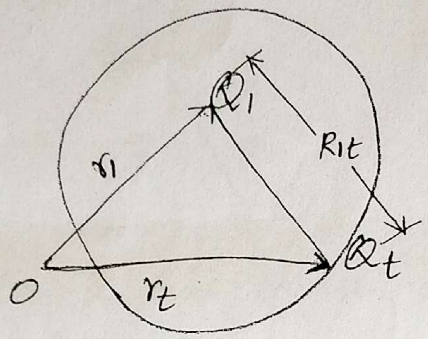
ELECTRIC FIELD INTENSITY (E)

* Electric field intensity (E) exists of field around a charge in which it exerts a force on a test charge.
* This region where a particular charge exerts a force on test charge located in that region.
* Electric field intensity equal to force exerted per unit charge.

$$\vec{E} = \frac{\vec{F}}{Q} \quad N/C \text{ (or) } V/m$$



$$\vec{a_R} = \frac{r_t - r_1}{|r_t - r_1|}$$

$$\vec{E} = \frac{\vec{F}}{Q}$$

$$\vec{F} = \frac{Q_1 Q_t}{4\pi\varepsilon_0 R_{1t}^2}\vec{a_R} \quad \text{[By Coulomb's law]}$$

$$\vec{E} = \frac{\vec{F}}{Q_1} = \frac{Q_t}{4\pi\varepsilon_0 R_{1t}^2}\vec{a_R}$$

$$\vec{E} = \frac{Q_1}{4\pi\varepsilon_0 R_{1t}^2}\frac{r_t - r_1}{|r_t - r_1|}$$

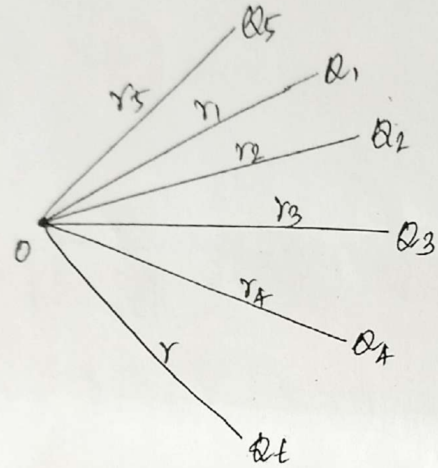
* unit of Electric field intensity N/C or V/m

Superposition principle: (or) Electric Field Intensity due to "N" charges

* If there are more than two point charges then each will be exert force on the test charge, then the net force on the test charge can be obtained by using superposition principle (or) Principle of superposition.

* Consider a point charge Q_t surrounded by number (1) of charges $Q_1, Q_2, Q_3 \ldots Q_n$ that the total force on Q_t is vector sum of all the forces exerted on Q_t due to each of the other point charges.

* Considering force exerted on Q_t due to Q_1 at according to principle of superposition effects of other charges supressed.



$$F_{Q_1 Q_t} = \frac{Q_1 Q_t}{4\pi\epsilon_0 R_{1t}^2} \frac{r_t - r_1}{|r_t - r_1|}$$

$$F_{Q_2 Q_t} = \frac{Q_2 Q_t}{4\pi\epsilon_0 R_{2t}^2} \frac{r_t - r_2}{|r_t - r_2|}$$

$$F_{Q_n} = \frac{Q_n Q_t}{4\pi\epsilon_0 R_{nt}^2} \frac{r_t - r_n}{|r_t - r_n|}$$

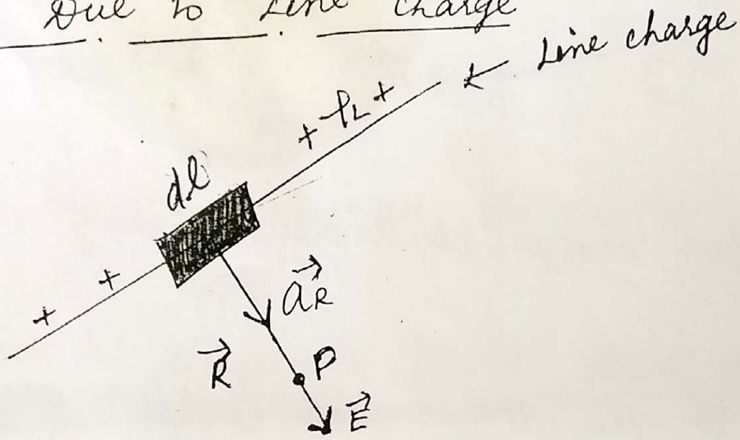
$$F_{total} = F_{Q_1 Q_t} + F_{Q_2 Q_t} + \ldots\ldots + F_{Q_n Q_t}$$

$$\boxed{F = \frac{Q_t}{4\pi\epsilon_0} \sum_{i=1}^{n} \frac{Q_i}{R_{it}^2} \frac{r_t - r_i}{|r_t - r_i|}}$$

Electric Field Intensity Due to Various charge Distributions

* The electric field intensity due to a point charge Q is given by $$\vec{E} = \frac{Q}{4\pi\epsilon_0 R^2}\vec{a}_R$$

