 \text{ mH}$$

$$C_1 = \frac{f_2 - f_1}{4 \pi R_K f_1 f_2}$$

$$C_1 = 12.75 \text{ nF}$$

$$L_2 = \frac{R_K (f_2 - f_1)}{4 \pi f_1 f_2}$$

$$L_2 = 0.1275 \text{ mH}$$

$$C_2 = \frac{1}{\pi R_K (f_2 - f_1)}$$

$$C_2 = 7.96 \ \mu F$$

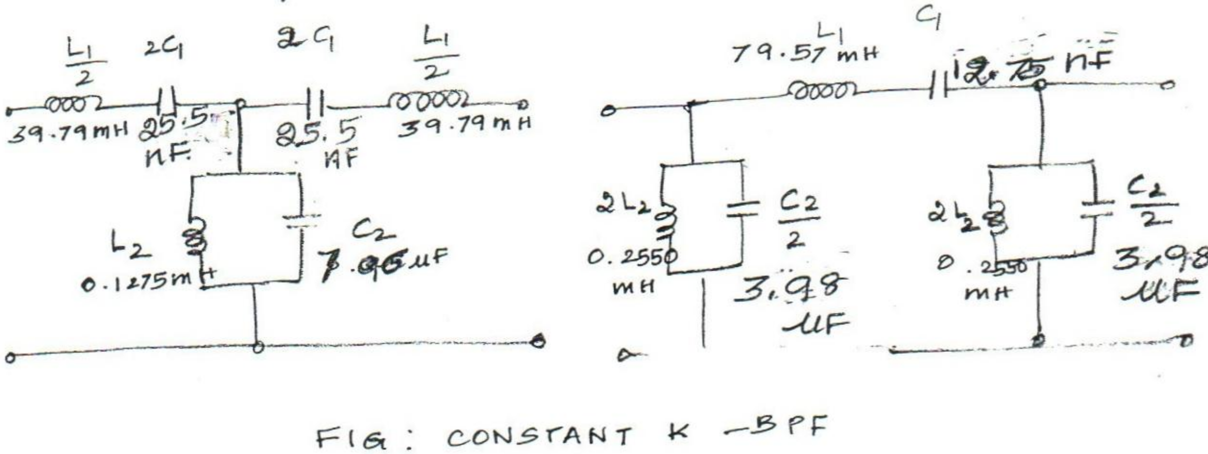
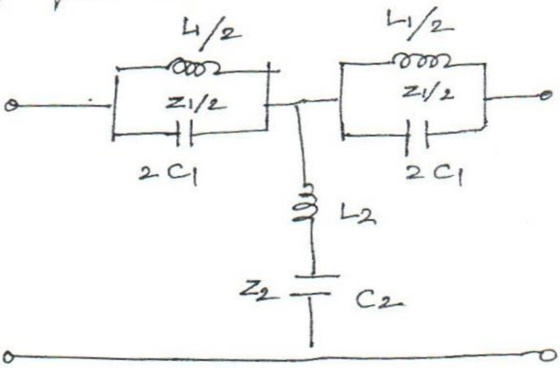


FIG: CONSTANT K - BPF

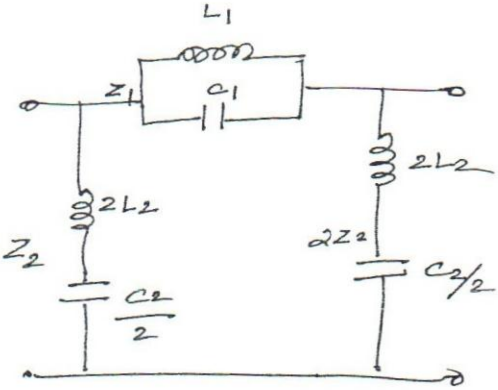
CONSTANT K – BAND STOP FILTER (BSF (or) BEF)

U.Q – NOV 2010 (8 marks), MAY 2012 (8 marks)
U.Q – NOV 2013 (8 marks)

BSF attenuates all frequencies between two designated frequencies and allows all other frequencies.



(a) T – SECTION



(b) π – SECTION

Series Impedance, $Z_1 = L_1 \| C_1 = \dfrac{j\omega L_1 \cdot \frac{1}{j\omega C_1}}{j\omega L_1 + \frac{1}{j\omega C_1}}$

$$Z_1 = \frac{\frac{L_1}{C_1}}{\frac{1-\omega^2 L_1 C_1}{j\omega C_1}} \Rightarrow \boxed{Z_1 = \frac{j\omega L_1}{1-\omega^2 L_1 C_1}} \quad \text{——— (1)}$$

Shunt Arm Impedance $\Rightarrow Z_2 = j\omega L_2 + \dfrac{1}{j\omega C_2}$

$$\boxed{Z_2 = \frac{1-\omega^2 L_2 C_2}{j\omega C_2}} \quad \text{——— (2)}$$

(i) Design Impedance

$$R_K^2 = Z_1 Z_2$$

$$\omega_0^2 = \frac{1}{L_1 C_1} = \frac{1}{L_2 C_2}$$

$$L_1 C_1 = L_2 C_2$$

$$R_K^2 = \frac{j\omega L_1}{1-\omega_2 L_1 C_1} \times \frac{1-\omega^2 L_2 C_2}{j\omega C_2}$$

At $\omega_1 = \omega_2 = \omega_0 \Rightarrow L_1 C_1 = L_2 C_2$

$$R_K^2 = \frac{L_1}{C_2}$$

$$\boxed{R_K = \sqrt{\frac{L_1}{C_2}} = \sqrt{\frac{L_2}{C_1}}} \quad \text{——(3)}$$

(ii) Cut off Frequencies

As in BPF, $Z_1 = -4Z_2$

$Z_1 = \pm j 2 R_K$ (as in BPF)

At $\omega = \omega_1$, $Z_1 = +j2R_K$.

$$\frac{j\omega L_1}{1-\omega_1^2 L_1 C_1} = +j2R_K$$

$$\omega_1 L_1 = 2R_K - 2\omega_1^2 L_1 C_1 R_K$$

$$2\omega_1^2 L_1 C_1 R_K + \omega_1 L_1 - 2R_K = 0$$

$$\omega_1 = \frac{-L_1 \pm \sqrt{L_1^2 + 16 L_1 C_1 R_K^2}}{4 L_1 C_1 R_K}$$

$$\omega_1 = \frac{-1 \pm \sqrt{1 + 16 \frac{C_1}{L_1} R_K^2}}{4 C_1 R_K}$$

Choose +ve sign,

$$f_1 = \frac{-1 + \sqrt{1+16\frac{C_1}{L_1} R_K^2}}{\ldots} \quad \text{——(4)}$$

Similarly $Z_1 = -j2R_K$

$$\boxed{f_2 = \frac{1 + \sqrt{1 + 16 R_K^2 \frac{C_1}{L_1}}}{8 \pi C_1 R_K}} \quad \text{— (5)}$$

(iii) Characteristic Impedance.

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

$Z_1 Z_2 = R_K^2$

$Z_1 = \frac{j\omega L_1}{1-\omega^2 L_1 C_1}$

$Z_1^2 = \frac{-\omega^2 L_1^2}{[1 - 2\omega^2 L_1 C_1 + \omega^4 L_1^2 C_1^2]}$

$$Z_{0T} = \sqrt{\frac{-\omega^2 L_1^2}{4(1-2\omega^2 L_1 C_1 + \omega^4 L_1^2 C_1^2)} + R_K^2}$$

(i) At $\omega = 0 \Rightarrow Z_{0T} = R_K$

(ii) At $\omega = \omega_0 \Rightarrow$ Stop band, Z_{0T} is imaginary

(iii) At $\omega = \omega_1$ & $\omega_2 \Rightarrow$ Stop Band, $Z_{0T} = 0$

(iv) At $\omega = \infty \Rightarrow$ Pass band, $Z_{0T} = R_K$

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{R_K^2}{Z_{0T}}$$

(i) At $\omega = 0 \Rightarrow Z_{0\pi} = \frac{R_K^2}{R_K} \Rightarrow Z_{0\pi} = R_K$

(ii) At $\omega = \omega_0 \Rightarrow Z_{0\pi} \rightarrow$ Imaginary

(iii) At $\omega = \omega_1, \omega_2 \Rightarrow Z_{0\pi} = \infty$

(iv) At $\omega = \infty \Rightarrow Z_{0\pi} = R_K$

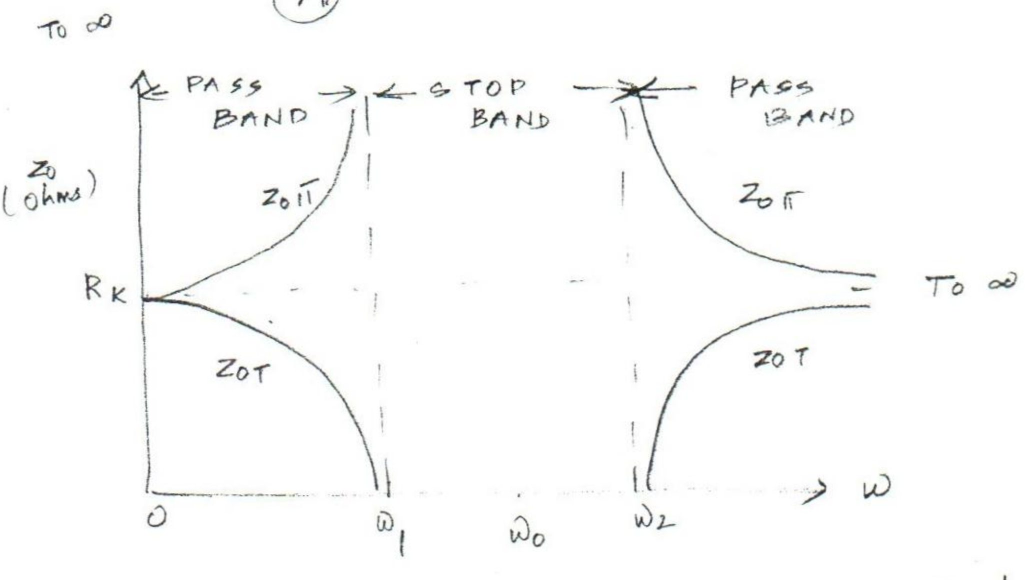


FIG : Zo variation of BSF in T and π sectio.

(iv) Design Equations.

At $\omega=\omega_1$ $Z_1 = +j2R_K$.

$$\omega_0^2 = \frac{1}{L_1C_1}$$

$$L_1C_1 = \frac{1}{\omega_0^2}$$

$$\frac{j\omega_1 L_1}{1-\omega_1^2 L_1C_1} = +j\,2R_K.$$

$$2\omega_1^2 L_1C_1 R_K + \omega_1 L_1 - 2R_K = 0$$

$$2R_K \frac{\omega_1^2}{\omega_0^2} + \omega_1 L_1 - 2R_K = 0$$

$$\omega_1 L_1 = 2R_K\left[1-\frac{\omega_1^2}{\omega_0^2}\right]$$

$$L_1 = 2\frac{R_K}{\omega_1}\left[1-\frac{\omega_1^2}{\omega_0^2}\right]$$

Since $\omega_0^2 = \omega_1\omega_2$

$$L_1 = \frac{2R_K}{\omega_1}\left[1-\frac{\omega_1^2}{\omega_1\omega_2}\right]$$

$$L_1 = \frac{2R_K}{\omega_1}\left[\frac{\omega_1\omega_2-\omega_1^2}{\omega_1\omega_2}\right] = 2\frac{R_K\,\cancel{\omega_1}(\omega_2-\omega_1)}{\cancel{\omega_1}\,\omega_1\omega_2}$$

(12)

$$f_1 f_2 = f_0$$

$$L_1 = \frac{2 R_K \cdot 2\pi (f_2 - f_1)}{\cancel{4} \pi^2 f_1 f_2}$$

$$\boxed{L_1 = \frac{R_K (f_2 - f_1)}{\pi f_1 f_2}} \quad \text{——— (6)}$$

$$\omega_0^2 = \frac{1}{L_1 C_1} \Rightarrow C_1 = \frac{1}{\omega_0^2 L_1}$$

$$\omega_0^2 = \omega_1 \omega_2$$

$$4\pi^2 f_0^2 = 4\pi^2 f_1 f_2$$

$$C_1 = \frac{1}{4\pi^2 f_1 f_2 \cdot \frac{R_K (f_2 - f_1)}{\pi f_1 f_2}}$$

$$\boxed{C_1 = \frac{1}{4\pi R_K (f_2 - f_1)}} \quad \text{——— (7)}$$

$$R_K^2 = \frac{L_2}{C_1}$$

$$L_2 = C_1 R_K^2$$

$$\boxed{L_2 = \frac{R_K}{4\pi (f_2 - f_1)}} \quad \text{——— (8)}$$

$$R_K^2 = \frac{L_1}{C_2} \qquad C_2 = \frac{L_1}{R_K^2}$$

$$\boxed{C_2 = \frac{(f_2 - f_1)}{\pi R_K f_2 f_1}} \quad \text{——— (9)}$$

Problems

1) Calculate the elements of a BSF to suppress harmonic sounds between 8.5 KHZ & 9.0 KHZ. The filter has to work between terminal impedances of 2000 ohms.

Sol :

$R_K = 2000$; $f_2 = 9$ KHZ ; $f_1 = 8.5$ KHZ

$$L_1 = \frac{R_K(f_2 - f_1)}{\pi f_1 f_2}$$ $\boxed{L_1 = 4.1609 \text{ mH}}$

$$L_2 = \frac{R_K}{4\pi(f_2 - f_1)}$$ $\boxed{L_2 = 318.3 \text{ mH}}$

$$C_1 = \frac{1}{4\pi R_K(f_2 - f_1)}$$ $\boxed{C_1 = 0.07958\ \mu F}$

$$C_2 = \frac{(f_2 - f_1)}{\pi R_K f_2 f_1}$$ $\boxed{C_2 = 1040 \text{ pF}}$

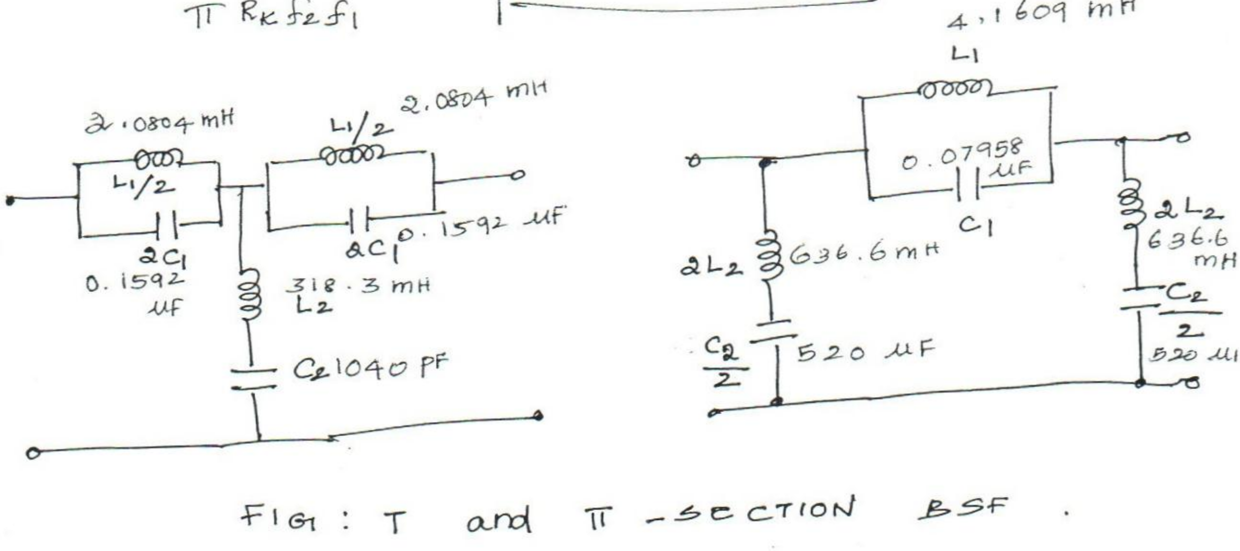


FIG : T and π -SECTION BSF.

DISADVANTAGES OF CONSTANT K - FILTERS. U.Q - MAY 2012 (2 MARKS)

① The characteristic impedance varies widely in the pass band from the desired value.

② The attenuation does not increase rapidly beyond cut off frequencies.

m - Derived Filters. U.Q - NOV 2010, NOV 2013 (2 marks) APRIL 2011, MAY 2014

The characteristic impedance varies widely in Pass band from desired value R_K. So impedance matching is not proper and reflection loss is present. The attenuation does not increase rapidly beyond Cut off frequencies. These advantages can be overcome by designing a filter having same Z_0 of prototype filter in Pass band but with very sharp attenuation characteristics in the stop band. Such a filter is called as m-derived filter.

For m = 0.6, the characteristic impedance is considerably constant throughout the pass band.

m - derived T - section

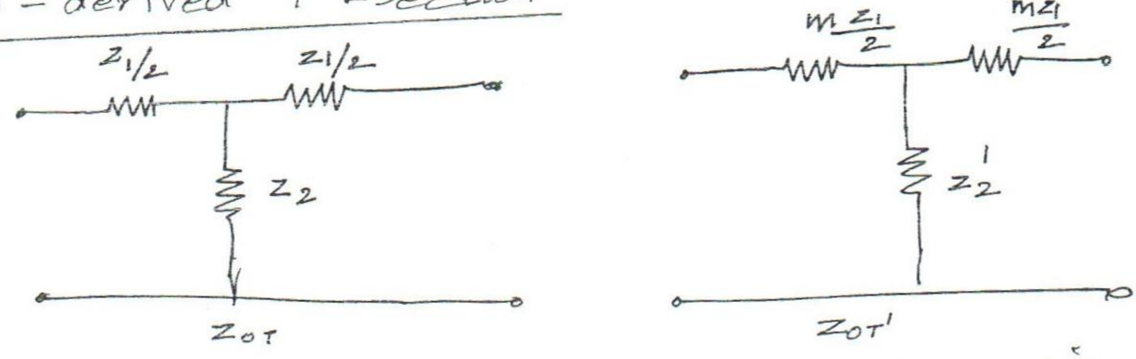


FIG. Prototype and m-derived T-section

Series arm $= Z_1$

shunt arm $= Z_2$

$$Z_{0T} = \sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}}$$

Series arm $= mZ_1$

shunt arm $= Z_2'$

$$Z_{0T}' = \sqrt{\frac{m^2 Z_1^2}{4} + mZ_1 Z_2'}$$

Both should be matched, $Z_{0T}^2 = Z_{0T}'^2$

$$\frac{Z_1^2}{4} + Z_1 Z_2 = \frac{m^2 Z_1^2}{4} + mZ_1 Z_2'$$

$$\frac{Z_1^2 + 4Z_1 Z_2}{4} = \frac{m^2 Z_1^2 + 4mZ_1 Z_2'}{4}$$

$$Z_1^2[1-m^2] + 4Z_1[Z_2 - mZ_2'] = 0$$

$$Z_1^2(1-m^2) + 4Z_1 Z_2 = 4mZ_1 Z_2'$$

$$Z_2' = \frac{Z_1^2(1-m^2) + 4Z_1 Z_2}{4mZ_1}$$

$$\boxed{Z_2' = \frac{Z_1(1-m^2)}{4m} + \frac{Z_2}{m}}$$

So, shunt arm has 2 impedances in series $\frac{Z_2}{m}$ & $\frac{(1-m^2)}{4m}Z_1$

Total impedance in series arm is $Z_1' = mZ_1$

(ie) $\frac{mZ_1}{2}$ in each series arm.

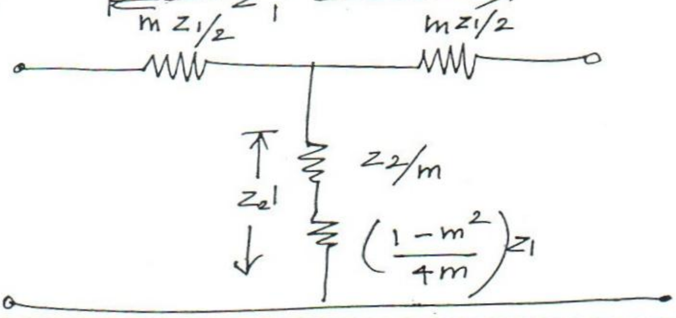
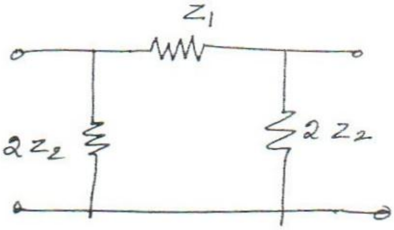


Fig. m-Derived T-section for both LPF & HPF

m – derived π – section.