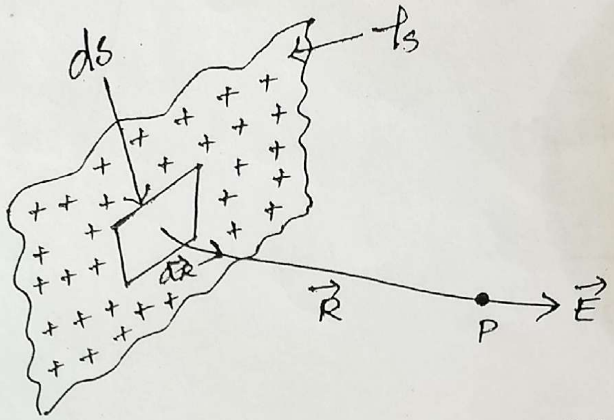
1.) $\vec{E}$ Due to Line charge



* Consider a line charge distribution having a charge density ρ_L.
* The charge dQ on the differential length dl is,
$$dQ = \rho_L dl$$
* Hence the differential electric field $d\vec{E}$ at point P due to dQ is given by
$$d\vec{E} = \frac{dQ}{4\pi\epsilon_0 R^2}\vec{a}_R$$
$$\vec{E} = \int \frac{\rho_L\, dl}{4\pi\epsilon_0 R^2}\vec{a}_R$$
* Hence the total $\vec{E}$ at a point P due to line charge can be obtained by integrating $d\vec{E}$ over the length of the charge.

$$\boxed{\vec{E} = \int_L \frac{\rho_L\, dl}{4\pi\epsilon_0 R^2}\vec{a}_R}$$

2. $\vec{E}$ Due to Surface charge



* Consider a surface charge distribution having a charge density ρ_s.
* The charge dQ on the differential surface area ds is

$$dQ = \rho_s \, ds.$$

* Hence the differential electric field $d\vec{E}$ at a point P due to dQ is given by.

$$d\vec{E} = \frac{dQ}{4\pi\varepsilon_0 R^2}\vec{a}_R$$

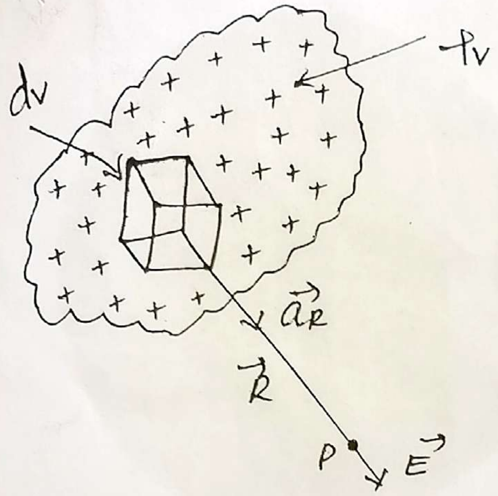
$$= \frac{\rho_s \, ds}{4\pi\varepsilon_0 R^2}\vec{a}_R$$

* The total $\vec{E}$ at a point P is to be obtained by integrating $d\vec{E}$ over the surface area on which charge is distributed.

$$\boxed{\vec{E} = \int_s \frac{\rho_s \, ds}{4\pi\varepsilon_0 R^2}\vec{a}_R}$$

3. $\vec{E}$ Due to Volume charge:

* Consider a volume charge distribution having a charge density ρ_v.



* The charge dQ on the differential volume dv is

$$dQ = \rho_v \, dv$$

* Hence the differential electric field $d\vec{E}$ at a point P due to dQ is given by

$$d\vec{E} = \frac{dQ}{4\pi\varepsilon_0 R^2}\vec{a_R} = \frac{\rho_v dv}{4\pi\varepsilon_0 R^2}\vec{a_R}$$

* The total $\vec{E}$ at a point P is to be obtained by integrating $d\vec{E}$ over the volume

$$\boxed{\vec{E} = \int_{vol} \frac{\rho_v dv}{4\pi\varepsilon_0 R^2}\vec{a_R}}$$

* Thus if there are all possible types of charge distributions, then the total $\vec{E}$ at a point is the vector sum of individual electric field intensities produced by each of the charges at a point under consideration.

$$\boxed{\vec{E}_{total} = \vec{E}_P + \vec{E}_L + \vec{E}_S + \vec{E}_V}$$

* where $\vec{E}_P$, $\vec{E}_L$, $\vec{E}_S$ & $\vec{E}_V$ are the field intensities due to point, line, surface and volume charge distributions respectively.

Electric field due to discrete charges:

[charges are not uniform]

$$E_1 = \frac{F Q_1 Q_t}{Q_1} = \frac{Q_t}{4\pi\varepsilon_0 R_{1t}^2} \frac{r_t - r_1}{|r_t - r_1|}$$

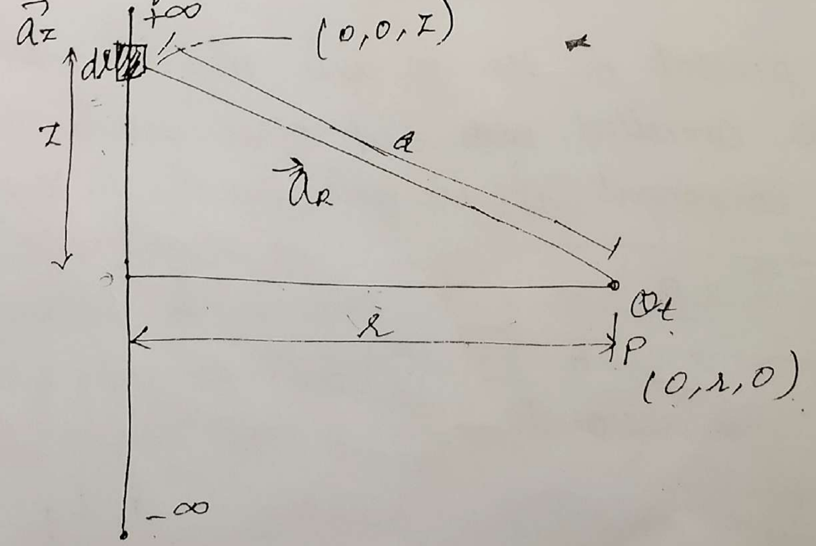
$$E_2 = \frac{Q_t}{4\pi\varepsilon_0 R_{2t}^2} \frac{r_t - r_2}{|r_t - r_2|}$$

$$E_n = \frac{Q_t}{4\pi\varepsilon_0 R_{nt}^2} \frac{r_t - r_n}{|r_t - r_n|}$$

$$E = \frac{Q_t}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{1}{R_{it}^2} \frac{r_t - r_i}{|r_t - r_i|}$$

Electric field due to Infinite Line charge:

* Consider an infinite long straight line carrying uniform line charge having density of ρ_l. Let this line lies in z-axis from $-\infty$ to ∞. Hence it is called infinite line charge.
* Consider a point P is on y-axis at which electric field intensity to be calculated. The distance of the point P from orgin is 'r'.



$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\vec{a_R}$$

charges are line form $Q = \int \rho_l \, dl$

$$\vec{E} = \int \frac{\rho_l}{4\pi\varepsilon_0 R^2}\vec{a_R}\, dl$$

$$\vec{E} = \int_{-\infty}^{\infty} \frac{\rho_l}{4\pi\varepsilon_0 R^2}\vec{a_R}\, dz$$

Note

$$\vec{r_P} = r\vec{a_y}$$

$$\vec{r}_{dl} = z\vec{a_z}$$

$$\vec{a_R} = \frac{\vec{r_P} - \vec{r_{dl}}}{|\vec{r_P} - \vec{r_{dl}}|}$$

$$\vec{E} = \int_{-\infty}^{\infty} \frac{\rho_l}{4\pi\varepsilon_0 R^2}\frac{r\vec{a_y} - z\vec{a_z}}{|r\vec{a_y} - z\vec{a_z}|}\, dz$$

$$\cancel{R = |r_P - r_{dl}|} = \sqrt{r^2 + z^2}$$

$$R^2 = r^2 + z^2.$$

$$\vec{E} = \int_{-\infty}^{\infty} \frac{\rho_l}{4\pi\varepsilon_0 (r^2+z^2)}\frac{r\vec{a_y} - z\vec{a_z}}{(r\vec{a_y} - z\vec{a_z})}\, dz$$

$$\vec{E} = \int_{-\infty}^{\infty} \frac{\rho_l}{4\pi\varepsilon_0 (r^2+z^2)}\frac{r\vec{a_y} - z\vec{a_z}}{\sqrt{r^2+z^2}^{\;1/2}}\, dz$$

$$\boxed{\vec{E} = \int_{-\infty}^{\infty} \frac{\rho_l (r\vec{a_y} - z\vec{a_z})}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}\, dz}$$

* The charges present in the z-axis induced electric field will be cancelled with each other within the axis, so z-component at the axis equal to zero.

$$\therefore \boxed{z\vec{a_z} = 0}$$

$$\boxed{z = r\tan\theta}$$

$dz = r\sec^2\theta \, d\theta$ differentiate.