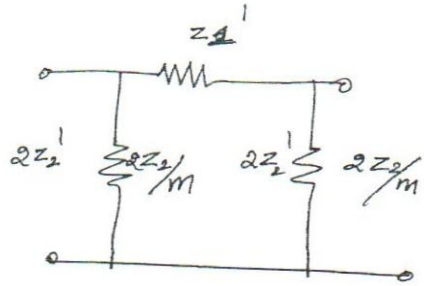


FIG. Prototype and m-derived π-section

Series arm $= Z_1$ Series arm $= Z_1'$

Shunt arm $= Z_2$ Shunt arm $= 2Z_2' = \dfrac{2Z_2}{m}$

$Z_{0\pi}$ $Z_{0\pi}'$

$Z_{0\pi}$ and $Z_{0\pi}'$ should be matched.

$$Z_{0\pi}^2 = \frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}} \qquad Z_{0\pi}'^2 = \frac{Z_1' Z_2'}{1 + \frac{Z_1'}{4Z_2'}} = \frac{Z_1' \frac{Z_2}{m}}{1 + \frac{Z_1'}{4 \cdot Z_2/m}}$$

$$\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}} = \frac{Z_1' Z_2/m}{1 + \frac{Z_1'}{4 Z_2/m}}$$

$$\frac{Z_1 \cancel{Z_2}}{1 + \frac{Z_1}{4Z_2}} = \frac{Z_1' \cancel{Z_2}}{m} \; \frac{1}{1 + \frac{mZ_1'}{4Z_2}}$$

$$\frac{Z_1}{4Z_2 + Z_1} = \frac{Z_1'}{m(4Z_2 + mZ_1')}$$

$$mZ_1(4Z_2 + mZ_1') = Z_1'(Z_1 + 4Z_2)$$

$$4mZ_1Z_2 + m^2 Z_1 Z_1' = Z_1 Z_1' + 4Z_1' Z_2$$

$$4mZ_1Z_2 + Z_1'(m^2 Z_1 - 4Z_2 - Z_1) = 0$$

$$Z_1' = \frac{-4mZ_1Z_2}{m^2Z_1 - 4Z_2 - Z_1}$$

$$Z_1' = \frac{4mZ_1Z_2}{Z_1 + 4Z_2 - m^2Z_1} = \frac{4mZ_1Z_2}{4Z_2 + Z_1(1-m^2)}$$

$\div$ Nr & Dr by $4m$

$$Z_1' = \frac{Z_1Z_2}{\frac{Z_2}{m} + \frac{(1-m^2)}{4m}\cdot Z_1}$$

Multiply Numerator & Denominator by $\frac{4m^2}{1-m^2}$

$$Z_1' = \frac{Z_1Z_2\cdot\frac{4m^2}{1-m^2}}{\frac{Z_2}{m}\left(\frac{4m^2}{1-m^2}\right) + mZ_1} = \frac{mZ_1\cdot\frac{4mZ_2}{1-m^2}}{mZ_1 + \left(\frac{4m}{1-m^2}\right)Z_2}$$

$$Z_1' = \underbrace{(mZ_1) \parallel \left(\frac{4m}{1-m^2}\right)Z_2}_{Z_1'}$$

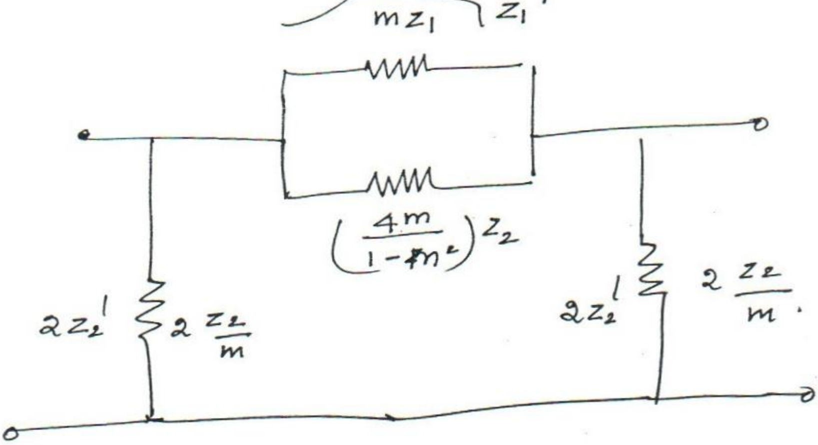
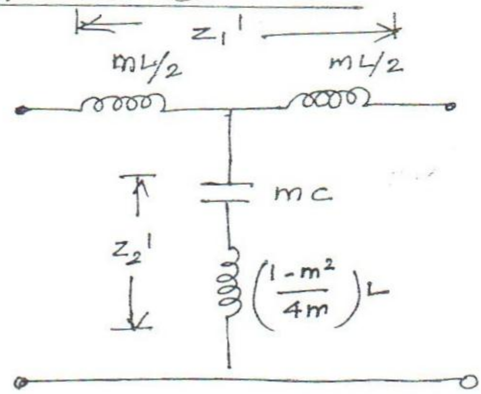
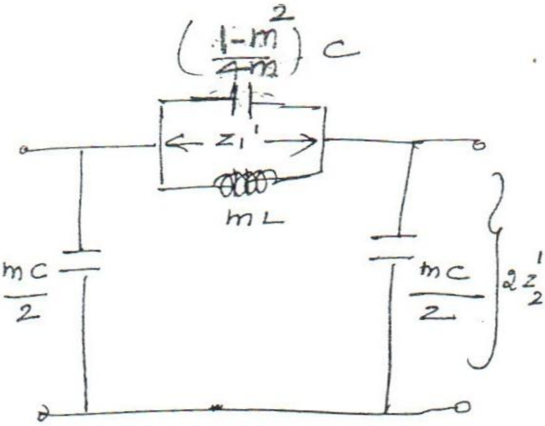


FIG. m – Derived π - section for both LPF & HPF m - derived LPF.





Frequency f_∞ at infinite attenuation.

The frequency at which attenuation becomes infinite is called as infinite frequence f_∞

At $\omega = \omega_\infty$, $\omega L' = \frac{1}{\omega C'}$

$$\omega_\infty \left(\frac{1-m^2}{4m}\right) L = \frac{1}{\omega_\infty mC}$$

$$\omega_\infty^2 = \frac{4}{LC(1-m^2)} = \frac{\omega_c^2}{(1-m^2)}$$

$$\omega_\infty = \frac{2}{\sqrt{LC(1-m^2)}}$$

$$2\pi f_\infty = \frac{2}{\sqrt{LC(1-m^2)}} = \frac{2\pi f_c}{\sqrt{1-m^2}}$$

But $\omega_c^2 = \frac{4}{LC}$

$$\boxed{f_\infty = \frac{f_c}{\sqrt{1-m^2}}}$$

$$\sqrt{1-m^2} = \frac{f_c}{f_\infty}$$

$$1-m^2 = \frac{f_c^2}{f_\infty^2}$$

$$m^2 = 1 - \frac{f_c^2}{f_\infty^2}$$

$$\boxed{m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2}}$$

T – type

$$z_1 = m z_1$$

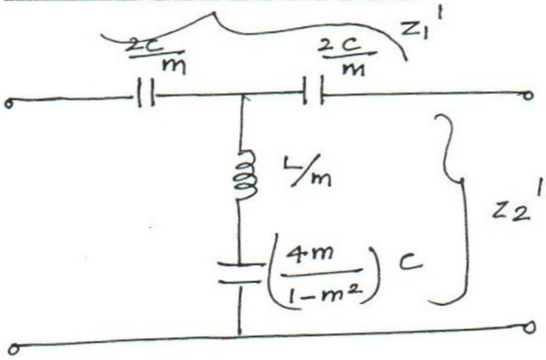
$$z_2 = \left(\frac{1-m^2}{4m}\right) z_1 + \frac{z_2}{m}$$

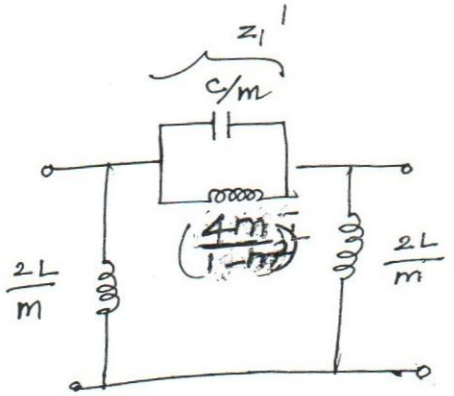
π – type

$$z_1 = (m z_1) \parallel \left(\frac{4m}{1-m^2}\right) z_2$$

$$Z_2 = \frac{z_2}{m}$$

m - derived HPF.





Infinite frequency.

At $\omega = \omega_\infty$, $\omega_\infty L' = \dfrac{1}{\omega_\infty C'}$

$$\omega_\infty^2 = \frac{1}{L'C'} = \frac{1}{\frac{L}{m}\left(\frac{4m}{1-m^2}\right)C}$$

$$\omega_\infty^2 = \frac{m(1-m^2)}{4mLC}$$

$$\omega_\infty^2 = \frac{(1-m^2)}{4LC} \qquad \omega_c^2 = \frac{1}{4LC}$$

$$\omega_\infty^2 = \omega_c^2(1-m^2)$$

$$\omega_\infty = \omega_c\sqrt{1-m^2}$$

$$\boxed{f_\infty = f_c\sqrt{1-m^2}}$$

$$\frac{f_\infty}{f_c} = \sqrt{1-m^2}$$

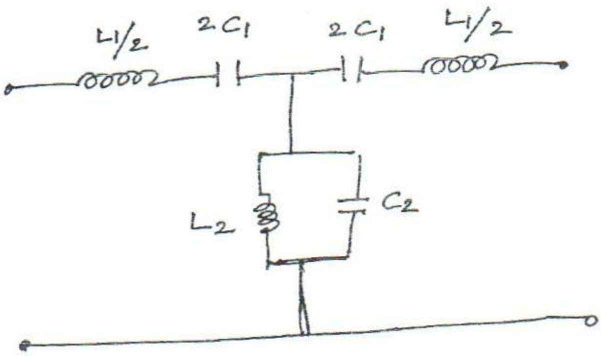
$$1-m^2 = \frac{f_\infty^2}{f_c^2}$$

$$m^2 = 1 - \frac{f_\infty^2}{f_c^2}$$

$$\boxed{m = \sqrt{1-\left(\frac{f_\infty}{f_c}\right)^2}}$$

U.Q - MAY 2013 (8 marks)
U.Q - MAY 2014 (8 marks)

m - derived BPF.



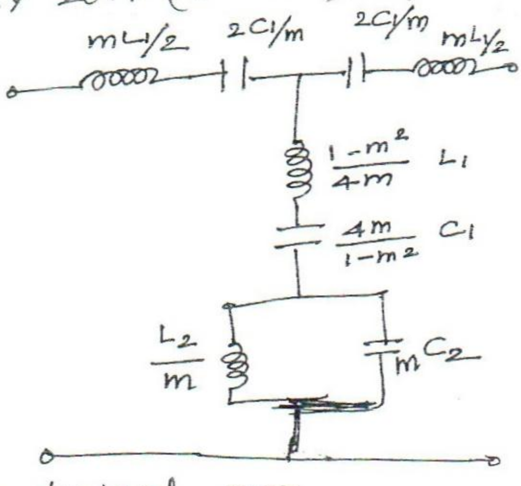


Fig ⇒ Prototype and m - derived BPF

$$f_{\infty 1} = \sqrt{\frac{(f_2 - f_1)^2}{4(1-m^2)} + f_1 f_2} - \frac{f_2 - f_1}{2\sqrt{1-m^2}}$$

$$f_{\infty 2} = \sqrt{\frac{(f_2 - f_1)^2}{4(1-m^2)} + f_1 f_2} + \frac{f_2 - f_1}{2\sqrt{1-m^2}}$$

$$f_{\infty 1} - f_{\infty 2} = \frac{f_2 - f_1}{\sqrt{1-m^2}}$$

$$m = \sqrt{\frac{(f_\infty^2 - f_2^2)(f_\infty^2 - f_1^2)}{(f_\infty^2 - f_1 f_2)}}$$

$$m = \sqrt{1 - \left(\frac{f_2 - f_1}{f_{\infty 1} - f_{\infty 2}}\right)^2}$$

m - derived BSF . U.Q - MAY 2013 (8 marks)

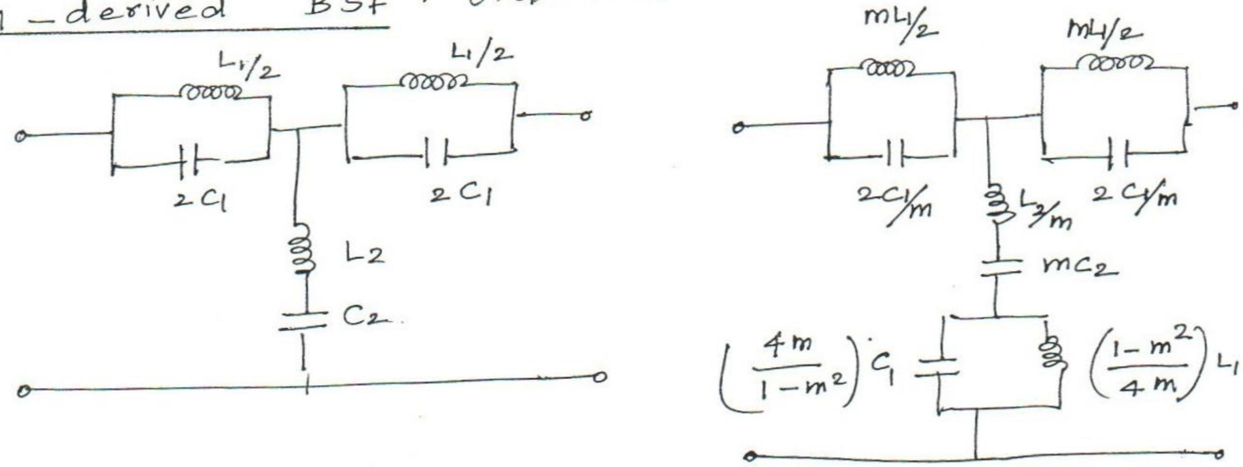


Fig → Prototype and m - derived BSF .

Problems .

1) Design a m - derived LPF whose design impedance is 500 Ω , f_c is 5000 Hz and $m = 0.65$.

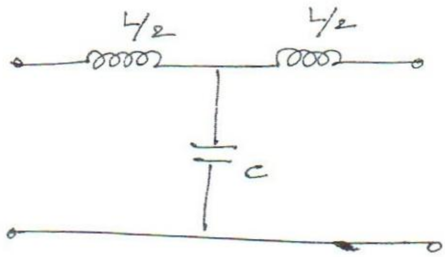
Sol :

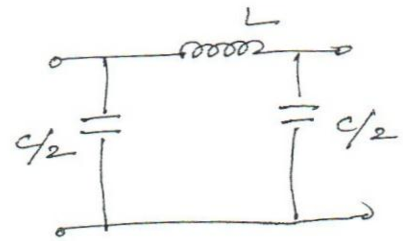
$R_K = 500\ \Omega$

$f_c = 5000$ Hz

$m = 0.65$

Constant K - LPF





$\frac{mL}{2} = 10.3 \text{ mH}$

$mC = 82.25 \text{ nF}$

$\left(\frac{1-m^2}{4m}\right)L = 7.063 \text{ mH}$

$\left(\frac{1-m^2}{4m}\right)C = 28.2 \text{ nF}$

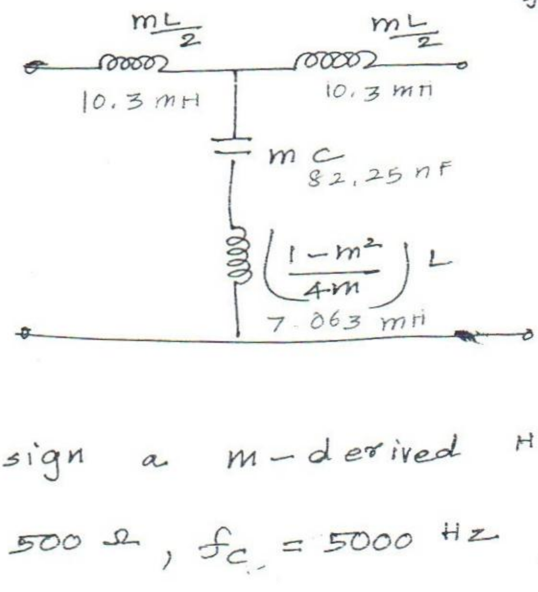
$L = \frac{R_K}{\pi f_c} = 31.8 \text{ mH}$

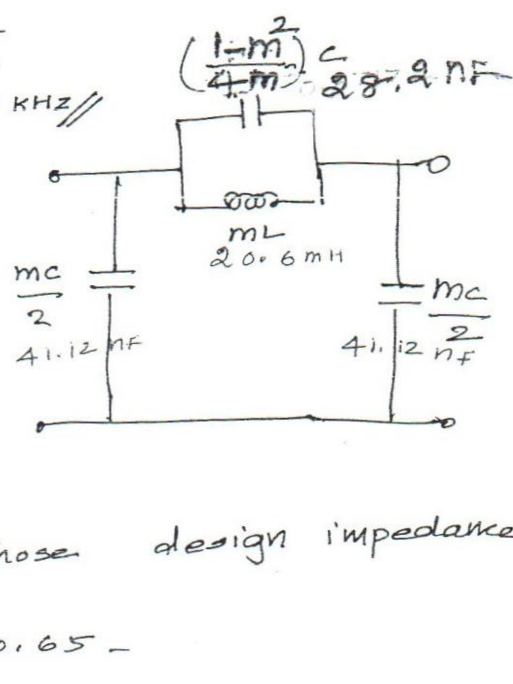
$C = \frac{1}{\pi R_K f_c} = 0.127 \ \mu F$

m - derived LPF.

$f_\infty = \frac{f_c}{\sqrt{1-m^2}}$

$f_\infty = 6.57 \text{ KHz}$





Q) Design a m - derived HPF whose design impedance is $500 \ \Omega$, $f_c = 5000$ Hz, $m = 0.65$.

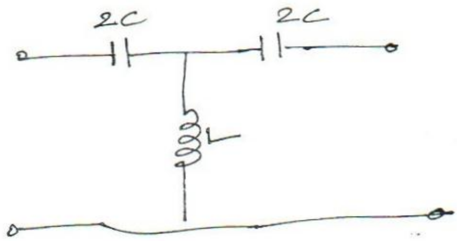
Sol :

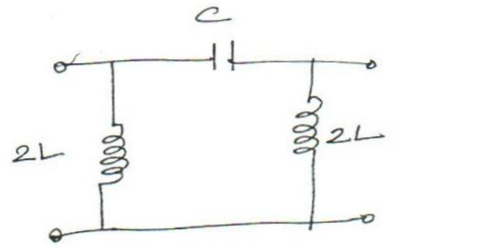
$R_K = 500 \ \Omega$

$f_c = 5000$ HZ

$m = 0.65$

CONSTANT K - HPF





$$L = \frac{R_K}{4\pi f_c} = 7.957 \text{ mH}$$

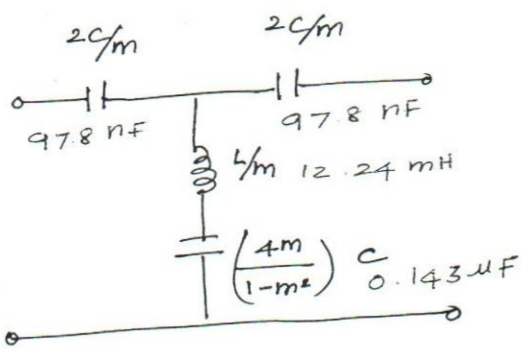
$$C = \frac{1}{4\pi f_c R_K} = 31.8 \text{ nF}$$

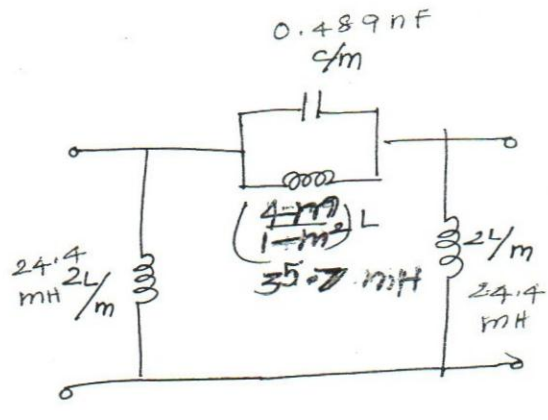
$C/m = 0.489$ nF ; $2C/m = 97.8$ nF ; $L/m = 12.24$ mH

$\left(\frac{4m}{1-m^2}\right) L = 35.7$ mH ; $2L/m = 24.4$ mH

$\left(\frac{4m}{1-m^2}\right) C = 0.143$ µF

m - derived HPF





$$f_\infty = f_c \sqrt{1-m^2}.$$

3) Design an m-derived T and π section low pass filters having a cut off frequency $f_c = 8$ KHZ, design impedance $R_K = 500$ ohms and frequency of infinite attenuation $f_\infty = 10$ KHZ.

ce

ite

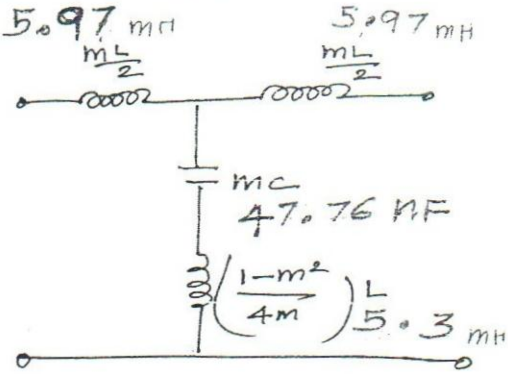
Sol.

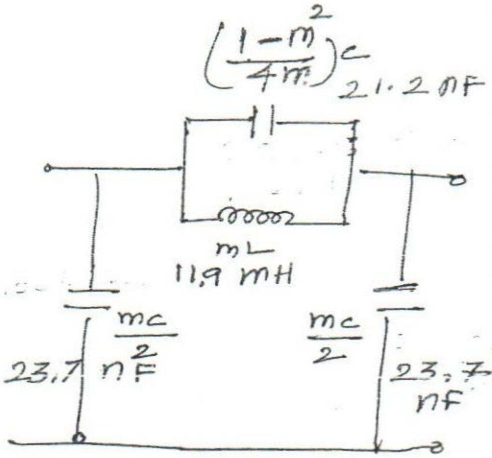
$$L = \frac{R_K}{\pi f_c} = \frac{600}{\pi \times 8 \times 10^3} = 19.1 \text{ mH}$$

$$C = \frac{1}{\pi f_c R_k} = 79.62 \text{ nF}$$

$$m = \sqrt{1 - \left(\frac{f_c}{f_\infty}\right)^2} = 0.6$$

m - derived LPF.





4). Design a m-derived HPF having design impedan[ce] of 500 ohms, cut off frequency 5 KHz and infin[ite] attenuation frequency of 4.5 KHz

Sol:

$f_c = 5$ KHz

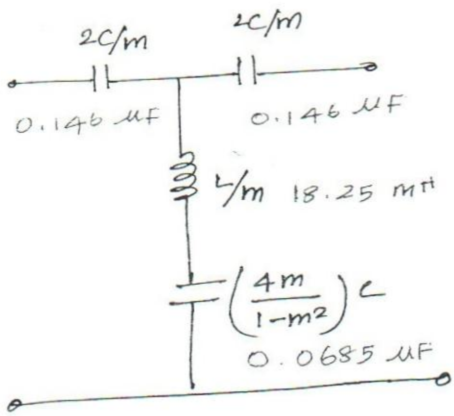
$f_\infty = 4.5$ KHz

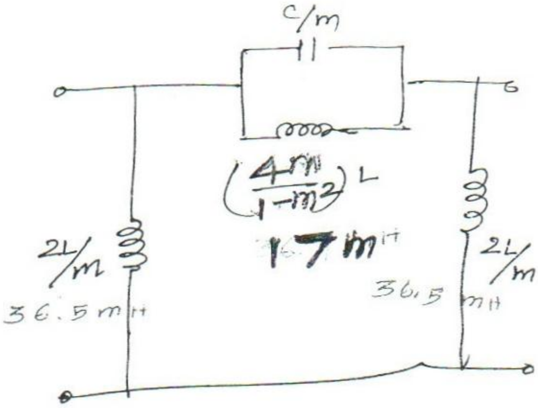
$R_K = 500\ \Omega$

$$L = \frac{R_K}{4\pi f_c} = 7.958 \text{ mH}$$

$$C = \frac{1}{4\pi f_c R_K} = 0.0318\ \mu\text{F}$$

$$m = \sqrt{1 - \left(\frac{f_\infty}{f_c}\right)^2} = 0.436$$





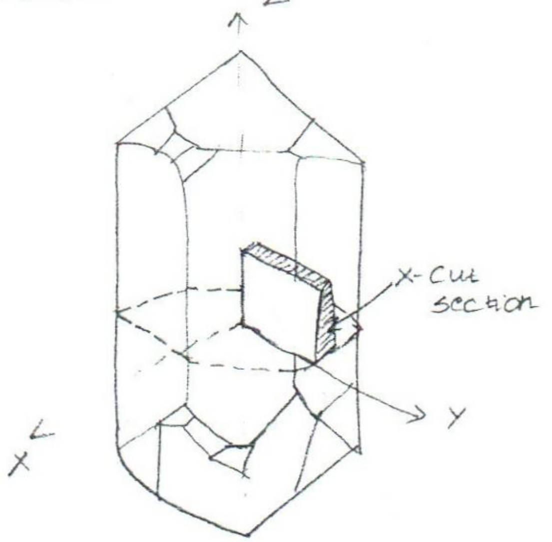
CRYSTAL FILTERS

U.Q – NOV 2012 (2 marks), U.Q – NOV 2013 (10 marks)
U.Q – NOV 2010 (8 marks), U.Q – MAY 2014 (10 marks)

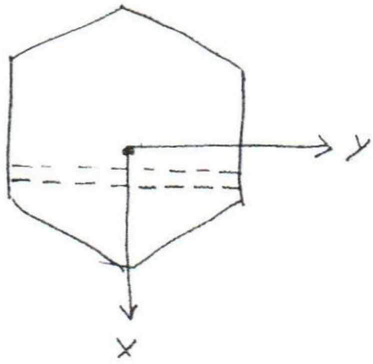
Certain substances such as quartz exhibit an effect commonly called piezoelectric effect. Such effect is that when a mechanical strain is applied to a suitably cut piece of quartz, it produces emf between two surfaces of that piece and conversely if emf is applied between two surfaces of the quartz piece a mechanical deformation is observed. Such effect is most widely used in piezoelectric microphones and other similar devices.