$$\theta = \tan^{-1}\left(\frac{z}{r}\right)$$

$$z = -\infty \; ; \; \theta = -\pi/2$$

$$z = \infty \; ; \; \theta = \pi/2$$

$$\vec{E} = \int_{-\pi/2}^{\pi/2} \frac{\rho l \, r \vec{a_y} \cdot r \sec^2\theta \, d\theta}{4\pi\varepsilon_0 (r^2 + r^2 \tan^2\theta)^{3/2}}$$

$$= \int_{-\pi/2}^{\pi/2} \frac{\rho l}{4\pi\varepsilon_0} \frac{r^2 \vec{a_y} \sec^2\theta \, d\theta}{(r^2)^{3/2} (1 + \tan^2\theta)^{3/2}}$$

$$= \frac{\rho l}{4\pi\varepsilon_0 r} \int_{-\pi/2}^{\pi/2} \frac{\sec^2\theta \, d\theta}{(\sec^2\theta)^{3/2}} \vec{a_y}$$

$$\frac{\sec^2\theta \, d\theta}{\sec^3\theta} \vec{a_y}$$

$$= \frac{\rho l}{4\pi\varepsilon_0 r} \int_{-\pi/2}^{\pi/2} \frac{1}{\sec\theta} \, d\theta \, \vec{a_y}$$

$$\vec{E} = \frac{\rho l}{4\pi\varepsilon_0 r} \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta \, \vec{a_y}$$

$$= \frac{\rho l}{4\pi\varepsilon_0 r} \vec{a_y} \left(\sin\theta\right]_{-\pi/2}^{\pi/2}$$

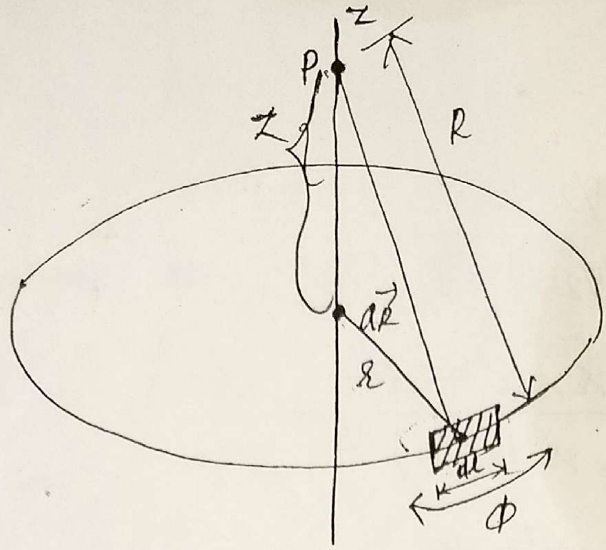
$$= \frac{\rho l}{4\pi\varepsilon_0 r} \vec{a_y} \, [2]$$

$$\frac{I \, a_\phi}{2\pi r}$$

$$\boxed{\vec{E} = \frac{\rho l}{2\pi\varepsilon_0 r} \vec{a_y}}$$

Electric field due to charge circular ring :-

* Consider a circular ring of radius r, placed in xy plane with centre at origin carrying a charge uniformly along its circumference.



* The charge density is ρ_l. Let point P is at a perpendicular distance from the ring
* Consider a small differential length dl on this ring. The charge of dl is dQ
* The Electric field intensity due to a charge is written as

$$\vec{E} = \frac{\vec{F}}{Q} = \frac{Q}{4\pi \epsilon_0 R^2} \vec{a_R}$$

$$\boxed{\vec{E} = \int \frac{\rho_l \, dl}{4\pi \epsilon_0 R^2} \vec{a_R}}$$

$\vec{a_R} \rightarrow$ Unit vector

Position vector of dl = $r\vec{a_r}$

" " of P = $z\vec{a_z}$

Limit is 0 to 2π

$R = \sqrt{r^2 + z^2}$ $\boxed{R^2 = r^2 + z^2.}$

$$\vec{a_R} = \frac{z\vec{a_z} - r\vec{a_r}}{|z\vec{a_z} - r\vec{a_r}|}$$

$$\vec{a_R} = \frac{z\vec{a_z} - r\vec{a_r}}{\sqrt{r^2 + z^2}}$$

$$\boxed{dl = r\, d\phi}$$

$$\vec{E} = \int_0^{2\pi} \frac{\rho l\, r\, d\phi}{4\pi\varepsilon_0 (z^2+r^2)} \frac{z\vec{az} - r\vec{ar}}{\sqrt{z^2+r^2}}$$

radial component $r\vec{ar} = 0$

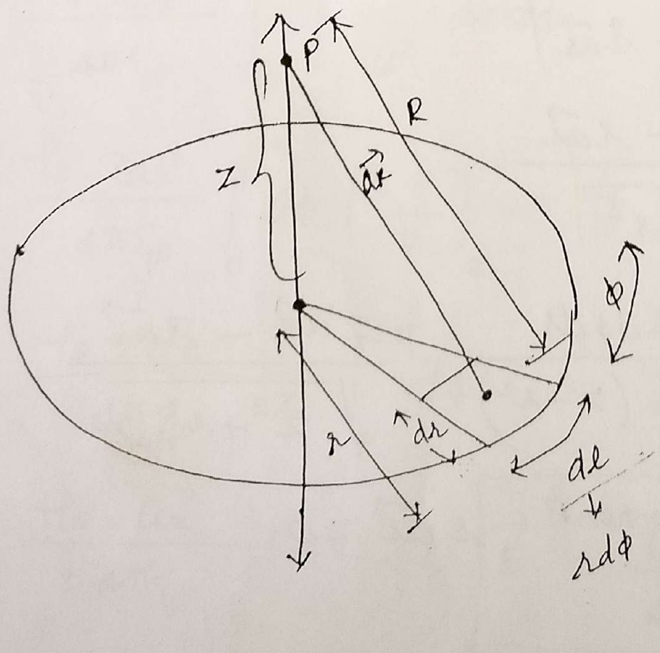
$$= \int_0^{2\pi} \frac{\rho l\, r\, d\phi}{4\pi\varepsilon_0} \frac{z\vec{az}}{(z^2+r^2)^{3/2}}$$