Another important property of the quartz crystal cut into slices is that each slice behaves as a resonant circuit electrically which has very large Q and it resonates at natural frequency of mechanical vibration of the slice of a quartz.

The complete crystal of the quartz in natural state with three axes for reference is shown in figure.



(a) Quartz crystal in natural shape with reference axes.

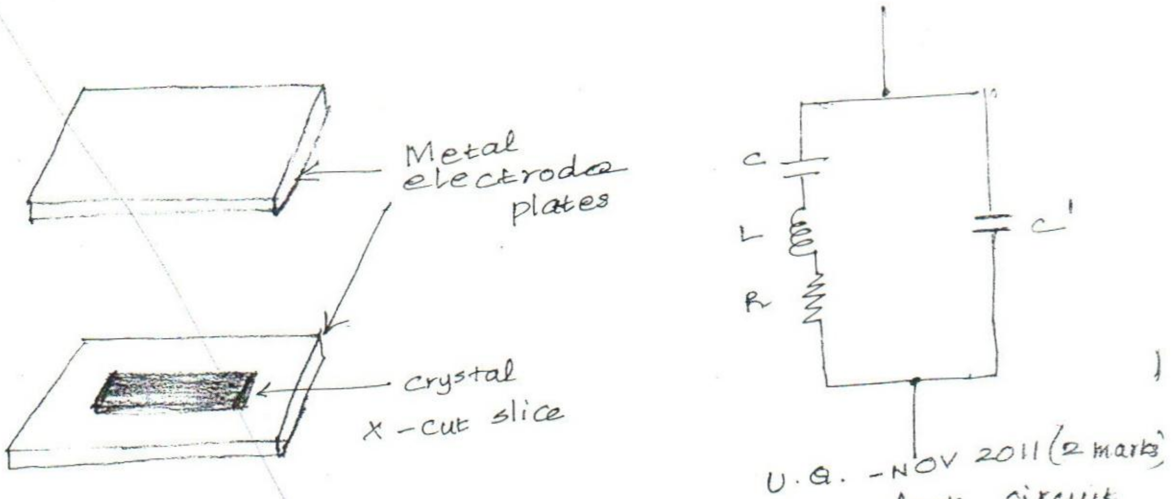


(b) X - cut slice section through X-Y plane

The vertical axis passing through top and bottom points is called optical axis or Z-axis. The axis at right angles to Z-axis parallel to any major face of crystal is called electrical axis or X-axis. while line perpendicular to any face of the crystal is called Y-axis. The slice cut from the crystal is generally described in terms of the angles between the slice and the three axes of the crystal & such slices can be used for various purposes in different applications.

In general a slice cut with its faces perpendicular to X-axis is called X-cut slice which is most extensively used in crystal filters.

When such a x-cut slice is placed between two flat metals, it behaves like a resonant circuit



(a) Crystal mounted between metal electrode plates

(b) Equivalent circuit of quartz crystal

Fig : Equivalent circuit of quartz crystal.

The elements R, L, C in the equivalent circuit are the electrical equivalents of corresponding mechanical properties of the crystal. The values of the elements in the equivalent circuit are determined by the physical constants and dimensions of the crystal material. C' is measure of electrical capacity between the faces of the crystal.

The series circuit consisting R, L and C resonates at the natural frequency of the crystal which is same as mechanical vibration frequency. Let the natural frequency be f_R. As C' is in parallel with series arm, the crystal also exhibits an antiresonant frequency. The antiresonant frequency is denoted by f_A.

The quality factor (Q) of crystal is infinite but practically it shows Q value as high as the Order of 5000 to 20,000. With normal type of inductances and capacitance used in circuits, the value of Q is obtained higher, about 200.

The impedance of circuit is given by,

$$Z = (jX_L - jX_c) \parallel (-jX_{c'})$$

Neglecting equivalent resistance R which is relatively small with extremely high Q.

$$Z = \left(j\omega L - \frac{j}{\omega C}\right) \parallel \left(\frac{-j}{\omega C'}\right)$$

$$Z = \frac{\frac{-j}{\omega C'}\left[j\omega L - \frac{j}{\omega C}\right]}{j\omega L - \frac{j}{\omega C} - \frac{j}{\omega C'}}$$

$$Z = \frac{\frac{-j}{\omega C'}\left[\frac{j\omega^2 LC - j}{\omega C}\right]}{\frac{j}{\omega}\left[\omega^2 L - \frac{1}{C} - \frac{1}{C'}\right]} = \frac{-1}{\omega CC'}\left[\frac{j(\omega^2 LC - 1)}{\omega^2 L - \frac{(C'+C)}{CC'}}\right]$$

According to the condition of resonance, to have resonance in the circuit, equate imaginary part to zero.

$$\frac{\omega_R^2 LC - 1}{\omega_R CC' \left[\omega_R^2 L - \left(\frac{C+C'}{CC'}\right)\right]} = 0$$

$$\omega_R^2 LC = 1$$

$$\omega_R^2 = \frac{1}{LC} \Rightarrow \omega_R = \frac{1}{\sqrt{LC}}$$

$$\boxed{f_R = \frac{1}{2\pi\sqrt{LC}}}$$

To have antiresonance equating susceptance to zero,

$$Y = \frac{1}{Z} = \frac{\omega CC' \left[\omega^2 L - \left(\frac{C+C'}{CC'}\right)\right]}{-j\,(\omega^2 LC - 1)}$$

$$Y = \frac{j\,\omega CC' \left[\omega^2 L - \left(\frac{C+C'}{CC'}\right)\right]}{\omega^2 LC - 1}$$

$$\omega_A^2 . L - \left(\frac{C+C'}{CC'}\right) = 0$$

$$\omega_A^2 L = \frac{C+C'}{CC'}$$

$$\omega_A^2 = \frac{1}{L\left[\frac{CC'}{C+C'}\right]}$$

$$\omega_A = \frac{1}{\sqrt{L\,C\left[\frac{C'}{C+C'}\right]}}$$

$$2\pi f_A = \frac{1}{\sqrt{LC\left[\frac{C'}{C+C'}\right]}}$$

$$f_A = \frac{\sqrt{\frac{C+C'}{C'}}}{2\pi\sqrt{LC}}$$

$$f_R = \frac{1}{2\pi\sqrt{LC}}$$

$$\boxed{f_A = f_R\sqrt{1+\frac{C}{C'}}}$$

Using Binomial theory,

$$f_A = f_R\left[1+\frac{C}{C'}\right]^{1/2} = f_R\left[1+\frac{1}{2}\frac{C}{C'}+\frac{(1/2)(-1/2)}{1\cdot 2}\left(\frac{C}{C'}\right)^2+\cdots\right]$$

For crystal C' is 125 times C. So $\frac{C}{C'}$ becomes very small. So terms of higher power of (C/C') can be neglected.

$$f_A = f_R\left(1+\frac{C}{2C'}\right)$$

$$C' = 125\,C$$

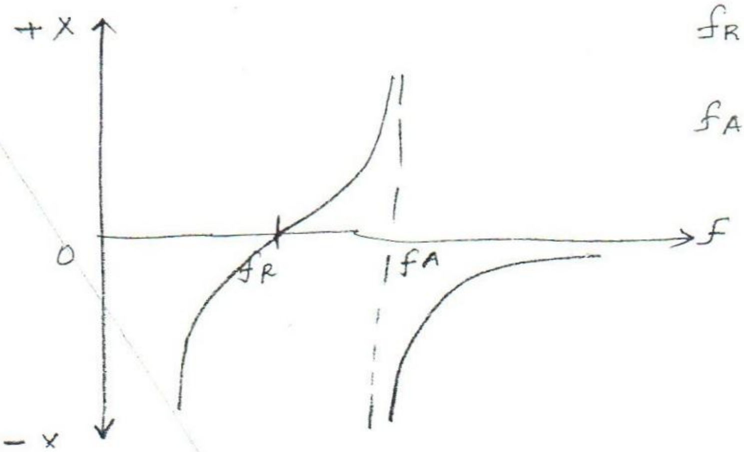
$$\frac{C}{C'} \simeq \frac{1}{125}$$

$$f_A \simeq 1.004\, f_R$$

The separation between resonant and anti-resonant frequency is calculated as,

$$f_A - f_R = f_R\left(1+\frac{C}{2C'}\right) - f_R$$

$$\boxed{f_A - f_R = f_R \frac{C}{2C'}}$$



$f_R \rightarrow$ Resonant frequency of series circuit

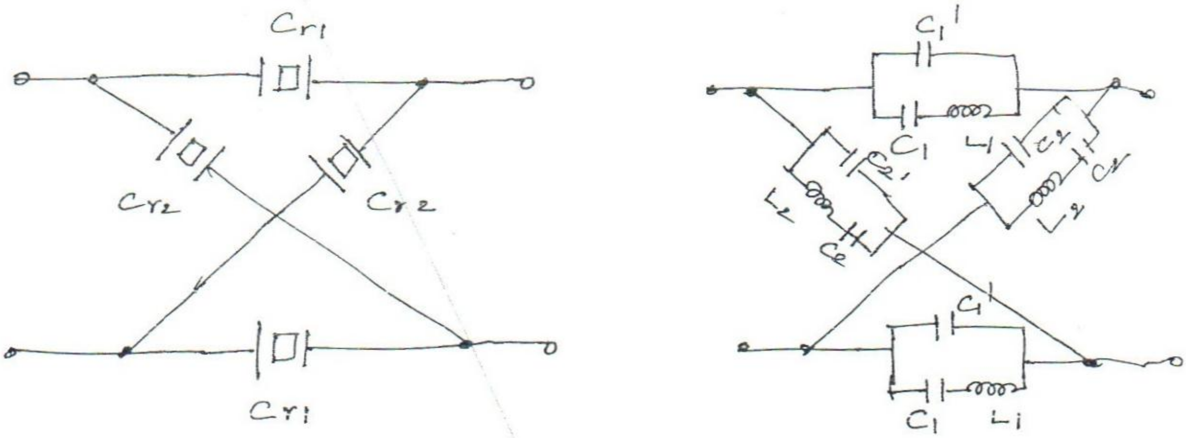
$f_A \rightarrow$ Resonant frequency of parallel circuit

Fig. $\rightarrow$ Reactance curve for quartz crystal.

In general, C is the equivalent series capacitance of crystal while C' is the parallel capacitance introduced by the metal electrodes between which crystal is mounted. Typically $C' >> C$ such that f_R and f_A lie close together along the frequency axis as shown in the reactance sketch. $f_A > f_R$. We can shift f_A closer to f_R by using adjustable capacitors in parallel to crystal increasing capacitance value of C' effectively.

CRYSTAL FILTER CIRCUIT USING LATTICE NETWORK

The main advantage of crystal filter is that it can either represent resonant or antiresonant circuit. Because of this property, the crystal is used to replace the normal elements of either bandpass or band elimination filter.



(a) Lattice crystal filter

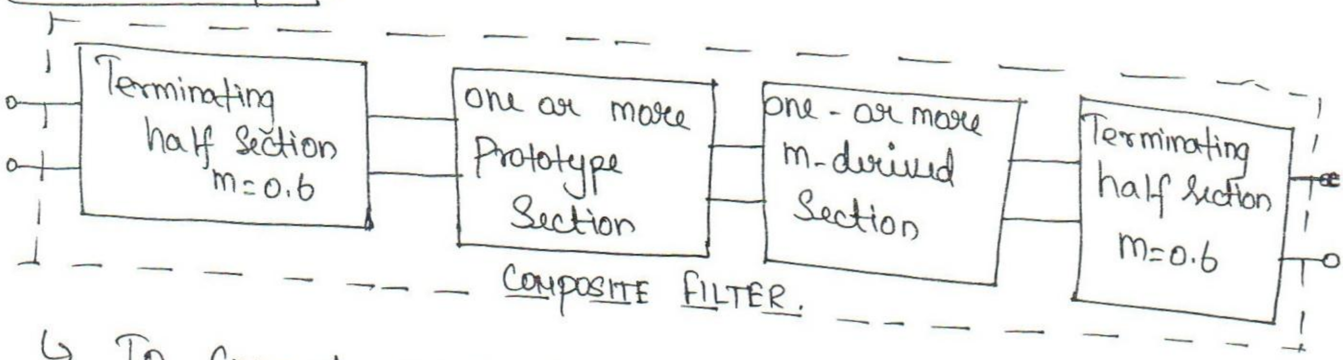
(b) Equivalent Circuit

Fig. → Lattice type Crystal BPF.

In the above lattice type crystal filter Cr1 and Cr2 are carefully selected matched pair. The crystals are cut in such a way that series resonant frequency of one pair corresponds with antiresonant frequency of other crystals.

COMPOSITE FILTERS

BLOCK DIAGRAM :—



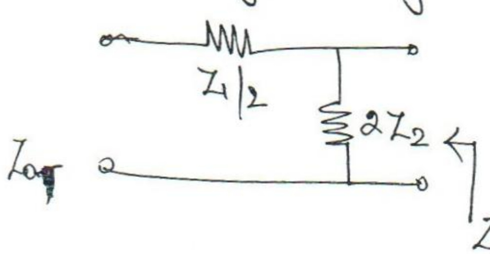
↳ To Connect no. of different Sections in the filter, It is important to match the impedances of the junction points.

↳ T- Section Should not connected to π- Section directly, as both sections have different impedances.

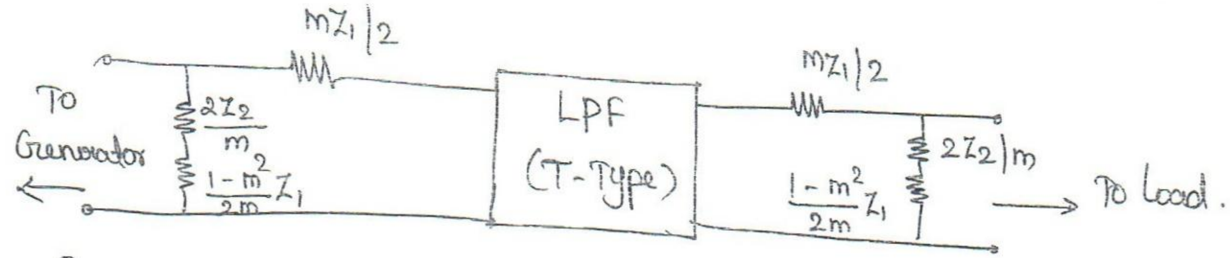
↳ Use matching Section to join T & π Section together. Such impedance matching is called as Half Section. The half Section is derived from Symmetrical T (or) π- Section

↳ prototype half Section can be obtained from Symmetrical T (and) π - Section by dividing these Sections centrally into two half Section.

Prototype half Section



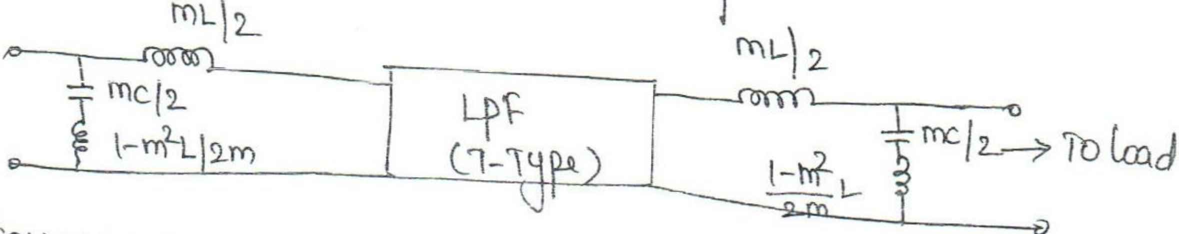
TERMINATING HALF SECTION for T- Configuration :- (LPF)



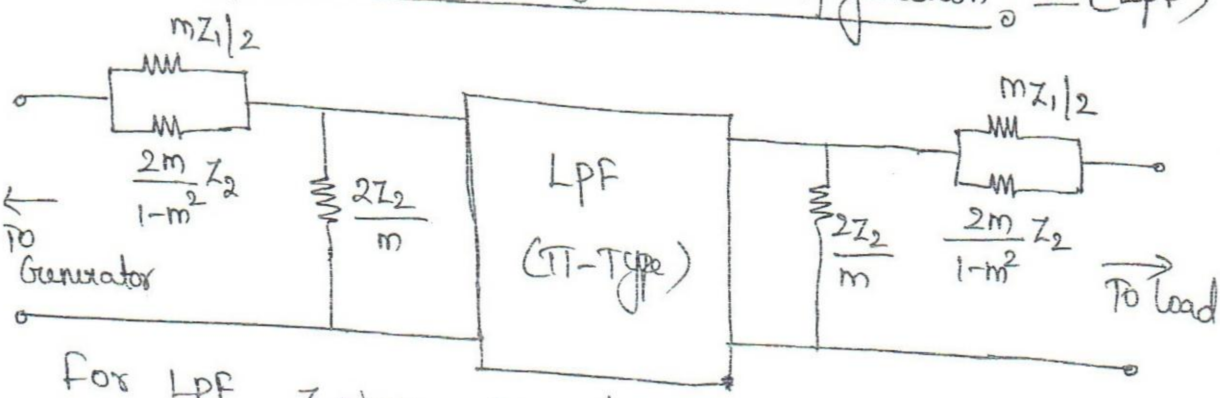
For LpF $Z_1 = j\omega L$; $Z_2 = \frac{1}{j\omega C} = \frac{-j}{\omega C}$

Terminating Half Section with $m = 0.6$ can be calculated as,

- $\frac{mZ_1}{2} = \frac{m(j\omega L)}{2} = j\omega\left(\frac{mL}{2}\right)$
- $\frac{mZ_2}{2m} = \frac{m\left(\frac{1}{j\omega C}\right)}{m} = \frac{1}{j\omega\left(\frac{mC}{2}\right)}$
- $\frac{1-m^2}{2m}Z_1 = \frac{1-m^2}{2m}(j\omega L)$
$= j\omega\left(\frac{1-m^2 L}{2m}\right)$



TERMINATING HALF SECTION for π- Configuration :- (LPF)

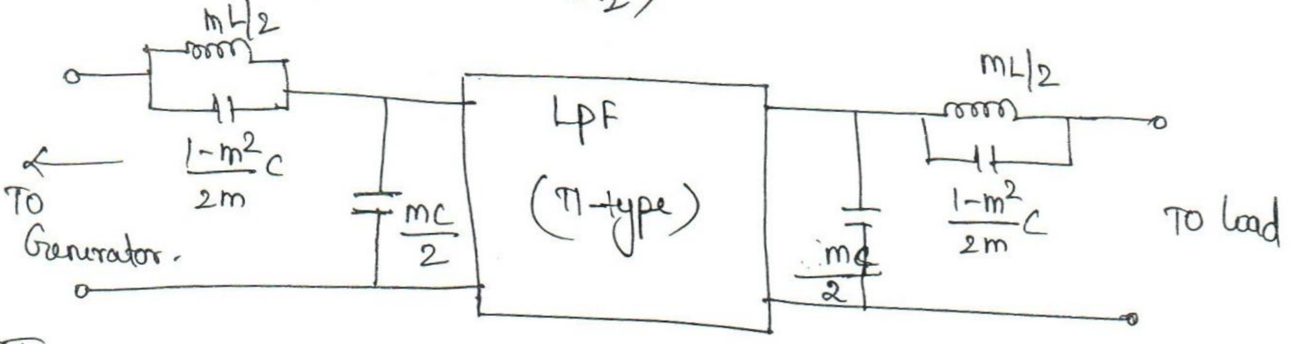


For LpF $Z_1 = j\omega L$, $Z_2 = \frac{1}{j\omega C}$

- $\frac{mZ_1}{2} = m\left(\frac{j\omega L}{2}\right) = j\omega\left(\frac{mL}{2}\right)$

$\odot \quad \frac{2m}{1-m^2} Z_2 = \frac{2m}{1-m^2}\left(\frac{1}{j\omega C}\right) = \frac{1}{j\omega\left(\frac{1-m^2}{2m}C\right)}$

$\odot \quad \frac{2Z_2}{m} = \frac{2\left(\frac{1}{j\omega C}\right)}{m} = \frac{1}{j\omega\left(\frac{mC}{2}\right)}$



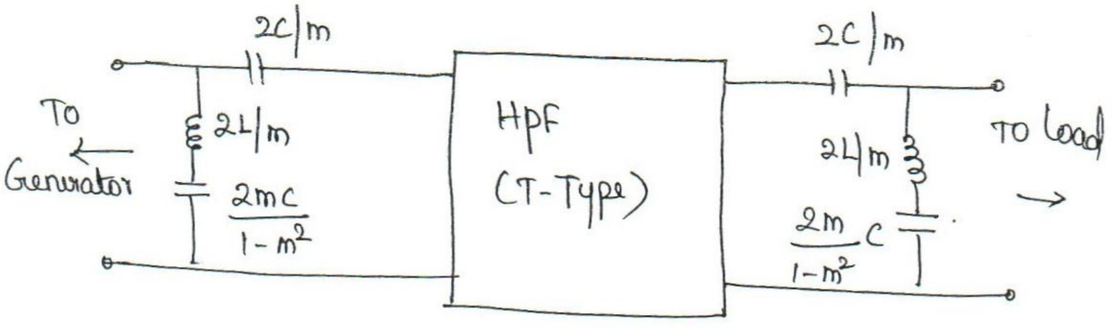
3) TERMINATING HALF Section for T-Configuration (HPF):-

In HPF $\quad Z_1 = \frac{1}{j\omega C}$ & $Z_2 = j\omega L$.