$$= \frac{\rho l\, z\vec{az}\, r}{4\pi\varepsilon_0 (z^2+r^2)^{3/2}} \int_0^{2\pi} d\phi$$

$$= \frac{\rho l\, z\vec{az}\, r}{\cancel{4}_2\pi\varepsilon_0 (z^2+r^2)^{3/2}} [\cancel{2}\pi]$$

$$\boxed{\vec{E} = \frac{\rho l\, z\vec{az}\, r}{2\varepsilon_0 (z^2+r^2)^{3/2}}}$$

Electric field intensity due to Infinite disc (or) Sheet charges: (or) Surface charge.



* Consider an infinite sheet or disc of charges having uniform charge density ($-\rho_s$) placed in xy plane.

* The point P at which electric field intensity ($\vec{E}$) to be calculated on z-axis.

* Consider the differential surface area ds carrying charge dQ.

* The normal direction to ds is z-direction.

The electric field intensity written as

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\,\vec{a_R}$$

$$= \frac{\int \rho_s\, ds}{4\pi\varepsilon_0 R^2}\,\vec{a_R}$$

limits of radius → 0 to ∞

" " circle → 0 to 2π

$$ds = dr \times d\phi$$

$$ds = r\,dr\,d\phi$$

$$\vec{a_R} = \frac{z\vec{a_z} - r\vec{a_r}}{|z\vec{a_z} - r\vec{a_r}|}$$

$$\vec{a_R} = \frac{z\vec{a_z} - r\vec{a_r}}{\sqrt{z^2+r^2}}$$

$$\vec{E} = \int_0^\infty \int_0^{2\pi} \frac{-\rho_s\, r\, d\phi\, dr}{4\pi\varepsilon_0 (z^2+r^2)} \cdot \frac{z\vec{a_z} - r\vec{a_r}}{\sqrt{z^2+r^2}}$$

Radial components $r\vec{a_r} = 0$

$$\vec{E} = \int_0^\infty \int_0^{2\pi} \frac{\rho_s r\, d\phi\, dr}{4\pi\varepsilon_0 (z^2+r^2)^{3/2}} \, z\vec{a_z}$$

$u^2 = z^2 + r^2$

$r = 0, \quad u = z.$

$r = \infty, \quad u = \infty$

① $\Rightarrow$ $\cancel{2}u\,du = \cancel{2}r\,dr$

$$\vec{E} = \int_z^\infty \int_0^{2\pi} \frac{\rho_s r.\, d\phi.\, dr z.\, \vec{a_z}}{4\pi\varepsilon_0 [u^2]^{3/2}}$$

$$\frac{\rho_s \vec{a_z}}{2\varepsilon_0}.$$

$$= \int_z^\infty \int_0^{2\pi} \frac{\rho_s r\, d\phi\, dr\, z\vec{a_z}}{4\pi\varepsilon_0 u^3}$$

$\cancel{u^2 = z^2 + r^2}$

$\cancel{2u\,du = 2r\,dr}$

$u\,du = r\,dr$

$$\vec{E} = \int_0^{2\pi} \int_z^\infty \frac{\rho_s.\, d\phi.\, u\,du\, z\vec{a_z}}{4\pi\varepsilon_0 u^3}$$

$$= \frac{\rho_s z\vec{a_z}}{4\pi\varepsilon_0} \int_0^{2\pi} \int_z^\infty \frac{1}{u^2}\, du\, d\phi$$

$$-\frac{1}{u}\Big|_z^\infty$$

$$= \frac{\rho_s z\vec{a_z}}{4\pi\varepsilon_0} \int_0^{2\pi} d\phi \left[-\frac{1}{u}\right]_z^\infty \Rightarrow \frac{1}{z}$$

$$-\left[\frac{1}{\infty} - \frac{1}{z}\right]$$

$$= \frac{\rho_s z\vec{a_z}}{4\pi\varepsilon_0} \frac{1}{z} \int_0^{2\pi} d\phi$$

$$\vec{E} = \frac{\rho_s \vec{a_z}}{2\,\cancel{4\pi}\varepsilon_0} \cancel{2\pi}, \quad \boxed{\vec{E} = \frac{\rho_s.\vec{a_z}}{2\varepsilon_0}}$$

Problem : 1.