$\odot \quad \frac{mZ_1}{2} = \frac{m\left(\frac{1}{j\omega C}\right)}{2} = \frac{1}{j\omega\left(\frac{2C}{m}\right)}$ $\qquad \odot \quad \frac{2Z_2}{m} = \frac{2(j\omega L)}{m} = j\omega\left(\frac{2L}{m}\right)$

$\odot \quad \cancel{\frac{2m}{1-m^2} Z_1 = \frac{2m}{1-m^2}(j\omega L) = j\omega\left(\frac{2m}{1-m^2}L\right)}$

$\odot \quad \frac{1-m^2}{2m} Z_1 = \frac{1-m^2}{2m}\left(\frac{1}{j\omega C}\right) = \frac{1}{j\omega\left(\frac{2m}{1-m^2}C\right)}$



④ TERMINATING Half Section for Π- Configuration (HPF):-

$Z_1 = 1/j\omega C$; $Z_2 = j\omega L$.

⊙ $\frac{mZ_1}{2} = \frac{m\left(\frac{1}{j\omega C}\right)}{2} = \frac{1}{j\omega\left(\frac{2C}{m}\right)}$ ⊙ $\frac{2Z_2}{m} = \frac{2(j\omega L)}{m} = j\omega\left(\frac{2L}{m}\right)$

⊙ $\frac{2m}{1-m^2} Z_1 = \frac{2m}{1-m^2}(j\omega L) = j\omega\left(\frac{2m}{1-m^2}L\right)$

Problems

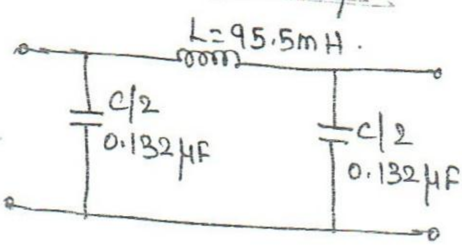
1) Design a composite LPF with Π- Section to work into 600Ω with cutoff frequency 2KHz and very high attenuation at 2.2KHz also terminate the filter properly.

Given data $R_K = 600\Omega$, $f_c = 2KHz$, $f_\infty = 2.2KHz$.

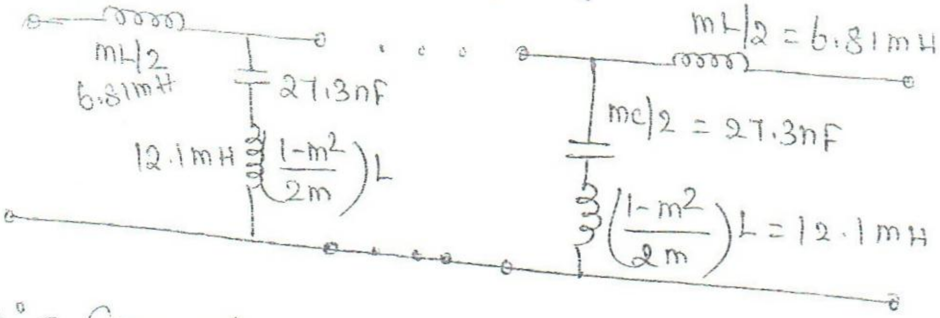
Step 1:- Design prototype Π Section LPF.

$$C = \frac{1}{\pi f_c R_K} = \frac{1}{\pi \times 2\times10^3 \times 600} = 2.65\times10^{-7} = 0.265\,\mu F$$

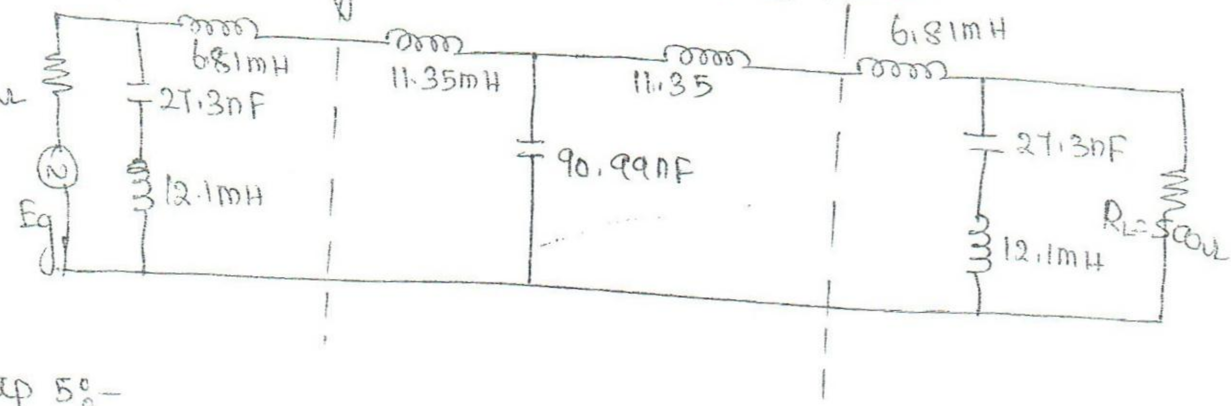
$$L = \frac{R_K}{\pi f_c} = \frac{600}{\pi\times 2\times10^3} = 95.5\,mH$$



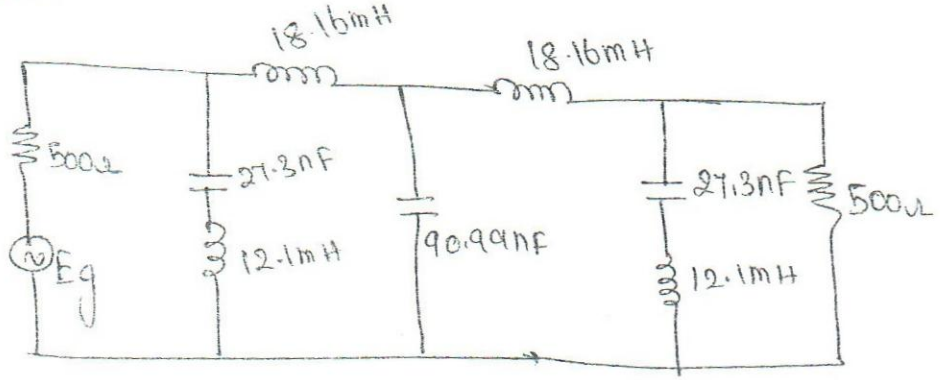
Step 3:- Design of terminating half sections with m = 0.6



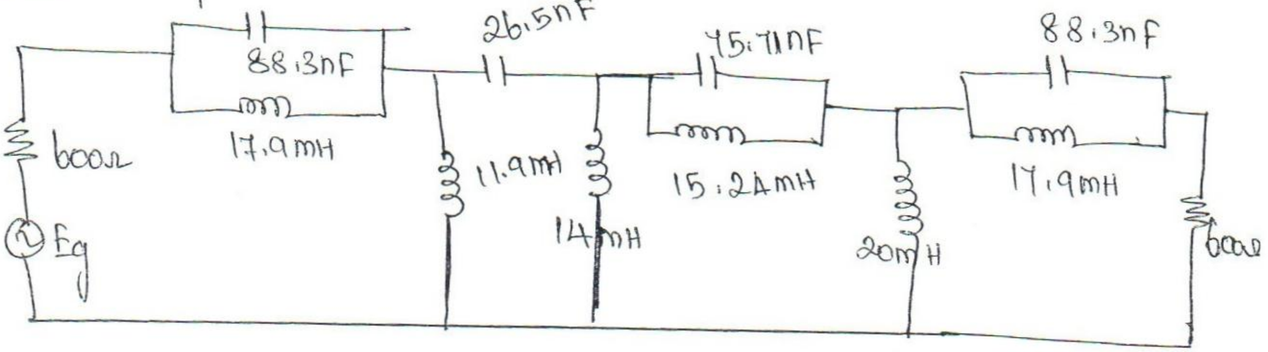
Step 4:- Composite filter with individual section



Step 5:-



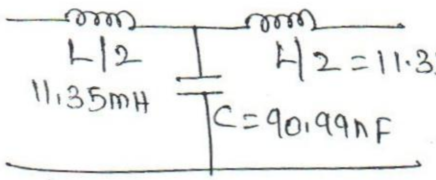
Step : 5 Composite Filter



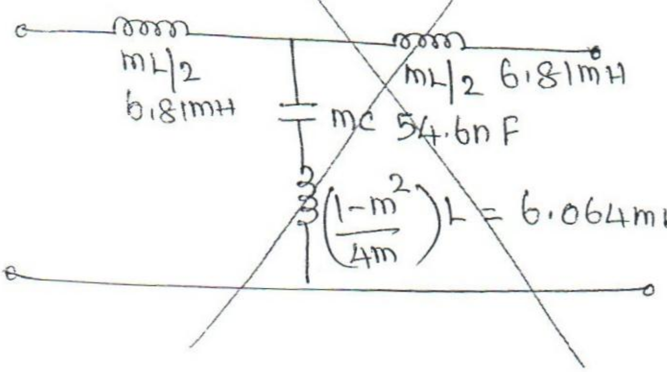
4) Design a Composite LPF Consisting a prototype T-Section and two terminating m-derived Section the cutoff frequency is 1KHz and load is 500Ω.

Step 1:- prototype LPF

$C = \frac{1}{\pi f_c R_K} = 90.94nF \approx 91nF$ $\quad L = \frac{R_K}{\pi f_c} = 22.73mH$



Step 2: m-derived T - LPF. (m=0.6)



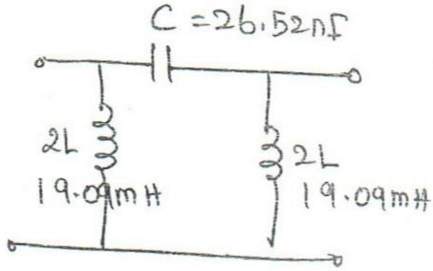
3) Design a HPF Composite Filter (Π-Sec) having design impedance 600Ω cutoff freq of 5KHZ m=0.35.

Given data → $R_K = 600\Omega$, $f_c = 5KHZ$, $m = 0.35$.

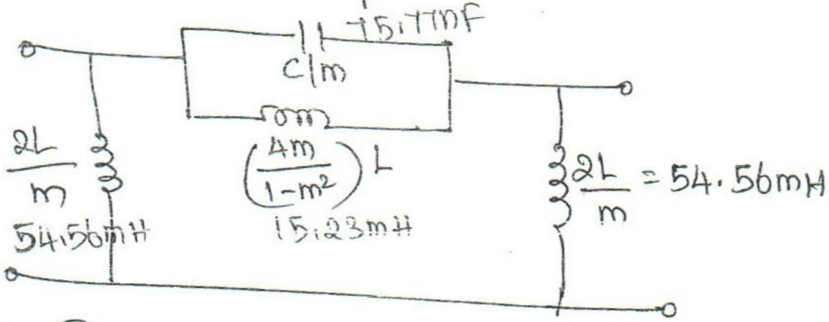
Step 1:- prototype HPF

$$C = \frac{1}{4\pi f_c R_c} = 26.52nF \text{ (or) } 0.026\mu F$$

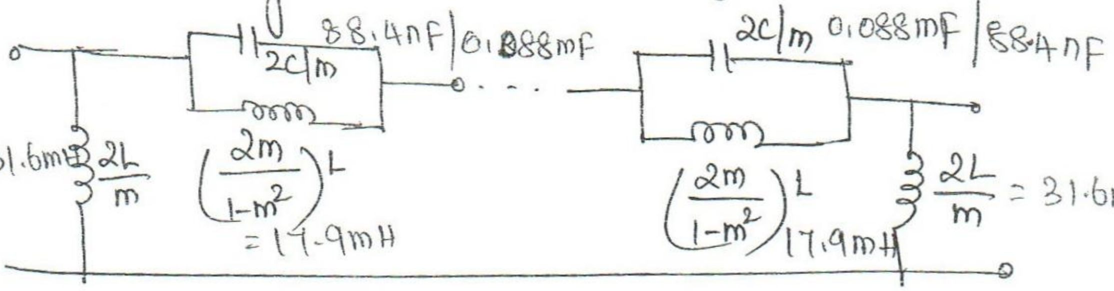
$$L = \frac{R_K}{4\pi f_c} = 9.55mH$$



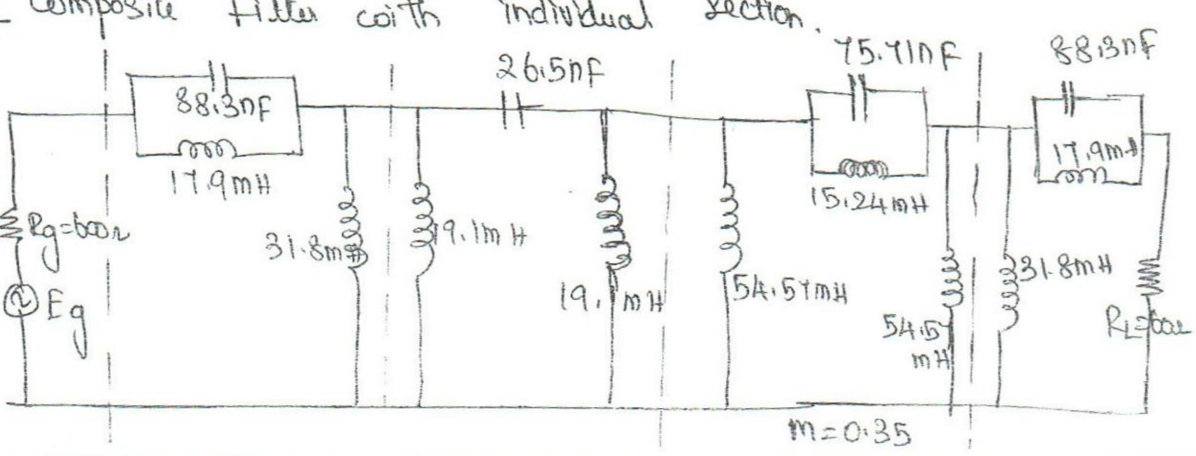
Step 2:- m-derived HPF



Step 3:- Terminating section with m=0.6



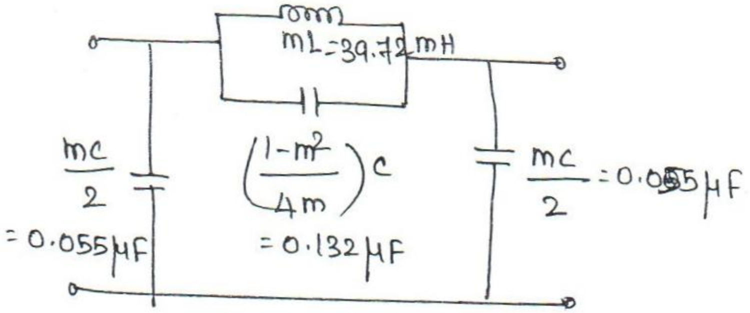
Step 4:- Composite filter with individual section.



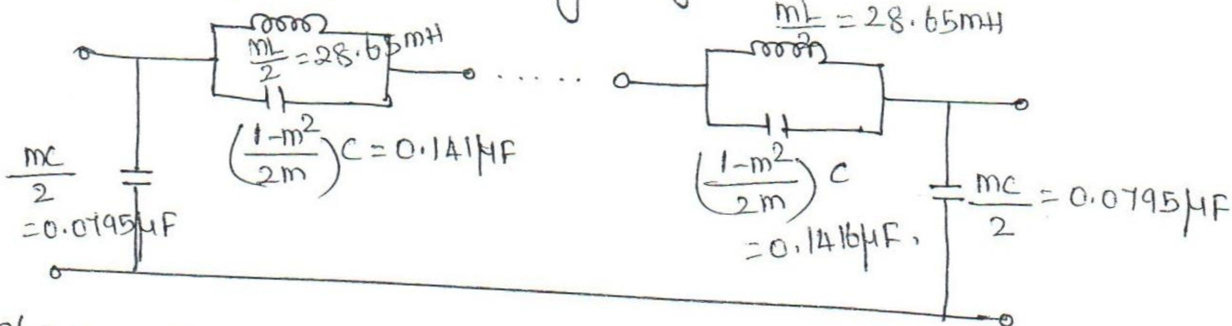
Step 2: Design m-derived Π Section LPF

$$m = \sqrt{1-\left(f_c/f_\infty\right)^2} = \sqrt{1-\left(\frac{2KHz}{2.2KHz}\right)^2}$$

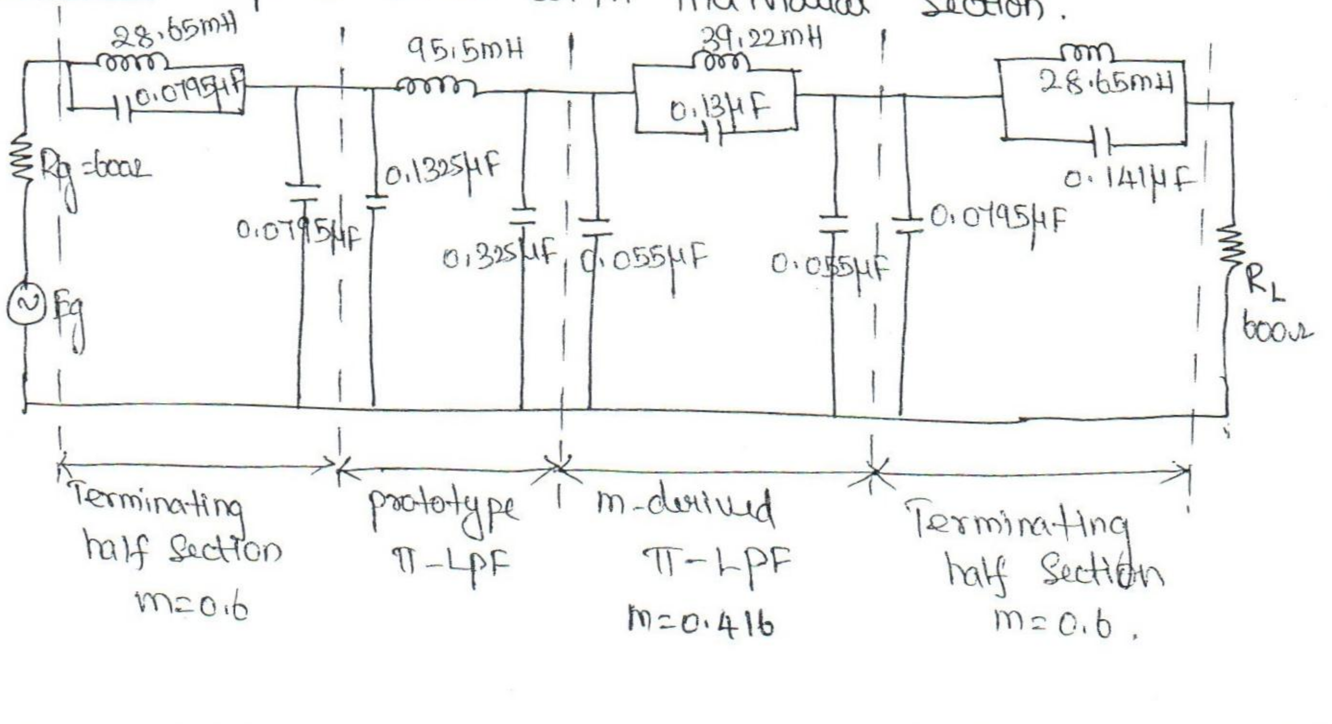
$m = 0.416$



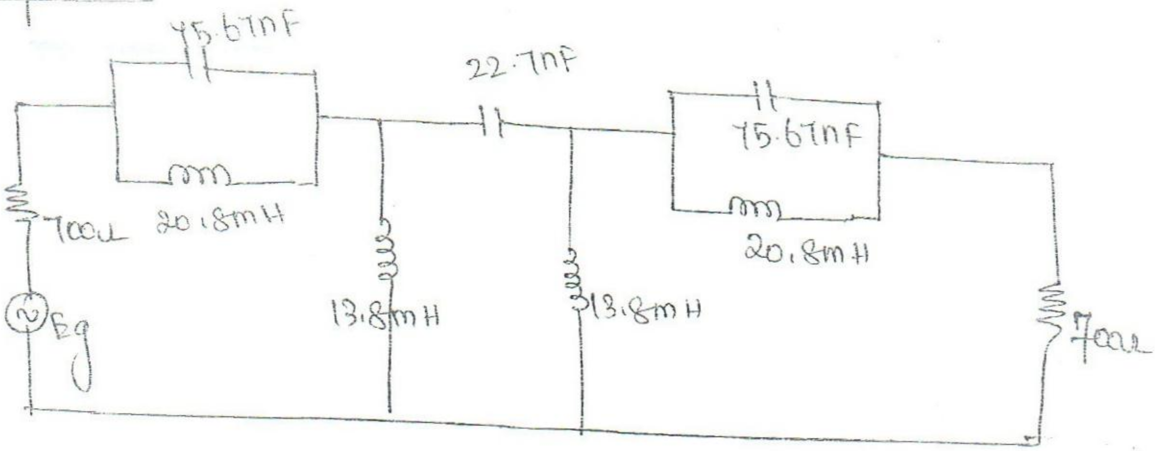
Step 3:- Design of terminating half sections with m=0.6.



Step 4:- Composite filter with individual Section.



Steps:

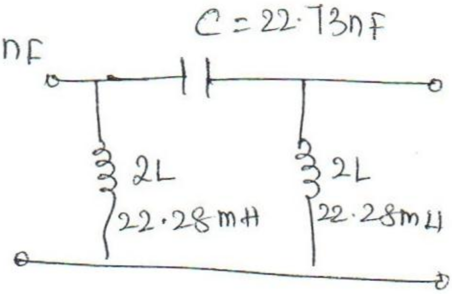


5) Design a Composite HPF with prototype π-Section and two terminating half section the cutoff frequency 5000 Hz are load of 700Ω.