Calculate electric field intensity at $P(3,-4,2)$ in free space called by

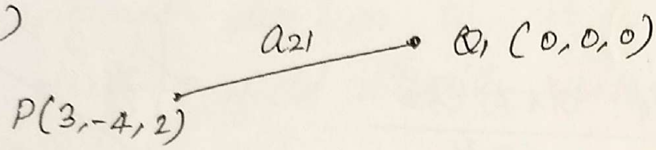
a) $Q_1 = 2\mu C$ at $(0,0,0)$

b) $Q_2 = 3\mu C$ at $(-1,2,3)$

c) $Q_1 = 2\mu C$ at $(0,0,0)$ and $Q_2 = 3\mu C$ at $(-1,2,3)$

Sol.

a)



unit vector $\vec{a}_{21} = \dfrac{r_1 - r_2}{|r_1 - r_2|}$

$$= \frac{3\vec{a_x} - 4\vec{a_y} + 2\vec{a_z}}{\sqrt{9+16+4}}$$

$$= \frac{3\vec{a_x} + 4\vec{a_y} + 2\vec{a_z}}{\sqrt{29}}$$

Electric field intensity at P due to Q_1

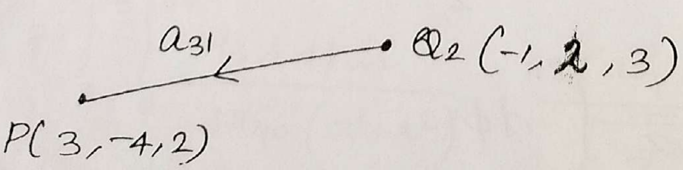
$$E = \frac{Q_1}{4\pi\varepsilon_0 r^2}\vec{a}_{21}$$

$$= \frac{2\times10^{-6}}{4\pi\times\dfrac{1}{36\pi\times10^9}\times(\sqrt{29})^2} \cdot \frac{3\vec{a_x} + 4\vec{a_y} + 2\vec{a_z}}{\sqrt{29}}$$

$$= \frac{2\times9\times10^3}{29\sqrt{29}}\left(3\vec{a_x} + 4\vec{a_y} + 2\vec{a_z}\right)$$

$$E = 345\vec{a_x} - 460\vec{a_y} + 230\vec{a_z} \text{ V/m}$$

b)



$$\vec{a}_{31} = \frac{\vec{a_x}(3-(-1)) + \vec{a_y}(-4-(2)) + \vec{a_z}(2-3)}{(3+1)^2 + (-4-2)^2 + (2-3)^2}$$

$$= \frac{4\vec{a_x} - 6\vec{a_y} - \vec{a_z}}{\sqrt{16+36+1}} = \frac{4\vec{a_x} - 6\vec{a_y} - \vec{a_z}}{\sqrt{53}}$$ (I.38)

Electric intensity at P due to Q2

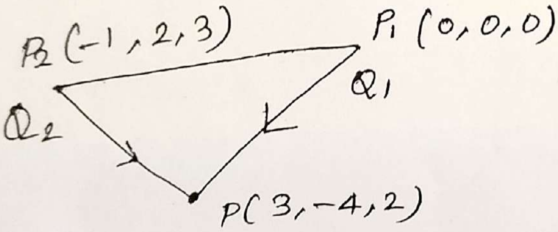
$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \vec{a}_{31}$$

$$= \frac{3\times10^{-6}}{4\pi \times \frac{1}{36\pi\times10^9} \times (\sqrt{53})^2} \frac{4\vec{a_x} - 6\vec{a_y} - \vec{a_z}}{\sqrt{53}}$$

$$= \frac{3\times9\times10^3}{53\sqrt{53}} (4\vec{a_x} - 6\vec{a_y} - \vec{a_z})$$

$$E = 280\vec{a_x} - 420\vec{a_y} - 70\vec{a_z} \text{ V/m}$$

c)



$$\vec{a}_{31} = \frac{3\vec{a_x} - 4\vec{a_y} + 2\vec{a_z}}{\sqrt{9+16+4}}$$

$$= \frac{3\vec{a_x} - 4\vec{a_y} + 2\vec{a_z}}{\sqrt{29}}$$

Electric field intensity at P due to Q1

$$E_1 = 345\vec{a_x} - 460\vec{a_y} + 230\vec{a_z} \text{ V/m}$$

Electric field intensity at P due to Q2

$$E_2 = 280\vec{a_x} - 420\vec{a_y} + 70\vec{a_z} \text{ V/m}$$

Total Electric field intensity at P due to Q1 and Q2

$$E = E_1 + E_2$$

$$= 345\vec{a_x} - 460\vec{a_y} + 230\vec{a_z} + 280\vec{a_x} - 420\vec{a_y} + 70\vec{a_z}$$

$$E = 65\vec{a_x} - 880\vec{a_y} + 160\vec{a_z} \text{ V/m}$$

ELECTRIC FLUX DENSITY (D)

Electric flux density or displacement density is defined as the electric flux per unit area.