Step 1:- prototype HPF

$$C = \frac{1}{4\pi R_k f_c} = \frac{1}{4\pi \times 700 \times 5000} = 22.73\,nF$$

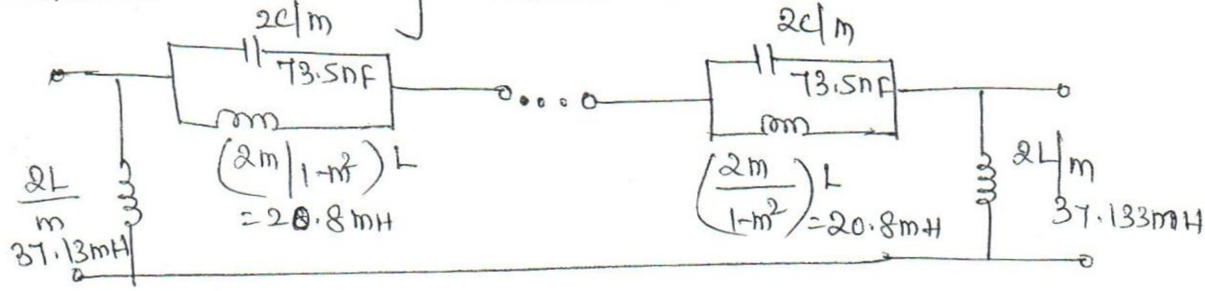
$$L = \frac{R_k}{4\pi f_c} = \frac{700}{4\pi \times 5000} = 11.140\,mH$$



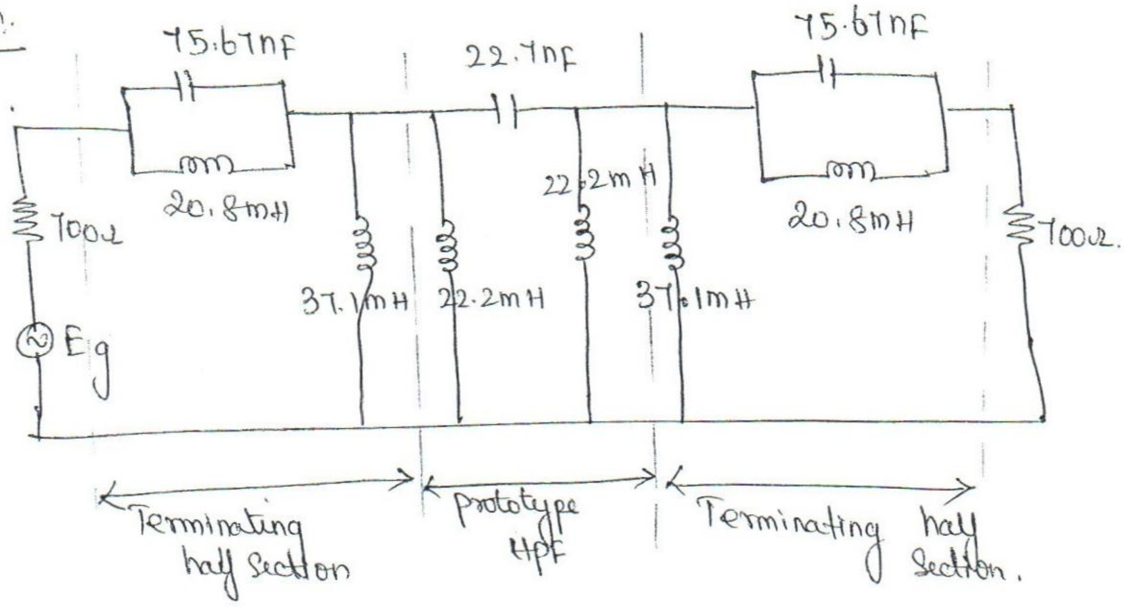
Step 2:- m derived π-HPF

(—)

Step 3:- Terminating Section with m = 0.6



Step 4:-



UNIT-V

Waveguides & Cavity Resonators:-

Waveguide:-

↳ A hollow metallic tube of uniform cross section txing Electromagnetic waves by successive reflections from inner walls of the tube is called waveguide.

↳ waveguides are madeup of Copper, Aluminium, brass.

Types:-

1) Rectangular 2) Circular 3) Elliptical.

Waveguides	Txion Lines
1) waveguides are used to transmit above 3GHz freq	used to transmit below 3GHz.
2) act as HPF, it allow to propagate a signals greater than 'fc'	act as All pass Filter (ie) Allow all frequency.
3) wave impedance $Z_0 = (E/H)$	Char. imp $Z_0 = \sqrt{Z/Y}$
4) wave propagation accordance with Field Theory	wave prop accordance with Circuit Theory

Advantages:-	Dis-advantages:-
1) Highly Shielded	1) High cost
2) High power txion capability	2) Not suitable for low freq as (10^{-6} to 10^{-9})
3) Low loss for μwave freq	

Formulas

1) Maxwell eqn

$\nabla \times H = j\omega \varepsilon E$

$\nabla \times E = -j\omega\mu H$

2) prop. const (γ)

$\gamma = \alpha + j\beta$

$\gamma = \sqrt{(\sigma + j\omega\varepsilon)(j\omega\mu)}$

($\sigma = 0$) for Non-conductive region

$\gamma = \sqrt{j\omega\varepsilon \times j\omega\mu}$

$\boxed{\gamma = j\omega\sqrt{\mu\varepsilon}}$

3) Wave Eqn

(i) $\nabla^2 E = \gamma^2 E$

$\nabla^2 E = -\omega^2 \mu\varepsilon E$

$$\boxed{\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = -\omega^2 \mu\varepsilon E}$$

(ii) $\nabla^2 H = \gamma^2 H$

$\nabla^2 H = -\omega^2 \mu\varepsilon H$

$$\boxed{\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} = -\omega^2 \mu\varepsilon H}$$

$\mu = \mu_0 \varepsilon_r$

$\varepsilon = \varepsilon_0 \varepsilon_r$

$\mu_0 = 4\pi \times 10^{-7}$ (permeability in free space)

$\varepsilon_0 = 8.856 \times 10^{-12}$ (permittivity in free space)

4) Electric field

$E = E_x \hat{a_x} + E_y \hat{a_y} + E_z \hat{a_z}$

$H = H_x \hat{a_x} + H_y \hat{a_y} + H_z \hat{a_z}$

5) Transverse Electric wave (TE)

$E_z = 0, \quad H_z \neq 0$

6) Transverse Mag. wave (TM)

$E_z \neq 0, \quad H_z = 0$

7) Transverse Electro Magnetic Wave (TEM)

$E_z = 0, \quad H_z = 0$

Note

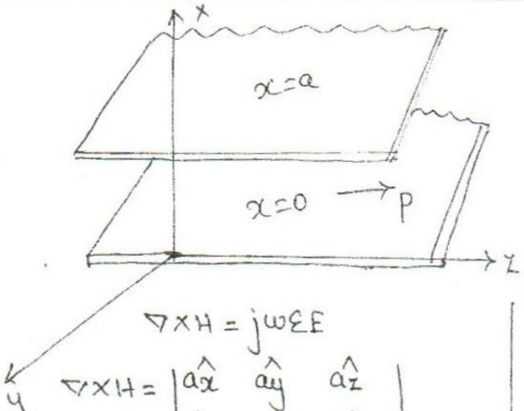
$E_{tan} = 0$

$H_{tan} \neq 0$

$H_{normal} = 0$

PARALLEL CONDUCTING PLATES

Applications of Restrictions to Maxwell Equations :—



① Maxwell Eqn

$$\nabla \times H = j\omega \varepsilon E$$

$$\nabla \times E = -j\omega \mu H$$

$$\nabla \times H = j\omega \varepsilon E$$

$$\nabla \times H = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix}$$

$$= \hat{a_x}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) + \hat{a_y}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) + \hat{a_z}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$$

$$\nabla \times H = j\omega \varepsilon\left[\hat{a_x} E_x + \hat{a_y} E_y + \hat{a_z} E_z\right]$$

Equating

$$\left.\begin{aligned} \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} &= j\omega \varepsilon E_x \\ \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} &= j\omega \varepsilon E_y \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} &= j\omega \varepsilon E_z \end{aligned}\right\} \rightarrow ①$$

$$\nabla \times E = -j\omega \mu H$$

$$\nabla \times E = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix}$$

$$\nabla \times E = \hat{a_x}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + \hat{a_y}\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) + \hat{a_z}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

$$\nabla \times E = -j\omega \mu\left[\hat{a_x} H_x + \hat{a_y} H_y + \hat{a_z} H_z\right]$$

Equating

$$\left.\begin{aligned} \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} &= -j\omega \mu H_x \\ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} &= -j\omega \mu H_y \\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} &= -j\omega \mu H_z \end{aligned}\right\} \rightarrow ②$$

(2) Wave eqn

$$\nabla^2 E = \gamma^2 E \quad \text{and} \quad \nabla^2 H = \gamma^2 H$$

$$\left.\begin{aligned} &\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = -\omega^2 \mu \varepsilon E \qquad \frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} = -\omega^2 \mu \varepsilon H \end{aligned}\right\} \rightarrow (3)$$

Assume prop is in Z-direction & Variation of field component in Z-direction is $e^{-\gamma z}$.

$$H_y = H_y^0 e^{-\gamma z}$$

$$\frac{\partial H_y}{\partial z} = -\gamma H_y^0 e^{-\gamma z}$$

$$\boxed{\frac{\partial H_y}{\partial z} = -\gamma H_y}$$

Illy $\frac{\partial H_x}{\partial z} = -\gamma H_x$

Illy

$$\left.\begin{aligned} \frac{\partial E_x}{\partial z} &= -\gamma E_x \\ \frac{\partial E_y}{\partial z} &= -\gamma E_y \end{aligned}\right\} \rightarrow (4)$$

Sub eqn (4) in (1) and (2) ($\because$ There is no variation in y-direction ie) derivative of $y = 0$).

$$\left.\begin{aligned} \gamma H_y &= j\omega\varepsilon E_x \\ -\gamma H_x - \frac{\partial H_z}{\partial x} &= j\omega\varepsilon E_y \\ \frac{\partial H_y}{\partial x} &= j\omega\varepsilon E_z \end{aligned}\right\} \rightarrow (5)$$

$$\left.\begin{aligned} \gamma E_y &= -j\omega\mu H_x \\ -\gamma E_x - \frac{\partial E_z}{\partial x} &= -j\omega\mu H_y \\ \frac{\partial E_y}{\partial x} &= -j\omega\mu H_z \end{aligned}\right\} \rightarrow (6)$$

$$\frac{\partial^2 E}{\partial x^2} + \gamma^2 E = -\omega^2 \mu \varepsilon E \quad \text{and} \quad \frac{\partial^2 H}{\partial x^2} + \gamma^2 H = -\omega^2 \mu \varepsilon H \rightarrow (7)$$

By Solving ⑤ & ⑥

Hint [$H_x, H_y, H_z, E_x, E_y, E_z \rightarrow$ Consider H_z & E_z as Known Variable & calculate H_x, H_y and E_x & H_y interms of H_z & E_z]

To find H_x: (Take H_x eqn from ⑤ ⑥)

$$\begin{bmatrix} -\bar{\gamma} H_x - \dfrac{\partial H_z}{\partial x} = j\omega\varepsilon E_y \\ \bar{\gamma} E_y = -j\omega\mu H_x \end{bmatrix}$$

$$E_y = \frac{-j\omega\mu H_x}{\bar{\gamma}}$$

Sub E_y in 1st eqn.

$$-\bar{\gamma} H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon\left[\frac{-j\omega\mu H_x}{\bar{\gamma}}\right]$$

$$-\bar{\gamma} H_x - \frac{\partial H_z}{\partial x} = \frac{\omega^2\varepsilon\mu H_x}{\bar{\gamma}}$$

$$-\bar{\gamma} H_x - \frac{\omega^2\varepsilon\mu H_x}{\bar{\gamma}} = \frac{\partial H_z}{\partial x}$$

$$-H_x\left[\frac{\bar{\gamma}^2 + \omega^2\varepsilon\mu}{\bar{\gamma}}\right] = \frac{\partial H_z}{\partial x}$$

$$H_x = \frac{-\bar{\gamma}}{\bar{\gamma}^2 + \omega^2\mu\varepsilon}\left(\frac{\partial H_z}{\partial x}\right)$$

$(h^2 = \bar{\gamma}^2 + \omega^2\mu\varepsilon)$

$$\boxed{H_x = \frac{-\bar{\gamma}}{h^2}\left(\frac{\partial H_z}{\partial x}\right)}$$

IIly

$$\boxed{E_x = \frac{-\bar{\gamma}}{h^2}\left(\frac{\partial E_z}{\partial x}\right)}$$

To find H_y: (Take H_y eqn from ⑤ & ⑥)

$$\begin{bmatrix} \bar{\gamma} E_x + \dfrac{\partial E_z}{\partial x} = j\omega\mu H_y \\ \bar{\gamma} H_y = j\omega\varepsilon E_x \end{bmatrix}$$

$$\therefore E_x = \frac{\bar{\gamma} H_y}{j\omega\varepsilon}$$

Sub E_x in 1st eqn.

$$\bar{\gamma}\left[\frac{\bar{\gamma} H_y}{j\omega\varepsilon}\right] + \frac{\partial E_z}{\partial x} = j\omega\mu H_y$$

$$\frac{\bar{\gamma}^2 H_y}{j\omega\varepsilon} - j\omega\mu H_y = -\frac{\partial E_z}{\partial x}$$

$$H_y\left[\frac{\bar{\gamma}^2 + \omega^2\mu\varepsilon}{j\omega\varepsilon}\right] = -\frac{\partial E_z}{\partial x}$$

$$H_y = \frac{-j\omega\varepsilon}{\bar{\gamma}^2 + \omega^2\mu\varepsilon}\left(\frac{\partial E_z}{\partial x}\right)$$

$$\boxed{H_y = \frac{-j\omega\varepsilon}{h^2}\left(\frac{\partial E_z}{\partial x}\right)}$$

IIly

$$\boxed{E_y = \frac{-j\omega\mu}{h^2}\left(\frac{\partial H_z}{\partial x}\right)}$$

Conclusion:

- H_x, H_y, E_x & E_y are expressed interms of E_z and H_z.
- There must be z-components either in E (or) H, otherwise all the components would be zero.
- If there is E-component in the direction of propagation of (E_z) and No H-components in this direction. Such waves are called as <u>E-waves (or) TM (Transverse Magnetic Waves)</u>
- If there is H-component in the direction of prop. of (H_z) and No E-components in this direction. Such waves are called as <u>H-waves (or) TE (Transverse Electric waves)</u>

Formula: $E_x = \frac{-\gamma}{h^2}\left(\frac{\partial E_z}{\partial x}\right)$, $H_x = \frac{-\gamma}{h^2}\left(\frac{\partial H_z}{\partial x}\right)$

$E_y = \frac{-j\omega\mu}{h^2}\left(\frac{\partial H_z}{\partial x}\right)$, $H_y = \frac{-j\omega\varepsilon}{h^2}\left(\frac{\partial E_z}{\partial x}\right)$

Note (X.)

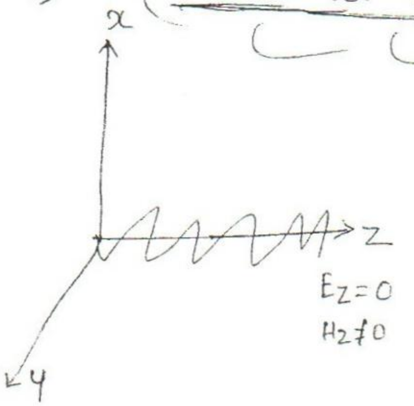
$E_{tan} = 0$

$H_{tan} \neq 0$

$E_{Normal} = 0$

16m

Transverse Electric Wave in parallel plates



Electric field	Magnetic field
$E_x = 0$	$H_x = ?$
$E_y = ?$	$H_y = 0$
$E_z = 0$	$H_z = ?$

In TE wave → H_z is direction of prop ($H_z \neq 0$)
→ $E_z = 0$

(*) $E_x = \frac{-\gamma}{h^2}\left(\frac{\partial \cancel{E_z}}{\partial x}\right)$ (*) $H_y = \frac{-j\omega\varepsilon}{h^2}\left(\frac{\partial \cancel{E_z}}{\partial x}\right)$

$\boxed{E_x = 0}$ $\boxed{H_y = 0}$

To find E_y:-

$E_y = E_{yo}\, e^{-\gamma z}$

wave eqn for component $\boxed{E_y = C_1 \sinh x + C_2 \cosh x}$

C_1, C_2 = arbitary constant

Expressed in time & direction

$\boxed{E_y = (C_1 \sinh x + C_2 \cosh x)\, e^{-\gamma z}}$

Boundary condition:-

$x = 0 \;,\; E_y = 0$

$x = a \;,\; E_y = 0$

1st condn $(x=0)$:-

$0 = 0 + C_2$

$\boxed{C_2 = 0}$

$\boxed{\therefore\; E_y = C_1 \sin hx\, e^{-\gamma z}}$

2nd condn $(x=a)$:-

$0 = C_1 \sin ha + 0$

$C_1 \sin ha = 0$

$\sin ha = 0 \Rightarrow \sin ha = m\pi$

$\left[\sin(1\times\pi) = 0 \quad \sin(2\times\pi) = 0 \quad \vdots\right]$

$\boxed{h = \frac{m\pi}{a}} \qquad m = 1, 2, 3 \ldots\ldots$

$\boxed{E_y = C_1 \sin\left(\frac{m\pi}{a}x\right) e^{-\gamma z}}$

To find H_x:-

$\nabla \times E = -j\omega\mu H$ (Maxwell eqn)

$$\begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu\left[H_x\hat{a_x} + H_y\hat{a_y} + H_z\hat{a_z}\right]$$

$$\begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \partial/\partial x & 0 & -\gamma \\ 0 & E_y & 0 \end{vmatrix} = -j\omega\mu\left[H_x\hat{a_x} + H_y\hat{a_y} + H_z\hat{a_z}\right]$$

$\hat{a_x}(0 + \gamma E_y) = -j\omega\mu H_x \hat{a_x}$

$$\boxed{H_x = \frac{-\gamma E_y}{j\omega\mu}}$$

Sub 'Ey' in Hx

$$\boxed{Hx = \frac{-\gamma}{j\omega\mu} C_1 \sin\left(\frac{m\pi}{a}x\right) e^{-\gamma z}}$$

To find H_z:-

$$\cancel{a_z}\left(\frac{\partial E_y}{\partial x} - 0\right) = -j\omega\mu H_z \cancel{a_z}$$

$$\boxed{H_z = \frac{-1}{j\omega\mu}\frac{\partial E_y}{\partial x}}$$

$$H_z = \frac{-1}{j\omega\mu}\frac{\partial}{\partial x}\left[C_1 \sin\left(\frac{m\pi}{a}x\right) e^{-\gamma z}\right]$$

$$H_z = \frac{-1}{j\omega\mu} \cdot C_1 \cos\left(\frac{m\pi}{a}x\right) e^{-\gamma z} \cdot \left(\frac{m\pi}{a}\right)$$

$$\boxed{H_z = -\frac{C_1}{j\omega\mu} \cos\left(\frac{m\pi x}{a}\right) e^{-\gamma z} \cdot \frac{m\pi}{a}}$$

Conclusion:- $\gamma = \alpha + j\beta$ $(\alpha = 0)$

$\boxed{\gamma = j\beta}$

Elec. field

$E_x = 0$

$E_y = C_1 \sin\left(\frac{m\pi}{a}\right) x \, e^{-j\beta z}$

$E_z = 0$

Mag. field

$H_x = \frac{-\beta}{\omega\mu} C_1 \sin\left(\frac{m\pi}{a}x\right) e^{-j\beta z}$

$H_y = 0$

$H_z = \frac{-C}{j\omega\mu}\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}x\right) e^{-j\beta z}.$

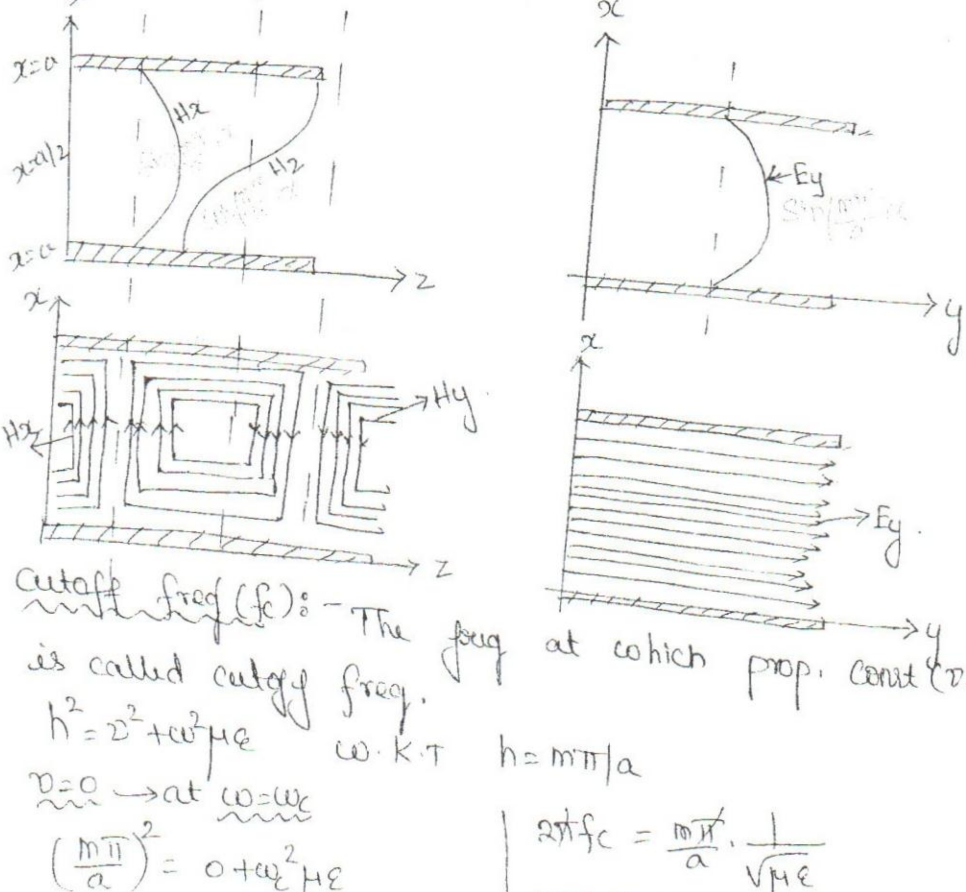
m = field of configuration (or) mode.

TE_{m0} = 2nd Subscript refer to integer varies with y

If m = 0 → then $E_y, H_x, H_z = 0$

∴ m = 1 → TE_{10} & TH_{10} → Lowest order Mode (or) Dominant mode.

$TE_{m,n}$ ⇒ m = No. of zero crossing in x-direction
n = No. of " " " y-direction (always n = 0) ∴ no comp in y-direction



Cutoff freq (fc):- The freq at which prop. const (γ = 0) is called cutoff freq.

$h^2 = \gamma^2 + \omega^2 \mu \varepsilon$ w.k.T $h = m\pi/a$

$\gamma = 0 \to$ at $\omega = \omega_c$

$$\left(\frac{m\pi}{a}\right)^2 = 0 + \omega_c^2 \mu \varepsilon$$

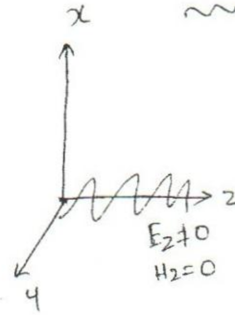
$$\omega_c^2 = \left(\frac{m\pi}{a}\right)^2 \cdot \frac{1}{\mu \varepsilon}$$

$$\omega_c = \frac{m\pi}{a} \cdot \frac{1}{\sqrt{\mu\varepsilon}}$$

$$2\pi f_c = \frac{m\pi}{a} \cdot \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\boxed{f_c = \frac{m}{2a}\left(\frac{1}{\sqrt{\mu\varepsilon}}\right)}$$

Transverse Magnetic Waves :-



Electric field	mag. field
$E_x = ?$	$H_x = 0$
$E_y = 0$	$H_y = ?$
$E_z = ?$ (Boundary)	$H_z = 0$

To find H_y :-

$$H_y = \left(C_3 \sinh x + C_4 \cosh x\right) e^{-\gamma z} \leftarrow H_y = H_y^{\circ} e^{-\gamma z}$$

$\boxed{H_{tan} \neq 0} \Rightarrow \therefore$ Boundary Condn cannot apply directly to H-component.