$$D = \frac{Q}{A} \text{ coulomb/meter}^2$$

For a sphere surface area

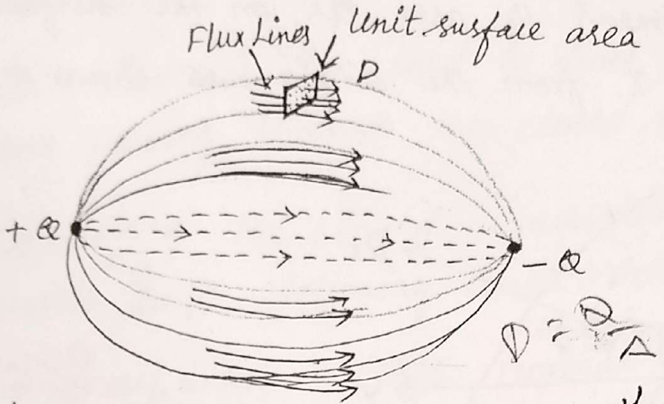
$A = 4\pi r^2$

$D = \frac{Q}{4\pi r^2}$

W.K.T $E = \frac{Q}{4\pi \epsilon r^2} = \frac{D}{\epsilon}$.

$D = \epsilon E$

$E = \frac{D}{\epsilon}$



* consider the two point charges as shown in fig. The flux lines originating from positive charge and terminating at negative charge.
* Consider a unit surface area as shown in the fig. The no. of flux lines are passing through this surface area.
* The net flux passing normal through the unit surface area is called the electric flux density. (D).

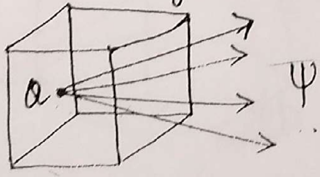
GAUSS'S LAW

The electric flux passing through any closed surface is equal to the total charge enclosed by the surface. $\psi = Q$

$$\boxed{\psi = Q}$$

where $\psi \rightarrow$ Total electric flux leaving normally from a closed surface

$Q \rightarrow$ Net charge enclosed by the closed surface.



Proof:

consider a charge Q at the origin of a spherical co-ordinate system, whose co-ordinates are r, θ, ϕ as shown in fig.

The electric field intensity due to the charge Q is

$$E = \frac{Q}{4\pi \epsilon_1 r^2}$$

The electric flux density $D = \epsilon E$

$D = \epsilon E$
$B = \mu H$

$$D = \frac{Q}{4\pi r^2}$$

Consider a small element of area ds on the surface the sphere at a distance r from the origin as shown in fig.

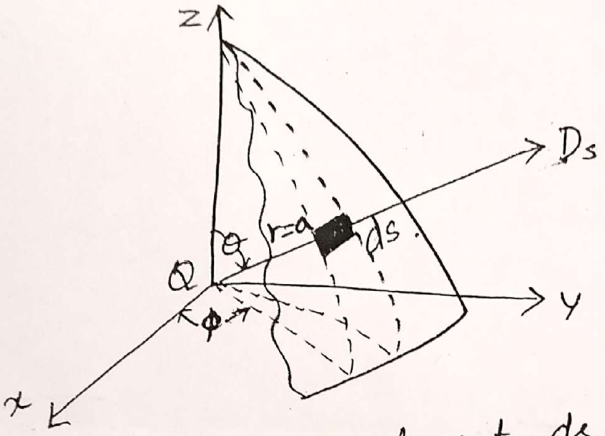


Fig. Surface element ds in the spherical co-ordinate

$$ds = r.\,d\theta.\,r\sin\theta.\,d\phi$$

$$= r^2.\sin\theta\, d\theta.\,d\phi$$

The electric flux $\psi = \int d\psi$

$$= \oint_S D.ds$$

$$\psi = \int_S \frac{Q}{4\pi r^2}\, r^2 \sin\theta\, d\theta.\,d\phi$$

$$= \frac{Q}{4\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \sin\theta\, d\theta.\,d\phi$$

$$= \frac{Q}{4\pi}\int_{\phi=0}^{2\pi}\left[\int_{\theta=0}^{\pi}\sin\theta\, d\theta\right] d\phi$$

$$= \frac{Q}{4\pi}\int_{0}^{2\pi}\left[-\cos\theta\right]_0^{\pi} d\phi$$

$$= \frac{Q}{4\pi}\int_0^{2\pi} 2\, d\phi = \frac{Q}{2\pi}\left[\phi\right]_0^{2\pi} = \frac{Q}{2\pi}\left[2\pi\right]$$

$$\boxed{\psi = Q}$$

Hence proved.

Applications of Gauss's law

* The Gauss's law can be used to find $\underline{E}$ or $\underline{D}$ for symmetrical charge distributions, such as point charge, an infinite line charge, an infinite sheet of charge and a spherical distribution of charge.

$D = \varepsilon E$

* Gauss's law is also used to find the charge enclosed or flux passing through the closed surface.

while selecting the closed Gaussian surface to apply the Gauss's law, following conditions must be satisfied,

1. $\vec{D}$ is every where either normal or tangential to the closed surface i.e. $\theta = \frac{\pi}{2}$ or π. So that $\vec{D} \cdot \vec{ds}$ becomes $D\,ds$ or zero respectively.
2. $\vec{D}$ is constant over the portion of the closed surface for which $\vec{D} \cdot \vec{ds}$ is not zero.