Apply Boundary condn

To find E_x :-

$\nabla \times H = j\omega\varepsilon E$ (Maxwell eqn)

$$\begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix} = j\omega\varepsilon\left[E_x \hat{a_x} + E_y \hat{a_y} + E_z \hat{a_z}\right]$$

$$\begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \partial/\partial x & 0 & -\gamma \\ 0 & H_y & 0 \end{vmatrix} = j\omega\varepsilon\left[E_x \hat{a_x} + E_y \hat{a_y} + E_z \hat{a_z}\right]$$

$$\cancel{a_x}\left(0 + \gamma H_y\right) = j\omega\varepsilon E_x \cancel{a_x}$$

$$\boxed{E_x = \frac{\gamma H_y}{j\omega\varepsilon}}$$

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To find E_z :-

$$\cancel{a_z}\left(\frac{\partial H_y}{\partial x} - 0\right) = j\omega\varepsilon E_z \cancel{a_z}$$

$$E_z = \frac{1}{j\omega\varepsilon}\frac{\partial H_y}{\partial x}$$

$$E_z = \frac{1}{j\omega\varepsilon}\frac{\partial}{\partial x}\left[C_3 \sin hx + C_4 \cos hx\right] e^{-\gamma z}$$

$$= \frac{1}{j\omega\varepsilon}\left[C_3 h \cos hx - C_4 h \sin hx\right] e^{-\gamma z}$$

$$\boxed{E_z = \frac{h}{j\omega\varepsilon}\left[C_3 \cos hx - C_4 \sin hx\right] e^{-\gamma z}}$$

Apply Boundary condition :-

1) $x = 0,\ E_z = 0$

$$0 = C_3 - 0$$

$$\boxed{C_3 = 0}$$

$$\boxed{E_z = \frac{-h}{j\omega\varepsilon}\left[C_4 \sin hx\right] e^{-\gamma z}}$$

2) Apply $x = a,\ E_z = 0$

$$0 = \sin ha$$

$$\sin ha = m\pi$$

$$\boxed{h = \frac{m\pi}{a}}$$

$$\boxed{\therefore E_z = \frac{-m\pi}{j\omega\varepsilon a}\left[C_4 \sin \frac{m\pi x}{a}\right] e^{-\gamma z}}$$

//

To find H_y :

$$E_z = \frac{1}{j\omega\varepsilon}\frac{\partial H_y}{\partial x}$$

$$\frac{\partial H_y}{\partial x} = j\omega\varepsilon E_z$$

Sub $\left(C_3 = 0 \text{ and } h = \frac{m\pi}{a}\right)$ B.C to H_y

$$H_y = \left(C_3 \sin hx + C_4 \cos hx\right) e^{-\gamma z}$$

$$\boxed{H_y = C_4 \cos\left(\frac{m\pi}{a}\right) x\, e^{-\gamma z}}$$

$\int$ on both side

$$Hy = \int j\omega\varepsilon E_z dx$$

$$= \int j\omega\varepsilon \left[\frac{-m\pi}{j\omega\varepsilon a} (C_4 \sin \frac{m\pi x}{a}) e^{-\gamma z}\right]$$

$$= \frac{j\omega\varepsilon m\pi}{j\omega\varepsilon a} \int C_4 \sin\left(\frac{m\pi x}{a}\right) e^{-\gamma z}$$

$$= -\frac{m\pi}{a} C_4 \left[-\frac{\cos \frac{m\pi x}{a}}{\frac{m\pi}{a}}\right] e^{-\gamma z}$$

$$\boxed{Hy = \left[C_4 \cos\left(\frac{m\pi}{a}\right)x\right] e^{-\gamma z}}$$

To find E_x:-

$$E_x = \frac{\gamma}{j\omega\varepsilon} Hy$$

$$\boxed{E_x = \frac{\gamma}{j\omega\varepsilon}\left[C_4 \cos\left(\frac{m\pi}{a}\right)x\right] e^{-\gamma z}}$$

To find E_z:-

$$E_z = \frac{1}{j\omega\varepsilon} \frac{\partial Hy}{\partial x}$$

$$E_z = \frac{1}{j\omega\varepsilon} \frac{\partial}{\partial x}\left[C_4 \cos\left(\frac{m\pi}{a}\right)x\right] e^{-\gamma z}$$

$$E_z = \frac{1}{j\omega\varepsilon} \cdot -\frac{m\pi}{a} C_4 \sin\left(\frac{m\pi}{a}\right) e^{-\gamma z}$$

$$\boxed{E_z = \frac{-m\pi}{a j\omega\varepsilon} C_4 \sin\left(\frac{m\pi}{a}\right) e^{-\gamma z}}$$

Conclusion:- Elec field ($\gamma = j\beta$)

$$E_x = \frac{\beta}{\omega\varepsilon}\left[C_4 \cos\left(\frac{m\pi}{a}\right)x\right] e^{-j\beta z}$$

$$E_y = 0$$

$$E_z = \frac{-m\pi}{j\omega\varepsilon a}\left[C_4 \sin\left(\frac{m\pi}{a}\right)x\right] e^{-j\beta z}$$

mag. field ($\gamma = j\beta$)

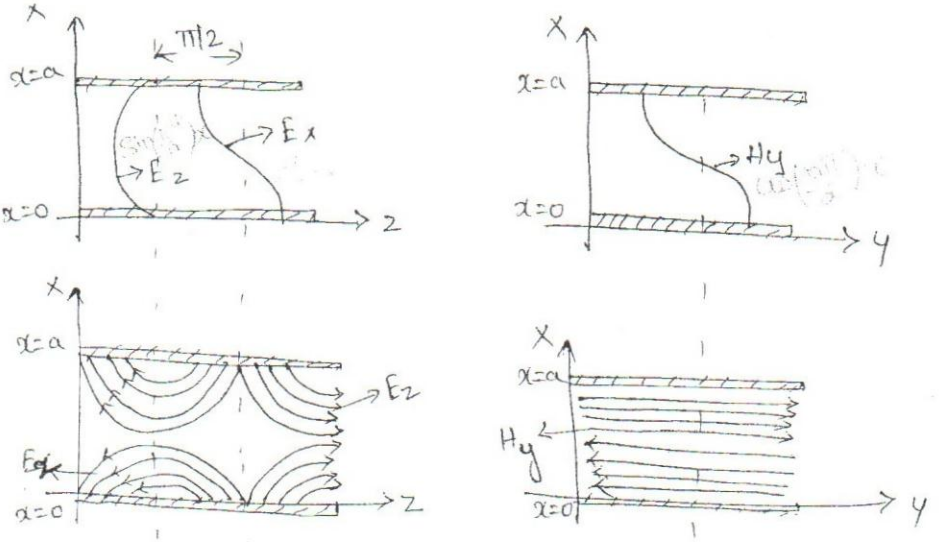
$$H_x = 0$$

$$Hy = \left[C_4 \cos\left(\frac{m\pi}{a}\right)x\right] e^{-j\beta z}$$

$$H_z = 0$$

7

$TM_{10} \rightarrow$ Lowest order Mode.



<u>calculation of cut-off freq (f_c):</u>

$$h^2 = \gamma^2 + \omega^2 \mu \varepsilon \qquad h = \frac{m\pi}{a}$$

$\gamma = 0 \rightarrow$ at $\omega = \omega_c$

$$\boxed{f_c = \frac{1}{2\cancel{\pi}} \cdot \frac{1}{\sqrt{\mu\varepsilon}} \left(\frac{m\cancel{\pi}}{a}\right)}$$

$$\boxed{f_c = \frac{m}{2a} \cdot \left(\frac{1}{\sqrt{\mu\varepsilon}}\right)}$$

16m

Transverse ElectroMagnetic Waves between ||el plates

* TEM is a wave where there is no component of E & H ie) $E_z = 0$ and $H_z = 0$.
* TEM obtained from $m=0$ (ie) if $m=0$ then All Comp $=0$
* $TEM_{00} \rightarrow$ Mode of TEM.

$$\boxed{TM/_{m=0} \Rightarrow TEM}$$

for TM.

$$E_x = \frac{\nu}{j\omega\varepsilon}\left[C_4 \cos\left(\frac{m\pi}{a}\right)x\right]e^{-j\beta z} \qquad H_x = 0$$

$$E_y = 0 \qquad H_y = \left[C_4 \cos\left(\frac{m\pi}{a}\right)x\right]e^{-j\beta z}$$

$$E_z = \frac{-m\pi}{a}\left[C_4 \sin\left(\frac{m\pi}{a}\right)x\right]e^{-j\beta z} \qquad H_z = 0$$

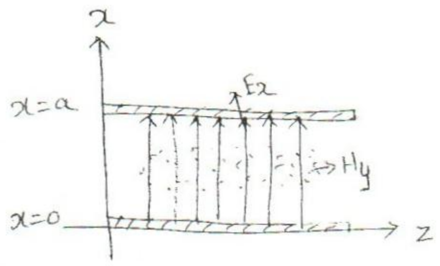
(Apply $m=0$)

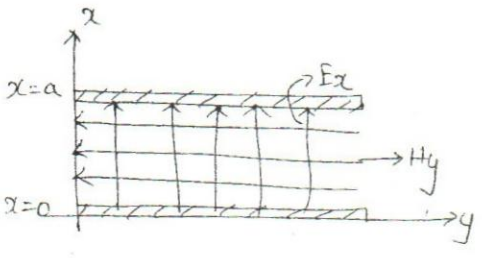
for TEM_{00}

$$E_x = \frac{\beta}{\omega\varepsilon} C_4 e^{-j\beta z} \qquad H_x = 0$$

$$E_y = 0 \qquad H_y = C_4 e^{-j\beta z}$$

$$E_z = 0 \qquad H_z = 0$$





Wave Impedance :-

In a waveguide, the wave is propagated by Electric & magnetic field .

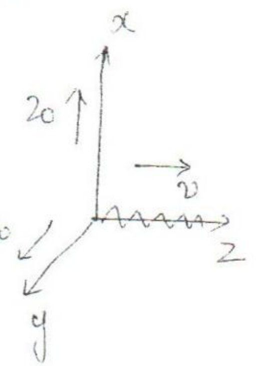
$\therefore$ wave impedance $\boxed{Z_0 = \frac{E}{H}}$

Free Space impedance $\eta = \sqrt{\mu/\varepsilon} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

$\mu_0 = 4\pi \times 10^{-7}$

$\varepsilon_0 = 8.854 \times 10^{-12}$

$\boxed{\therefore \eta = 377\,\Omega}$



Wave impedance for TE waves:-

$Z_{0(TE)} = \frac{E}{H} = \frac{E_y}{H_x}$ [Don't consider z]

$= \frac{C_1 \sin\left(\frac{m\pi}{a}\right)x \cdot e^{-j\beta z}}{\frac{-\beta}{\omega\mu} C_1 \sin\left(\frac{m\pi}{a}\right)x \cdot e^{-j\beta z}}$ (or) $\frac{\frac{+j\omega\mu}{h^2}\left(\frac{\partial H_z}{\partial x}\right)}{\frac{+\gamma}{h^2}\left(\frac{\partial H_z}{\partial x}\right)} = \frac{j\omega\mu}{\gamma}$

$= \frac{j\omega\mu}{j\beta}$

$Z_0 = -\frac{\omega\mu}{\beta}$

$|Z_0| = \frac{\omega\mu}{\beta} = \frac{2\pi f\mu}{\beta} = \frac{2\pi f\,\mu}{2\pi\sqrt{\mu\varepsilon}\sqrt{f^2 - f_c^2}}$

$= \frac{\sqrt{\mu}}{\sqrt{\varepsilon}\left(\sqrt{1-\left(\frac{f_c}{f}\right)^2}\right)} = \sqrt{\frac{\mu}{\varepsilon}} \cdot \frac{1}{\sqrt{1-(f_c/f)^2}}$

$$\boxed{Z_{0(TE)} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}}$$

1) $f < f_c \rightarrow Z_0 =$ imag (No propagation)
2) $f = f_c \rightarrow Z_0 = \infty$
3) $f > f_c \rightarrow Z_0 =$ real

Wave Impedance of TM waves:

$$Z_{0(TM)} = \frac{E}{H} = \frac{E_x}{H_y} \quad \text{(or)}$$

$$= \frac{\frac{\beta C_4}{\omega\varepsilon}\cos\left(\frac{m\pi}{a}\right)x \cdot e^{-j\beta z}}{C_4 \cos\left(\frac{m\pi}{a}\right)x \cdot e^{-j\beta z}} \quad \text{(or)} \quad \frac{\frac{-\gamma}{h^2}\left(\frac{\partial E_z}{\partial x}\right)}{\frac{-j\omega\varepsilon}{h^2}\left(\frac{\partial E_z}{\partial x}\right)} = \frac{\gamma}{j\omega\varepsilon} = \frac{j\beta}{j\omega\varepsilon}$$

$$Z_{0(TM)} = \frac{\beta}{\omega\varepsilon} = \frac{2\pi\sqrt{\mu\varepsilon}\sqrt{f^2-f_c^2}}{2\pi f\varepsilon} = \frac{\sqrt{\mu}\cdot f\sqrt{1-\left(\frac{f_c}{f}\right)^2}}{f\sqrt{\varepsilon}}$$

$$Z_{0(TM)} = \sqrt{\frac{\mu}{\varepsilon}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\boxed{Z_{0(TM)} = \eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

1) $f < f_c \rightarrow Z_0 =$ img (No prop)
2) $f = f_c \rightarrow Z_0 = 0$
3) $f > f_c = Z_0 = \eta$

Wave Impedance of TEM wave:

$$Z_{0(TEM)} = \frac{E}{H} = \frac{E_x}{H_y}$$

$$= \frac{\frac{\beta}{\omega\varepsilon} \cancel{C_4}\, \cancel{e^{-j\beta z}}}{\cancel{C_4} \cdot \cancel{e^{-j\beta z}}} = \frac{\beta}{\omega c}$$

$$= \frac{2\pi\sqrt{\mu\varepsilon}\sqrt{f^2-f_c^2}}{2\pi f c} = \frac{\cancel{2\pi}\sqrt{\mu\varepsilon}\cancel{f}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}{\cancel{2\pi}\cancel{f}c}$$

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$$Z_{O(TEM)} = \eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

W.K.T $f_c = \frac{m}{2a}\left(\frac{1}{\sqrt{\mu\varepsilon}}\right)$ when $m = 0 \rightarrow f_c = 0$

$$Z_{O\,TEM} = \eta\sqrt{1-0}$$

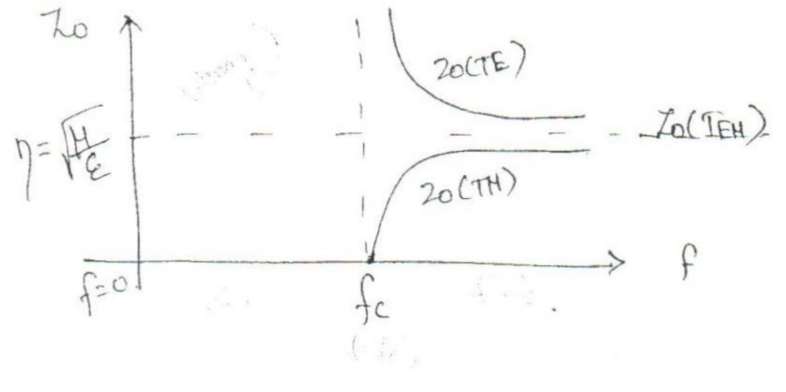
$$\boxed{Z_{O(TEM)} = \eta}$$

Conclusion:-

$$Z_{O(TE)} \times Z_{O(TM)} = \eta\sqrt{1-\left(\frac{f_c}{f}\right)^2} \times \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$= \eta^2$$

$$\boxed{Z_{O(TE)} \times Z_{O(TM)} = Z_{O(TEM)}}$$



RECTANGULAR WAVEGUIDE:-

A hollow metallic tube of uniform rectangular cross section txing electromagnetic waves are called as Rect. waveguide.

Maxwell Eqn in Rectangular waveguide

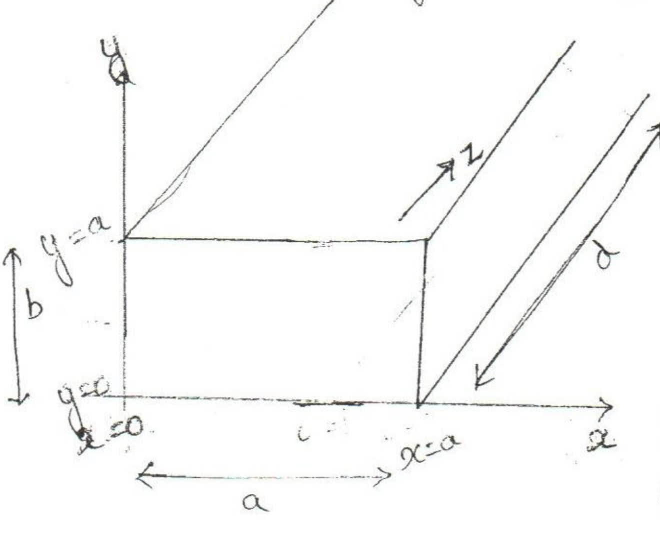
$$E_x = \frac{-\gamma}{h^2}\left(\frac{\partial E_z}{\partial x}\right) - \frac{j\omega\mu}{h^2}\left(\frac{\partial H_z}{\partial y}\right)$$

$$E_y = \frac{-\gamma}{h^2}\left(\frac{\partial E_z}{\partial y}\right) + \frac{j\omega\mu}{h^2}\left(\frac{\partial H_z}{\partial x}\right)$$

$$H_x = \frac{-\gamma}{h^2}\left(\frac{\partial H_z}{\partial x}\right) + \frac{j\omega\varepsilon}{h^2}\left(\frac{\partial E_z}{\partial y}\right)$$

$$H_y = \frac{-\gamma}{h^2}\left(\frac{\partial H_z}{\partial y}\right) - \frac{j\omega\varepsilon}{h^2}\left(\frac{\partial E_z}{\partial x}\right)$$

(TM) - Transverse Magnetic waves in the Rectangular Waveguide:-



a = width
b = height
d = length.

$a >> b$

$b = a/2$

Boundary condn:

(i) $x = 0$, $x = a$

(ii) $y = 0$, $y = b$

TM condition:- ($E_z \neq 0$, $H_z = 0$)

Maxwell eqn

$$E_x = -\frac{\gamma}{h^2}\left(\frac{\partial E_z}{\partial x}\right) - \frac{j\omega\mu}{h^2}\cancel{\left(\frac{\partial H_z}{\partial y}\right)}$$

$$E_y = -\frac{\gamma}{h^2}\left(\frac{\partial E_z}{\partial y}\right) + \frac{j\omega\mu}{h^2}\cancel{\left(\frac{\partial H_z}{\partial x}\right)}$$

$$H_x = -\frac{\gamma}{h^2}\cancel{\left(\frac{\partial H_z}{\partial x}\right)} + \frac{j\omega\varepsilon}{h^2}\left(\frac{\partial E_z}{\partial y}\right)$$

$$H_y = -\frac{\gamma}{h^2}\cancel{\left(\frac{\partial H_z}{\partial y}\right)} - \frac{j\omega\varepsilon}{h^2}\left(\frac{\partial E_z}{\partial x}\right)$$

Apply TM condn ($H_z = 0$, $E_z \neq 0$)

All above eqn interms of 'E_z'.

1) To find E_z:-

$E_z = XY$ $\quad X = C_1 \cos Ax + C_2 \sin Ax$

$Y = C_3 \cos By + C_4 \sin By$

$\because$ Rect. waveguide has two boundary

$$\boxed{E_z = (C_1 \cos Ax + C_2 \sin Ax)(C_3 \cos By + C_4 \sin By)\, e^{-\gamma z}} \rightarrow (1)$$

$C_1, C_2, C_3, C_4 \Rightarrow$ arbitary constant

Apply 1st boundary condn:-

(1) $x = 0 \rightarrow E_z = 0$ (Sub in (1))

$0 = (C_1 + 0)(C_3 \cos By + C_4 \sin By)$

$\boxed{C_1 = 0}$

$$\boxed{E_z = C_2 \sin Ax\,(C_3 \cos By) + C_4 \sin By)\, e^{-\gamma z}} \rightarrow (2)$$

(2) $x = a \rightarrow E_z = 0$

$0 = C_2 \sin Aa\,(C_3 \cos By + C_4 \sin By)$

$\sin Aa = 0$

$Aa = m\pi$

$\boxed{A = \frac{m\pi}{a}}$

Apply 2^{nd} Boundary condition

① $y=0 \rightarrow E_z=0$ (sub in ②)

$$0 = C_2 \sin Ax (C_3)$$

$$\boxed{C_3=0}$$

$$\boxed{E_z = C_2 C_4 \sin Ax \sin By\, e^{-\gamma z}}$$

② $y=b \rightarrow E_z=0$

$$0 = C_2 C_4 \sin Ax \sin Bb$$

$$\sin Bb = 0$$

$$Bb = n\pi$$

$$\boxed{B = \frac{n\pi}{b}}$$

$$E_z = (\underbrace{C_2 C_4}_{}\sin Ax \sin By) e^{-\gamma z}$$

$$\boxed{\begin{aligned} E_z &= (C \sin Ax \sin By)\, e^{-\gamma z} \\ E_z &= C \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \end{aligned}}\ e^{-\gamma z}$$

To find: E_x, E_y, H_x, H_y

$$E_x = \frac{-\gamma}{h^2}\left(\frac{\partial E_z}{\partial x}\right) = \frac{-\gamma}{h^2}\frac{\partial}{\partial x}(C \sin Ax \sin By) e^{-\gamma z}$$

$$= \frac{-\gamma}{h^2} C \cdot A \cos Ax \sin By\, e^{-\gamma z}$$

$$E_y = \frac{-\gamma}{h^2}\left(\frac{\partial E_z}{\partial y}\right) = \frac{-\gamma}{h^2}\frac{\partial}{\partial y}(C \sin Ax \sin By) e^{-\gamma z}$$

$$= \frac{-\gamma}{h^2} C \cdot B \sin Ax \cos By\, e^{-\gamma z}$$

$$H_x = \frac{j\omega\varepsilon}{h^2}\left(\frac{\partial E_z}{\partial y}\right) = \frac{j\omega\varepsilon}{h^2}\frac{\partial}{\partial y}(C \sin Ax \sin By) e^{-\gamma z}$$

$$= \frac{j\omega\varepsilon}{h^2} \cdot C \cdot B \sin Ax \cos By\, e^{-\gamma z}$$

$$H_y = \frac{-j\omega\varepsilon}{h^2}\left(\frac{\partial E_z}{\partial x}\right) = \frac{-j\omega\varepsilon}{h^2}\frac{\partial}{\partial x}(C \sin Ax \sin By) e^{-\gamma z}$$

$$= \frac{-j\omega\varepsilon}{h^2} C \cdot A \cos Ax \cdot \sin By\, e^{-\gamma z}$$

Now Sub $\boxed{A = \frac{m\pi}{a} \quad \& \quad B = \frac{n\pi}{b}}$

and $\gamma = \alpha + j\beta \quad (\alpha = 0)$

$\boxed{\gamma = j\beta}$

$\therefore$ Field component of TM waves in Rectangular waveguide propagated in z-direction,

$$E_x = \frac{-j\beta}{h^2} C \left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-j\beta z}$$

$$E_y = \frac{-j\beta}{h^2} C \left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-j\beta z}$$

$$E_z = C \sin\left(\frac{m\pi}{a}x\right) \cdot \sin\left(\frac{n\pi}{b}y\right) \cdot e^{-j\beta z}$$

$$H_x = \frac{j\omega\varepsilon}{h^2} C \left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a}x\right) \cdot \cos\left(\frac{n\pi}{b}y\right) e^{-j\beta z}$$

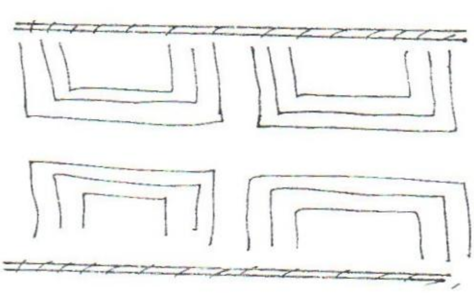
$$H_y = \frac{-j\omega\varepsilon}{h^2} C \left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-j\beta z}$$

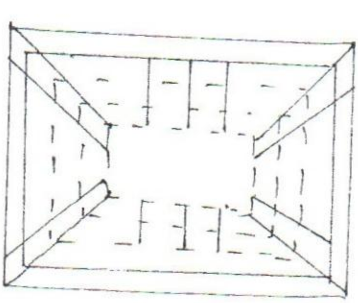
$$H_z = 0$$

where $h^2 = A^2 + B^2$

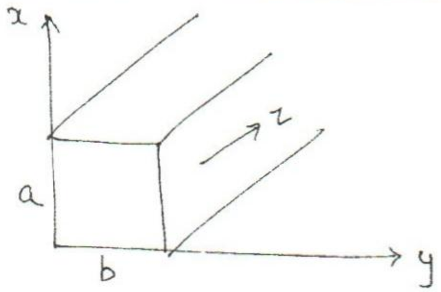
$h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2, (m, n = 1)$

TM modes:





(TE): Transverse Elec. Wave in Rectangular Waveguide



Boundary condn:

(I) $x=0, x=a$

(II) $y=0, y=b$

TE condition

$E_z=0$ and $H_z \neq 0$

Maxwell eqn:

$$E_x = \frac{-\gamma}{h^2}\left(\cancel{\frac{\partial E_z}{\partial x}}\right) - \frac{j\omega\mu}{h^2}\left(\frac{\partial H_z}{\partial y}\right) \qquad E_y = \frac{-\gamma}{h^2}\left(\cancel{\frac{\partial E_z}{\partial y}}\right) + \frac{j\omega\mu}{h^2}\left(\frac{\partial H_z}{\partial x}\right)$$

$$H_x = \frac{-\gamma}{h^2}\left(\frac{\partial H_z}{\partial x}\right) + \frac{j\omega\varepsilon}{h^2}\left(\cancel{\frac{\partial E_z}{\partial y}}\right) \qquad H_y = \frac{-\gamma}{h^2}\left(\frac{\partial H_z}{\partial y}\right) - \frac{j\omega\varepsilon}{h^2}\left(\cancel{\frac{\partial E_z}{\partial x}}\right)$$

All above eqn interms of 'H_z'.

1) To find H_z:

$H_z = XY$ $\qquad X = C_1\cos Ax + C_2\sin Ax$

$\qquad\qquad\qquad Y = C_3\cos By + C_4\sin By$

$$\boxed{H_z = (C_1\cos Ax + C_2\sin Ax)(C_3\cos By + C_4\sin By)e^{-\gamma z}} \rightarrow (1)$$

C_1, C_2, C_3, C_4 - arbitary const.

Directly Boundary condn cannot applicable to H_z, H_x, H_y.
($\because H_{tan} \neq 0$)

So apply Boundary condn to E_x & E_y

1st Boundary condn

(1) $x=0$, $\rightarrow E_y=0$

$$E_y = \frac{j\omega\mu}{h^2}\left(\frac{\partial H_z}{\partial x}\right)$$

2

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial}{\partial x}\left(C_1\cos Ax + C_2\sin Ax\right)\left(C_3\cos By + C_4\sin By\right)$$

$$E_y = \frac{j\omega\mu}{h^2}\left(-C_1 A\sin Ax + C_2 A\cos Ax\right)\left(C_3\cos By + C_4\sin By\right)$$

① $x=0 \rightarrow E_y=0$

$$0 = \frac{j\omega\mu}{h^2}(C_2 A)(C_3\cos By + C_4\sin By)$$

$$\boxed{C_2=0}$$

$$\boxed{E_y = \frac{j\omega\mu}{h^2}(-C_1 A\sin Ax)(C_3\cos By + C_4\sin By)e^{-\gamma z}}$$

② $x=a, \rightarrow E_y=0$

$$0 = \frac{j\omega\mu}{h^2}(-C_1 A\sin Aa)(C_3\cos By + C_4\sin By)$$

$\sin Aa = 0$

$Aa = m\pi$

$$\boxed{A = \frac{m\pi}{a}}$$

Apply 2nd Boundary condn:.

$$E_x = -\frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$E_x = -\frac{j\omega\mu}{h^2}\frac{\partial}{\partial y}\left(C_1\cos Ax + C_2\sin Ax\right)\left(C_3\cos By + C_4\sin By\right)$$

$$E_x = \frac{-j\omega\mu}{h^2}C_1\cos Ax\left(-C_3 B\sin By + C_4 B\cos By\right)$$

① $y=0 \rightarrow E_x=0$

$$0 = -\frac{j\omega\mu}{h^2} C_1 \cos Ax \, (C_4 B)$$

$\boxed{C_4 = 0}$ $\boxed{E_x = -\frac{j\omega\mu}{h^2} C_1 \cos Ax \, (-C_3 B \sin By)}$

② $y = b \rightarrow E_x = 0$

$$0 = -\frac{j\omega\mu}{h^2} C_1 \cos Ax (-C_3 B \sin Bb)$$

$$\sin Bb = 0$$

$$Bb = n\pi$$

$$\boxed{B = \frac{n\pi}{b}}$$

Sub all the results in H_z

Sub $C_2 = 0$, $C_4 = 0$, $A = \frac{m\pi}{a}$ & $B = \frac{n\pi}{b}$ in eqn ①

$$\therefore H_z = C_1 C_3 \cos Ax \cos By \, e^{-\gamma z}$$

$$H_z = C \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-\gamma z}$$

<u>To find E_x, E_y, H_x, H_y:</u>

$$E_x = -\frac{j\omega\mu}{h^2}\left(\frac{\partial H_z}{\partial y}\right)$$

$$= -\frac{j\omega\mu}{h^2} \frac{\partial}{\partial y}\left(C \cos Ax \cos By\right) e^{-\gamma z}$$

$$= -\frac{j\omega\mu}{h^2} \cdot C \cos Ax \, (-B \sin By) e^{-\gamma z}$$

$$E_x = \frac{j\omega\mu}{h^2} C B \cos Ax \sin By \, e^{-\gamma z}$$

$$E_y = \frac{j\omega\mu}{h^2}\left(\frac{\partial H_z}{\partial x}\right)$$

$$= \frac{j\omega\mu}{h^2}\left(C \cos Ax \cos By\right) e^{-\gamma z}$$

$$= \frac{j\omega\mu}{h^2} \cdot C - A \sin Ax \cos By \, e^{-\gamma z}$$

$$E_y = -\frac{j\omega\mu}{h^2} C \cdot A \sin Ax \cos By \, e^{-\gamma z}$$

$$H_x = \frac{-\gamma}{h^2}\left(\frac{\partial H_z}{\partial x}\right) = \frac{-\gamma}{h^2}\frac{\partial}{\partial x}\left(C \cos Ax \cos By\right) e^{-\gamma z}$$

$$= \frac{-\gamma}{h^2}\left(C \cdot -A \sin Ax \cos By\right) e^{-\gamma z}$$

$$H_x = \frac{\gamma}{h^2} C A \sin Ax \cos By \, e^{-\gamma z}.$$

$$H_y = \frac{-\gamma}{h^2}\left(\frac{\partial H_z}{\partial y}\right) = \frac{-\gamma}{h^2}\frac{\partial}{\partial y}\left(C \cos Ax \cos By\right) e^{-\gamma z}$$

$$= \frac{-\gamma}{h^2}\left(C \cdot \cos Ax \cdot -B \sin By\right) e^{-\gamma z}$$

$$H_x = \frac{\gamma}{h^2} C \cdot B \cos Ax \sin By \, e^{-\gamma z}$$

∴ Field of TE waves are Sub $A = \frac{m\pi}{a}$, $B = \frac{n\pi}{b}$, $\gamma = j\beta$

$$\boxed{\begin{aligned}
E_x &= \frac{j\omega\mu}{h^2} C \cdot \left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \, e^{-j\beta z} \\
E_y &= -\frac{j\omega\mu}{h^2} C \left(\frac{m\pi}{a}\right) \sin\left(\frac{n\pi}{b}\right)x \cos\left(\frac{n\pi}{b}\right)y \, e^{-j\beta z} \\
E_z &= 0 \\
H_x &= \frac{j\beta}{h^2} C \left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \, e^{-j\beta z} \\
H_y &= \frac{j\beta}{h^2} C \left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \, e^{-j\beta z} \\
H_z &= C \cos\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \, e^{-j\beta z}.
\end{aligned}}$$

$$h^2 = A^2 + B^2$$

$$h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

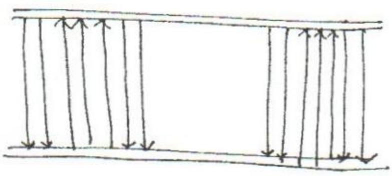
Conclusion:

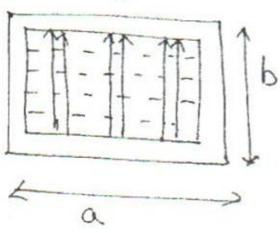
Case (i) when $m = n = 0$, E & H field components Vanish. Hence TE_{00} & TM_{00} → does not exist.

Case (ii) $m = 1, n = 0$, $E_x = H_y = 0$, E_y & H_x is present. Hence TE_{10} is possible → Dominant Mode.

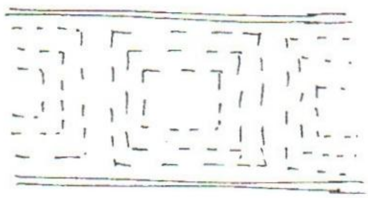
Case (iii) $m = 0, n = 1$, $E_y = H_x = 0$, E_x, H_y is present. Hence TE_{01} is possible → Dominant Mode

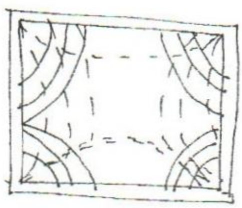
TE_{10} Mode:





TE_{11} Mode:





Cut of freq:

$$f_c = \frac{1}{2\pi} \cdot \frac{h}{\sqrt{\mu\epsilon}}$$

$$h^2 = A^2 + B^2$$

$$h = \sqrt{A^2 + B^2}$$

$$= \frac{1}{2\pi\sqrt{\mu\epsilon}}\left[\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}\right]$$

$$\Rightarrow \boxed{f_c = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

CIRCULAR WAVEGUIDE
(or) cylindrical

The waveguides with circular cross-section are called cylindrical/circular waveguides.

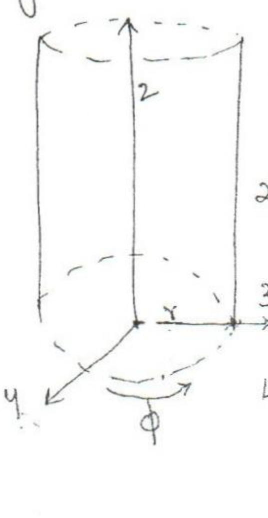
Practically rectangular waveguides are widely used. Circular waveguides are used as

* Isolator
* Circulators
* Circular Cavity Resonators

<u>Drawbacks of circular Waveguide:</u>

1) Polarization of Electromagnetic Energy is Not constant throughout the waveguide. Hence losses are more
2) Size of circular waveguide is very big compared to Rect. waveguide operating at same frequency.
3) Difference between cut-off frequency of dominant & successive higher order modes are high.

<u>Maxwell eqns:- Soln</u>



$$1)\ E_r = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial r} - \frac{j\omega\mu}{r h^2}\frac{\partial H_z}{\partial \phi}$$

$$2)\ E_\phi = \frac{-\gamma}{r \cdot h^2}\frac{\partial E_z}{\partial \phi} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial r}$$

$$3)\ H_r = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial r} + \frac{j\omega\varepsilon}{r \cdot h^2}\frac{\partial E_z}{\partial \phi}$$

$$4)\ H_\phi = \frac{-\gamma}{r h^2}\frac{\partial H_z}{\partial \phi} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial r}$$

(Y)

Wave Equation:-

$$\frac{\partial^2 H}{\partial r^2} + \frac{1}{r}\frac{\partial H}{\partial r} + \frac{1}{r^2}\frac{\partial^2 H}{\partial \phi^2} + h^2 H = 0$$

$$\frac{\partial^2 E}{\partial r^2} + \frac{1}{r}\frac{\partial E}{\partial r} + \frac{1}{r^2}\frac{\partial^2 E}{\partial \phi^2} + h^2 E = 0$$

Transverse Magnetic (TM) Waves in Cylindrical Waveguides

TM Condition:- $H_z = 0$, $E_z \neq 0$

To find E_z:—

Consider wave equation

$$\frac{\partial^2 E}{\partial r^2} + \frac{1}{r}\frac{\partial E}{\partial r} + \frac{1}{r^2}\frac{\partial^2 E}{\partial \phi^2} + h^2 E = 0$$

By variable separable Method,

$$\boxed{E_z = X(r)\cdot \Psi(\phi) = X\Psi}$$

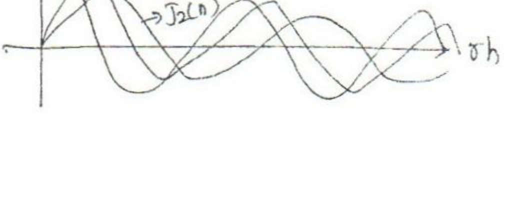
$$\frac{\partial^2 X\Psi}{\partial r^2} + \frac{1}{r}\frac{\partial (X\Psi)}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial^2 (X\Psi)}{\partial \phi^2} + h^2 (X\Psi) = 0$$

The Soln is

$$\boxed{\Psi = A_n \cos n\phi + B_n \sin n\phi}$$

The Standard Bessel differential Equation is,

$$\boxed{X = C_n J_n(rh)}$$



J_n - Bessel function of n-order.

Sub x & y in E_z

$$\boxed{E_z = C_n J_n(rh)\left(A_n \cos n\phi + B_n \sin n\phi\right)} e^{-\gamma z}$$

C_n = constant.

where $A_n \cos n\phi + B_n \sin n\phi = \sqrt{A_n^2 + B_n^2} \cdot \cos\left[\underbrace{n\phi + \tan^{-1}\left(\frac{A_n}{B_n}\right)}_{\Downarrow}\right]$

(By trigonometric identity)

$$= \sqrt{A_n^2 + B_n^2} \cdot \cos n\phi$$

$$E_z = \left[\underbrace{C_n J_n(rh) \cdot \sqrt{A_n^2 + B_n^2}}_{C_0} \cdot \cos n\phi\right] e^{-\gamma z}$$

$$\boxed{E_z = C_0 J_n(rh) \cos n\phi} e^{-\gamma z}$$ [where $C_0 = C_n \sqrt{A_n^2 + B_n^2}$]

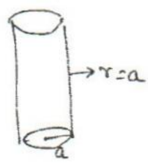
Boundary Conditions:-

$r = a$, $\phi = 0$ to 2π $\longrightarrow$ $E_z = 0$ and $E_\phi = 0$

$0 = C_0 J_n(ah) \cos n\phi$

$$\boxed{J_n(ah) = 0}$$

a = Inner radius of circular waveguide.



* $J_n(ah)$ is oscillatory function.
* There are infinite no. of roots for $J_n(ah)$.
* The values of these roots for $J_n(ah)$ = zero are called Eigen values. These roots are denoted by 'X_{nm}'.

$\therefore J_n(X_{nm}) = 0$

$$\boxed{X_{nm} = ah}.$$

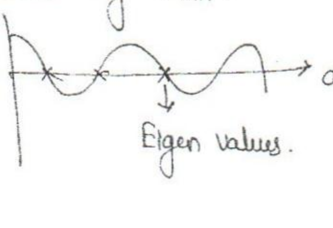


Table of various Values for X_{nm}.

n\m	1	2	3
0	$\boxed{2.405}$	5.52	8.645
1	3.832	7.106	10.173
2	5.136	8.416	11.62
3	6.38	9.716	13.015
4	7.588	11.065	14.372

dominant Mode.

$$\boxed{TM_{01} = TM_{nm}}$$

n = order of Bessel function

m = roots of $J_n(ah) = 0$

Dominant Mode of TM Mode:-

* The Mode for which the max value of cutoff wavelength (or) min value of cutoff freq is called Dominant Mode.
* If f_c low

$\downarrow$

X_{nm} low

ie) lowest value = 2.405

$\downarrow$

$\therefore$ TM waves of TM_{01} mode is called as Dominant Mode in Circular waveguide.

To find E_r, E_ϕ, H_r and H_ϕ:-

$$\left.\begin{aligned}
E_r &= \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial r} - \frac{j\omega\mu}{rh^2}\cancel{\frac{\partial H_z}{\partial \phi}} \\
E_\phi &= \frac{-\gamma}{rh^2}\frac{\partial E_z}{\partial \phi} + \frac{j\omega\mu}{h^2}\cancel{\frac{\partial H_z}{\partial r}} \\
H_r &= \frac{-\gamma}{h^2}\cancel{\frac{\partial H_z}{\partial r}} + \frac{j\omega\varepsilon}{rh^2}\frac{\partial E_z}{\partial \phi} \\
H_\phi &= \frac{-\gamma}{rh^2}\cancel{\frac{\partial H_z}{\partial \phi}} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial r}
\end{aligned}\right\}$$

Sub $H_z = \phi$ (TM condition)

To find E_r:

$$E_r = \frac{-v}{h^2}\frac{\partial E_z}{\partial r}$$

$$E_r = \frac{-v}{h^2}\frac{\partial}{\partial r}\left(C_0 J_n(rh)\cos n\phi\right)$$

$$= \frac{-v}{\cancel{h^2}} \cdot C_0 \cdot \cancel{h} \cdot J_n'(rh)\cos n\phi$$

$$\boxed{E_r = \frac{-v}{h} C_0 J_n'(rh)\cos n\phi}$$

To find E_ϕ:

$$E_\phi = \frac{-v}{r \cdot h^2}\frac{\partial E_z}{\partial \phi}$$

$$E_\phi = \frac{-v}{r \cdot h^2}\frac{\partial}{\partial \phi}\left(C_0 J_n(rh)\cos n\phi\right)$$

$$= \frac{-v}{r \cdot h^2} C_0 J_n(rh)(-n\sin n\phi)$$

$$\boxed{E_\phi = \frac{v}{rh^2} C_0 \cdot n J_n(rh)\sin n\phi}$$

To find H_r:-

$$H_r = -\frac{j\omega\varepsilon}{rh^2}\frac{\partial E_z}{\partial \phi}$$

$$= -\frac{j\omega\varepsilon}{rh^2}\frac{\partial}{\partial \phi}\left(C_0 J_n(rh)\cos n\phi\right)$$

$$= \frac{-j\omega\varepsilon}{rh^2} C_0 J_n(rh) \cdot n(-\sin n\phi)$$

$$\boxed{H_r = \frac{+j\omega\varepsilon}{rh^2} C_0 \cdot n \cdot J_n(rh)\sin n\phi}$$

To find H_ϕ:

$$H_\phi = -\frac{j\omega\varepsilon}{h^2}\frac{\partial}{\partial r}\left(C_0 J_n(rh)\cos n\phi\right)$$

$$= \frac{j\omega\varepsilon}{\cancel{h^2}} \cdot C_0 \cdot \cancel{h}\, J_n(rh)\cos n\phi$$

$$\boxed{H_\phi = \frac{j\omega\varepsilon}{h^2} C_0 J_n(rh)\cos n\phi}$$

Conclusion:-

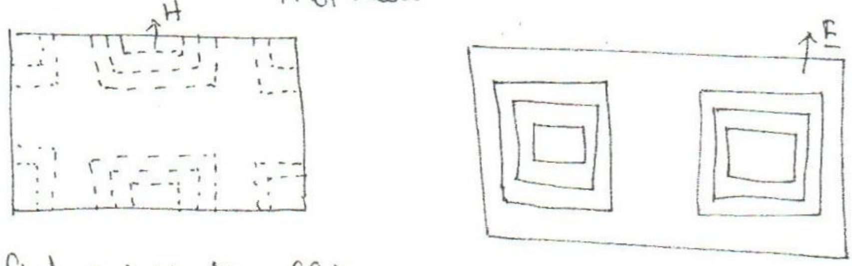
Circular waveguide TM wave field components are,

$$E_r = \frac{-v}{h} C_0 J_n'(rh)\cos n\phi \qquad H_r = \frac{j\omega\varepsilon}{rh^2} C_0 n J_n(rh)\sin n\phi$$

$$E_\phi = \frac{v}{rh^2} C_0 n J_n(rh)\sin n\phi \qquad H_\phi = \frac{j\omega\varepsilon}{h^2} C_0 J_n(rh)\cos n\phi$$

$$E_z = C_0 J_n(rh)\cos n\phi \qquad H_z = 0$$

TM01 Mode



To find cutoff freq (fc):-

$$h^2 = \gamma^2 + \omega^2 \mu \varepsilon$$

$$\boxed{\gamma = \sqrt{h^2 - \omega^2 \mu \varepsilon}}$$

at $\omega = \omega_c \to \gamma = 0$

$$0 = \sqrt{h^2 - \omega_c^2 \mu \varepsilon}$$

$$h^2 - \omega_c^2 \mu \varepsilon = 0$$

$$\omega_c^2 \mu \varepsilon = h^2$$

W.K.T $ha = X_{nm}$

$$\left[h = \frac{X_{nm}}{a} \right]$$

$$\omega_c^2 \mu \varepsilon = \left(\frac{X_{nm}}{a} \right)^2$$

$$\omega_c = \frac{1}{\sqrt{\mu \varepsilon}} \left(\frac{X_{nm}}{a} \right)$$

$$\boxed{f_c = \frac{1}{2\pi \sqrt{\mu \varepsilon}} \left(\frac{X_{nm}}{a} \right)}$$

(or)

$$\boxed{f_c = \frac{c}{2\pi} \left(\frac{X_{nm}}{a} \right)}$$

To find cutoff wavelength:-

$$\lambda_c = \frac{c}{f_c} = \frac{\cancel{c}}{\frac{\cancel{c}}{2\pi} \left(\frac{X_{nm}}{a} \right)}$$

$$\boxed{\lambda_c = \frac{2\pi a}{X_{nm}}}$$

$$\left(\lambda_c = \frac{2\pi a}{2.405} \right) \quad X_{nm} = 2.405$$

(8) Transverse Electric Wave in Cylindrical Waveguide (TE)

TE Condition:

$$E_z = 0, \; H_z \neq 0$$

To find H_z:

Wave eqn $\quad \frac{\partial^2 H_z}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial H_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 H_z}{\partial \phi^2} + h^2 H_z = 0$

$$\boxed{H_z = XY = X(r) \cdot Y(\phi)}$$

$$\frac{\partial^2 (XY)}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial (XY)}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2}(XY) + h^2 (XY) = 0$$

Soln is $\quad \boxed{Y = A_n \cos n\phi + B_n \sin n\phi}$

The Standard Bessel fn eqn is

$$\boxed{X = C_n J_n(rh)} \qquad J_n(rh) \text{ - Bessel fn of n-order.}$$

$$\boxed{H_z = XY = C_n J_n(rh) (A_n \cos n\phi + B_n \sin n\phi)}$$

$$A_n \cos n\phi + B_n \sin n\phi = \sqrt{A_n^2 + B_n^2} \cdot \cos \underbrace{\left[n\phi + \tan^{-1}\left(\frac{A_n}{B_n}\right)\right.}$$

$$= \sqrt{A_n^2 + B_n^2} \cos n\phi$$

$$H_z = \underbrace{C_n J_n(rh) \cdot \sqrt{A_n^2 + B_n^2}}_{C_0'} \cos n\phi$$

$$\left(C_0' = C_n \cdot \sqrt{A_n^2 + B_n^2} \right)$$

$$\boxed{H_z = C_0' J_n(rh) \cdot \cos n\phi}$$

Boundary condition:

$r = a$, $\phi = 0$ to $2\pi \rightarrow E_z = 0$ and $E_\phi = 0$

So boundary condn not directly applicable to 'H_z'.

To find E_r, E_ϕ, H_r and H_ϕ:

$$E_r = \frac{-\gamma}{h^2}\cancel{\frac{\partial E_z}{\partial r}} - \frac{j\omega\mu}{rh^2}\frac{\partial H_z}{\partial \phi}$$

$$E_\phi = \frac{-\gamma}{rh^2}\cancel{\frac{\partial E_z}{\partial \phi}} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial r}$$

$$H_r = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial \phi} + \frac{j\omega\varepsilon}{rh^2}\cancel{\frac{\partial E_z}{\partial \phi}}$$

$$H_\phi = \frac{-\gamma}{rh^2}\frac{\partial H_z}{\partial \phi} + \frac{j\omega\varepsilon}{h^2}\cancel{\frac{\partial E_z}{\partial r}}.$$

$(E_z = 0)$

To find E_r:

$$E_r = \frac{-j\omega\mu}{rh^2}\left(\frac{\partial H_z}{\partial \phi}\right)$$

$$= \frac{-j\omega\mu}{rh^2}\frac{\partial}{\partial \phi}(C_0' J_n(rh)\cos n\phi)$$

$$= \frac{-j\omega\mu}{rh^2}\cdot C_0'\cdot J_n(rh)\cdot -n\cos n\phi$$

$$\boxed{E_r = \frac{+j\omega\mu n}{rh^2}C_0' J_n(rh)\cos n\phi}$$

To find E_ϕ:

$$E_\phi = \frac{j\omega\mu}{h^2}\frac{\partial}{\partial r}(C_0' J_n(rh)\cos n\phi)$$

$$= \frac{j\omega\mu}{h^2}\cdot C_0' J_n(rh)\cdot h\cos n\phi$$

$$\boxed{E_\phi = \frac{j\omega\mu}{h}C_0' J_n'(rh)\cos n\phi}$$

To find H_r:

$$H_r = \frac{-v}{h^2} \frac{\partial H_z}{\partial r}$$

$$= \frac{-v}{h^2} \frac{\partial}{\partial r}\left(C_0' J_n(rh) \cos n\phi \right)$$

$$= \frac{-v}{h^2} \cdot C_0' J_n'(rh) . \cancel{h} \cos n\phi$$

$$\boxed{H_r = \frac{-v}{h^2} C_0' J_n'(rh) \cos n\phi}$$

To find H_ϕ:

$$H_\phi = \frac{-v}{rh^2} \frac{\partial H_z}{\partial \phi}$$

$$= \frac{-v}{rh^2} \frac{\partial}{\partial \phi}\left(C_0' J_n(rh) \cos n\phi \right)$$

$$= \frac{-v}{rh^2} \cdot C_0' J_n(rh) (-n \sin n\phi)$$

$$\boxed{H_\phi = \frac{v}{rh^2} n \, C_0' J_n(rh) \sin n\phi}$$

Apply Boundary condn:

At the surface $r=a \rightarrow E_\phi = 0$ (tangential component of Elec. field at surface of waveguide).

$$E_\phi = \frac{j\omega\mu}{h} C_0' J_n'(ah) \cos n\phi$$

$$0 = \frac{j\omega\mu}{h} C_0' J_n'(ah) \cos n\phi$$

$\boxed{J_n'(ah) = 0}$ The values of these roots for which $J_n'(ah) = 0$ are called Eigen values which is denoted by X'_{nm}.

$$X'_{nm} = 0 \Rightarrow J_n'(X'_{nm}) = 0$$

$$ah = X'_{nm}$$

$$\boxed{h = \frac{X'_{nm}}{a}}$$

Table for X'_{nm} for various values of n, m :

n|m	1	2	3
0	3.832	7.016	10.173
1	$\boxed{1.841}$	5.331	8.536
2	3.054	3.706	9.49
3	4.201	8.015	11.340
4	5.317	9.282	12.652

Dominant Mode for TE waves is TE_{11}. Since among various nm values X'_{11} is the lowest value.

To find cutoff freq (f_c):

$$h^2 = \gamma^2 + \omega^2 \mu \varepsilon$$

$\gamma = 0$ at $\omega = \omega_c$

$$h^2 = \omega_c^2 \mu \varepsilon$$

$$\boxed{\omega_c = \frac{h}{\sqrt{\mu\varepsilon}}}$$

w.k.t $h = \frac{X'_{nm}}{a}$

$$\omega_c = \frac{X'_{nm}}{a\sqrt{\mu\varepsilon}}$$

$$\boxed{f_c = \frac{1}{2\pi} \frac{X'_{nm}}{a\sqrt{\mu\varepsilon}}}$$

To find cutoff wavelength (λ_c):

$$\lambda_c = \frac{c}{f_c} = \frac{1/\sqrt{\mu\varepsilon}}{\frac{1}{2\pi} \cdot \frac{X'_{nm}}{a\sqrt{\mu\varepsilon}}}$$

$$\boxed{\lambda_c = \frac{2\pi a}{X'_{nm}}}$$

λ_c at dominant Mode

$$\lambda_c = \frac{2\pi a}{1.841}$$

Cavity Resonators

It is a metallic enclosure that Confines the EH Energy. They are formed by shorting the two ends of the Section of a waveguide. The EH energy inside the cavity determine Equivalent Inductance & Capacitance.

The Energy dissipated by finite conductivity of the wall determine its Equivalent Resistance. The Resonator has Infinite no. of Resonant Modes and each mode has its own Resonant frequency. The Mode having Lowest Resonant freq is called Dominant Mode.

Types of Resonators in Microwave

1) Rectangular cavity Resonators
2) Circular /cylindrical cavity Resonators
3) Semi - cylindrical cavity Resonators

Cavity Resonators:-(definition)

2M

Cavity Resonators are formed by placing perfect Conducting Sheets on the rectangular/circular waveguide on the two end of the sections and all the Sides are Surrounded by conducting walls. Thus forms cavity Resonators

Resonator (dyn)

↳ Electromagnetic Energy confined within a Mettalic Enclosure they act as an Resonators.

↳ It is Storing component made up of `LC' components.

Lumped Resonators → used for Low freq. generators

Distributed Resonators → used for Microwave freq Generators

↳ Both txion Lines & Waveguides built the Microwave Cavity Resonators.

Applications :- (X) 2m

1) Reflex Klystron Amplifier (Microwave Amplifier)
2) Klystron oscillator (Microwave Generators)
3) Radar Applications
4) Light House (UHF freq Generation)

} Rect. cavity Resonator

5) Microwave freq Meter } → Circular cavity Resonator.

Resonators performance parameters:

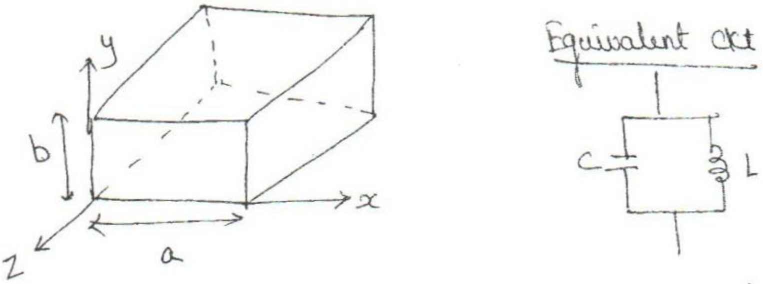
① Resonant frequency (f_r) → The freq at which Resonator attains max Energy ie) Twice of Electric / Magnetic Energy.

② Quality factor (Q) → Measure of Frequency Selectivity of Resonator

$$Q = 2\pi \times \frac{\text{Max. Energy Stored}}{\text{Energy dissipated / cycle}} = \frac{f_0}{\text{Bandwidth}}.$$

(iii) Input Impedance → Specifying matching with i/p (or) o/p circuits.

Rectangular Cavity Resonators :



Rect. cavity Resonators are Short ckted at both ends to avoid radiation losses from open end of waveguide. Due to Short ckted ends, Cavity (or) closed box formed.

Resonant freq of Rect. Cavity Resonator (f_r).

$$h^2 = \gamma^2 + \omega^2 \mu \varepsilon \qquad h^2 = A^2 + B^2$$

$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \gamma^2 + \omega^2 \mu \varepsilon \qquad A = \frac{m\pi}{a} \ \& \ B = \frac{n\pi}{b} \text{ (for Rect. W.G)}$$

If wave propagated $\gamma = \alpha + j\beta$ $(\alpha = 0)$

$\gamma = j\beta$

at $\omega = \omega_r$

$$\omega_r^2 \mu \varepsilon + \gamma^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\omega_r^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \gamma^2$$

$$\omega_r^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - (j\beta)^2$$

$$\omega_r^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \beta^2$$

Sub $\beta = \frac{p\pi}{d}$ where p = No. of half wavelength along Z-direction

$p = 1, 2, 3 \ldots$

$$\omega_r^2 = \frac{1}{\mu\varepsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]$$

$$2\pi f_r = \frac{1}{\sqrt{\mu\varepsilon}}\left[\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2}\right]$$

$$f_r = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$$

$$f_r = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$$

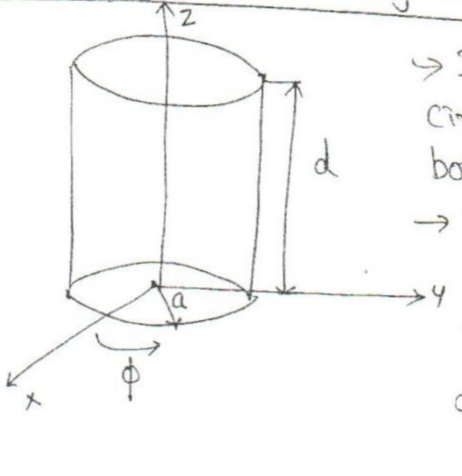
Velocity of light $V = \frac{1}{\sqrt{\mu\varepsilon}}$

$$\boxed{f_r = \frac{V}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}}$$

dominant Mode of the Resonator depends on dimensions of the cavity $b < a < d$ (or) $d > a > b$

$\therefore$ Dominant Mode = TE_{101}

Cylindrical (or) Circular Cavity Resonators:



→ It is constructed from Circular waveguide by Shorting both the ends.

→ Mechanical tuning of the Resonant freq is done with the help of movable top wall.

a = radius of the cylinder.

To find Resonant freq (f_r):-

$$h^2 = \gamma^2 + \omega^2 \mu \varepsilon$$

$$h = \frac{X_{nm}}{a} \quad \text{(for circular)}$$

$$\left(\frac{X_{nm}}{a}\right)^2 = \gamma^2 + \omega^2 \mu \varepsilon$$

$\gamma = j\beta$ at $\omega = \omega_r$

$$\omega_r^2 \mu \varepsilon = \left(\frac{X_{nm}}{a}\right)^2 - \gamma^2$$

$$-\gamma^2 = (-j\beta)^2 = \beta$$

$$\omega_r^2 \mu \varepsilon = \left(\frac{X_{nm}}{a}\right)^2 + \beta^2$$

$$\beta = \frac{p\pi}{d}$$

$$\omega_r^2 = \frac{1}{\mu \varepsilon}\left[\left(\frac{X_{nm}}{a}\right)^2 + \left(\frac{p\pi}{a}\right)^2\right]$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{X_{nm}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2}}$$

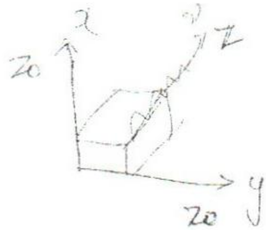
$d > a$

For TE_{nmp}, $f_r = \frac{V}{2\pi}\sqrt{\left(\frac{X'_{nm}}{a}\right)^2 + \left(\frac{p\pi}{a}\right)^2}$

$$V = \frac{1}{\sqrt{\mu\varepsilon}}$$

for TM_{nmp}, $f_r = \frac{V}{2\pi}\sqrt{\left(\frac{X_{nm}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$

Wave Impedance for Rect. Waveguide:-



$$Z_0 = \frac{E}{H}$$

For TE:-

$$Z_{0(TE)} = \frac{E_x}{E_y} = \frac{-\frac{\gamma}{h^2}\cancel{\left(\frac{\partial E_z}{\partial x}\right)} - \frac{j\omega\mu}{h^2}\left(\frac{\partial H_z}{\partial y}\right)}{-\frac{\gamma}{h^2}\left(\frac{\partial H_z}{\partial y}\right) - \cancel{\frac{j\omega\varepsilon}{h^2}\left(\frac{\partial E_z}{\partial x}\right)}}$$

In TE $\rightarrow E_z = 0$

$$Z_{0(TE)} = \frac{\cancel{-}\frac{j\omega\mu}{\cancel{h^2}}\cancel{\left(\frac{\partial H_z}{\partial y}\right)}}{\cancel{-}\frac{\gamma}{\cancel{h^2}}\cancel{\left(\frac{\partial H_z}{\partial y}\right)}} = \frac{j\omega\mu}{\gamma} = \frac{\cancel{j}\omega\mu}{\cancel{j}\beta} = \frac{\omega\mu}{\beta}$$

$$Z_{0(TE)} = \frac{\omega\mu}{\beta}$$

$$= \frac{\cancel{\omega}\mu}{\cancel{\omega}\cdot\sqrt{\mu\varepsilon}\cdot\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$= \sqrt{\frac{\mu}{\varepsilon}}\cdot\frac{1}{\sqrt{1-(f_c/f)^2}}$$

$$\boxed{Z_{0(TE)} = \frac{\eta}{1-(f_c/f)^2}} \quad \left[\eta = \sqrt{\mu/\varepsilon}\,\right]$$

For TM:-
$$Z_{o(TM)} = \frac{E_x}{E_y} = \frac{-\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \cancel{\frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}}}{\cancel{-\frac{\gamma}{h^2}\frac{\partial H_z}{\partial y}} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}}$$

TM condn $\rightarrow (H_z = 0)$

$$Z_{o(TM)} = \frac{\cancel{-}\frac{\gamma}{\cancel{h^2}}\cancel{\frac{\partial E_z}{\partial x}}}{\cancel{-}\frac{j\omega\varepsilon}{\cancel{h^2}}\cancel{\frac{\partial E_z}{\partial x}}} = \frac{\gamma}{j\omega\varepsilon}$$

$$Z_{o(TM)} = \frac{\cancel{j}\beta}{\cancel{j}\omega\varepsilon} = \frac{\cancel{\omega}\sqrt{\mu\varepsilon}\cdot\sqrt{1-(f_c/f)^2}}{\cancel{\omega}\varepsilon}$$

$$Z_{o(TM)} = \sqrt{\mu/\varepsilon}\cdot\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\boxed{Z_{o(TM)} = \eta\cdot\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

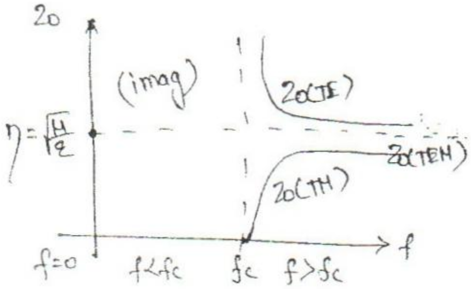
for TEM:-

$$Z_{o(TE)} \times Z_{o(TM)} = \frac{\eta}{\cancel{\sqrt{1-(f_c/f)^2}}} \times \eta\,\cancel{\sqrt{1-(f_c/f)^2}}$$

$$Z_{o(TE)} \times Z_{o(TM)} = \eta^2$$

$$\boxed{Z_{o(TE)} \times Z_{o(TM)} = Z_{o(TEM)}^2}$$

$$\boxed{Z_{o(TEM)} = \eta}$$



PROBLEMS:

1) A cylindrical Cu tube of diameter 3cm is air filled. Calculate cutoff frequency, TE_{01}, TM_{01}, TE_{11}, TM_{11} Modes.

Given data

$d = 3 \times 10^{-2}$ m

$a = \frac{d}{2} = 1.5 \times 10^{-2}$ m (radius)

formula:

TE $f_c = \frac{V}{2\pi}\left(\frac{X'_{nm}}{a}\right)$

TM $f_c = \frac{V}{2\pi}\left(\frac{X_{nm}}{a}\right)$

w.k.T $X_{nm} = ha$

for TM_{nm}

TM_{01}

TM_{11}

n|m	1
0	2.405
1	3.832

for TE_{nm}

$\left.\begin{matrix} TE_{01} \\ TE_{11} \end{matrix}\right\}$

n|m	1
0	3.823
1	1.841

$TM_{01} \rightarrow f_c = \frac{3 \times 10^8}{2 \times \pi}\left(\frac{2.405}{1.5 \times 10^{-2}}\right)$

$\boxed{f_c = 7.65 GHz}$

$TM_{11} \rightarrow f_c = \frac{3 \times 10^8}{2\pi}\left(\frac{3.832}{1.5 \times 10^{-2}}\right)$

$\boxed{f_c = 12.19 GHz}$

TE_{01} $f_c = \frac{3 \times 10^8}{2\pi}\left(\frac{3.823}{1.5 \times 10^{-2}}\right)$

$\boxed{f_c = 12.17 GHz}$

$TM_{11} \rightarrow \boxed{f_c = 5.86 GHz}$

2) A circular waveguide has an internal diameter of 6cm for 9 GHz Signal propagted in it TE_{11} Mode. Calculate cutoff freq, cutoff wavelength, Characteristic impedance & group wavelength / Guided wavelength.

<u>Gin</u>

$d = 6cm$, $a = 3cm$, $f = 9 \times 10^9 Hz$.

$TE_{11} \longrightarrow$ So $\begin{bmatrix} X'_{nm} = 1.841 \\ X'_{11} \end{bmatrix}$

① $f_c = \frac{V}{2\pi}\left(\frac{X'_{nm}}{a}\right)$

$f_c = \frac{3\times10^8}{2\times\pi\times3\times10^{-2}} \times 1.841 = 2.93\,GHz$

② $\lambda_c = \frac{C}{f_c} = \frac{3\times10^8}{2.93\times10^{12}}$

$\lambda_c = 0.1255m$

③ $Z_{O(TE)} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$

$\eta = \sqrt{\mu/\varepsilon}$

$\eta = \sqrt{\frac{4\pi\times10^{-7}}{8.854\times10^{-12}}}$

$\eta = 376.73$

$= \frac{376.73}{\sqrt{1-\left(\frac{2.93\times10^9}{9\times10^9}\right)^2}}$

$Z_{O(TE)} = 390.79\,\Omega$

④ Group wavelength (λg)

$$\lambda g = \frac{\lambda}{\sqrt{1-(fc/f)^2}} \qquad \lambda = \frac{c}{f} = \frac{3\times10^8}{9\times10^9} = 0.033m$$

$$\lambda g = \frac{0.033}{\sqrt{1-\left(\frac{2.93\times10^9}{9\times10^9}\right)^2}}$$

$$\boxed{\lambda g = 0.0342m}$$

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3) For freq of 6GHz, plane of Seperation of 3cm, find Vp, Vg for dominant Mode.

Gin

Dominant Mode in ||el plates = TE_{10}

$f = 6GHz$, $a = 3cm$

$a = 0.03m$

$$Vg = v\cdot\sqrt{1-(fc/f)^2}$$

$$Vp = c\cdot\sqrt{1-(fc/f)^2}$$

$$fc = \frac{mv}{2a} = \frac{1\times3\times10^8}{2\times0.03} \qquad TE_{mn} = TE_{10}$$

$$\boxed{fc = 5GHz}$$

$$Vg = 3\times10^8 \times \sqrt{1-\left(\frac{5\times10^9}{6\times10^9}\right)}$$

$$\boxed{Vg = 1.65\times10^8 m/s}$$

$$Vp = 3\times10^8 \Big/ \sqrt{1-\left(\frac{5\times10^9}{6\times10^9}\right)}$$

$$\boxed{Vp = 5.42\times10^8 m/s}$$

4) Calculate cutoff wavelength of TM_{11} mode in std Rect. w.G if $a = 4.5cm$.

<u>Gn</u>

$a = 4.5cm$ <u>In Rect. w.G</u>

$a = 0.045m$ $a >> b$ $b = a/2$

$b = a/2 = 0.0225m$

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$TM_{11} = TM_{mn}$

$$f_c = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{0.045}\right)^2 + \left(\frac{1}{0.0225}\right)^2}$$

$$\boxed{f_c = 7.45 GHz}$$

$$\lambda_c = \frac{c}{f_c} = \frac{3\times10^8}{7.45\times10^9}$$

$$\boxed{\lambda_c = 0.0402m}$$

5) A TEM wave at 1MHz propagates in the region b/n conducting planes which is filled with dielectric material of $\mu_r = 1$, $\varepsilon_r = 2$. Find phase const, char. wave imp.

$$\beta = \omega\sqrt{\mu\varepsilon}\sqrt{1 - \cancel{\left(\frac{f_c}{f}\right)^2}} \qquad f = f_c$$

$$\beta = 2\pi f\sqrt{\mu\varepsilon} = 2\pi\times10^6 \times \sqrt{\mu_0\mu_r\cdot\varepsilon_0\cdot\varepsilon_r}$$

$$= 2\pi\times10^6 \times \sqrt{4\pi\times10^{-7}\times1\times8.854\times10^{-12}\times2}$$

$$\boxed{\beta = 0.0296 rad}.$$

$$Z_{0(TEM)} = \eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu\cdot\mu_r}{\mu\cdot\mu\varepsilon}} = \sqrt{\frac{4\pi\times10^{-7}\times1}{8.85\times10^{-12}\times2}}$$

$$\boxed{Z_{0(TEM)} = 266.39\,\Omega}$$