1.) Infinite Line charge

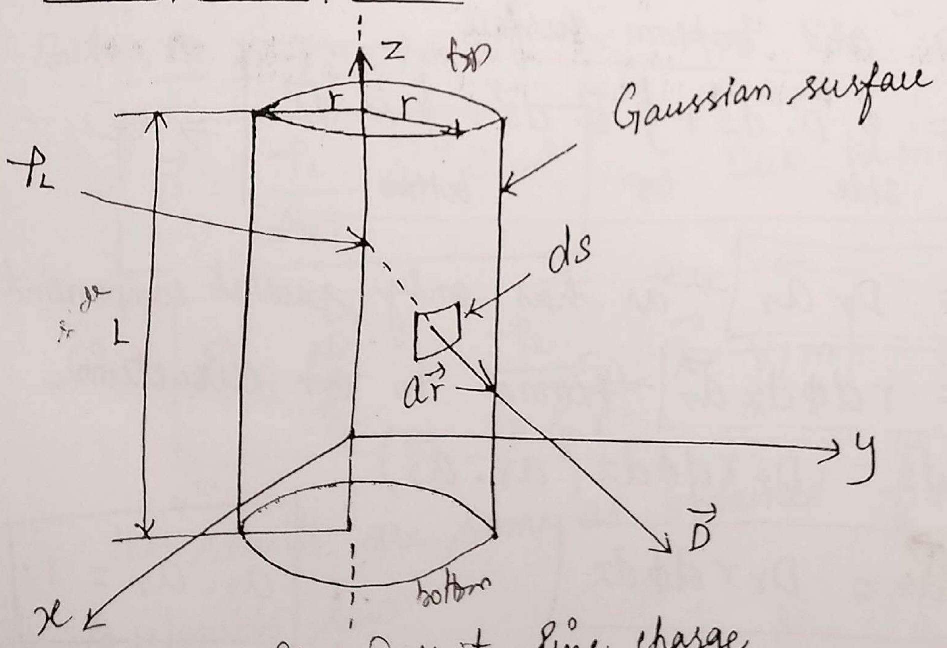


Fig. Infinite line charge

(I.4.2) * Consider an infinite line charge of density $+\rho_L$ C/m lying along z-axis from $-\infty$ to ∞.

* Consider the Gaussian surface as the right circular cylinder with z-axis as its axis and radius 'r' as shown in fig. The length of the cylinder is L.

* The flux density at any point on the surface is direct radially outwards i.e in the $\vec{a}_r$ direction according to cylindrical co-ordinate system.

* Consider differential surface area ds as shown which is at a radial distance r from the line charge. The direction normal to ds is $\vec{a}_r$.

* As the line charge is along z-axis, there can not be any component of $\vec{D}$ in z direction. So $\vec{D}$ has only radial component

$$\boxed{Q = \oint_s \vec{D} \cdot \vec{ds}} \rightarrow (1)$$

(iii) The integration is to be evaluated for side surface, top surface and bottom surface.

$$\therefore \boxed{Q = \oint_{side} \vec{D} \cdot \vec{ds} + \oint_{top} \vec{D} \cdot \vec{ds} + \oint_{bottom} \vec{D} \cdot \vec{ds}} \rightarrow (2)$$

$\boxed{\vec{D} = D_r \vec{a}_r} \rightarrow (3)$ as has only radial component

$\boxed{\vec{ds} = r\,d\phi\,dz\,\vec{a}_r} \rightarrow (4)$ normal to $\vec{a}_r$ direction.

$$\therefore \vec{D} \cdot \vec{ds} = D_r\, r\, d\phi\, dz\, (\vec{a}_r \cdot \vec{a}_r)$$

$$\boxed{\vec{D} \cdot \vec{ds} = D_r\, r\, d\phi\, dz} \qquad \therefore \boxed{\vec{a}_r \cdot \vec{a}_r = 1}$$

Now D_r is constant over the side surface.

* As $\vec{D}$ has only radial component and no component

along $\vec{a_z}$ and $-\vec{a_z}$, hence integrations over top and bottom surface is zero.

$$\therefore \boxed{\oint_{top} \vec{D}\cdot d\vec{s} = \oint_{bottom} \vec{D}\cdot d\vec{s} = 0}$$

about Boundary Conditions

$$Q = \oint_{side} \vec{D}\cdot d\vec{s}$$

$$= \oint_{side} D_r \, r\, d\phi\, dz$$

$$= \int_{z=0}^{L} \int_{\phi=0}^{2\pi} D_r\, r\, d\phi\, dz$$

$$= r.\, D_r . [z]_0^L \, [\phi]_0^{2\pi}$$

$$Q = 2\pi r D_r L \Rightarrow \boxed{D_r = \frac{Q}{2\pi r L}}$$

W.K.T $\boxed{\vec{D} = D_r \vec{a_r}}$

so $\boxed{\vec{D} = \frac{Q}{2\pi r L} \vec{a_r}}$

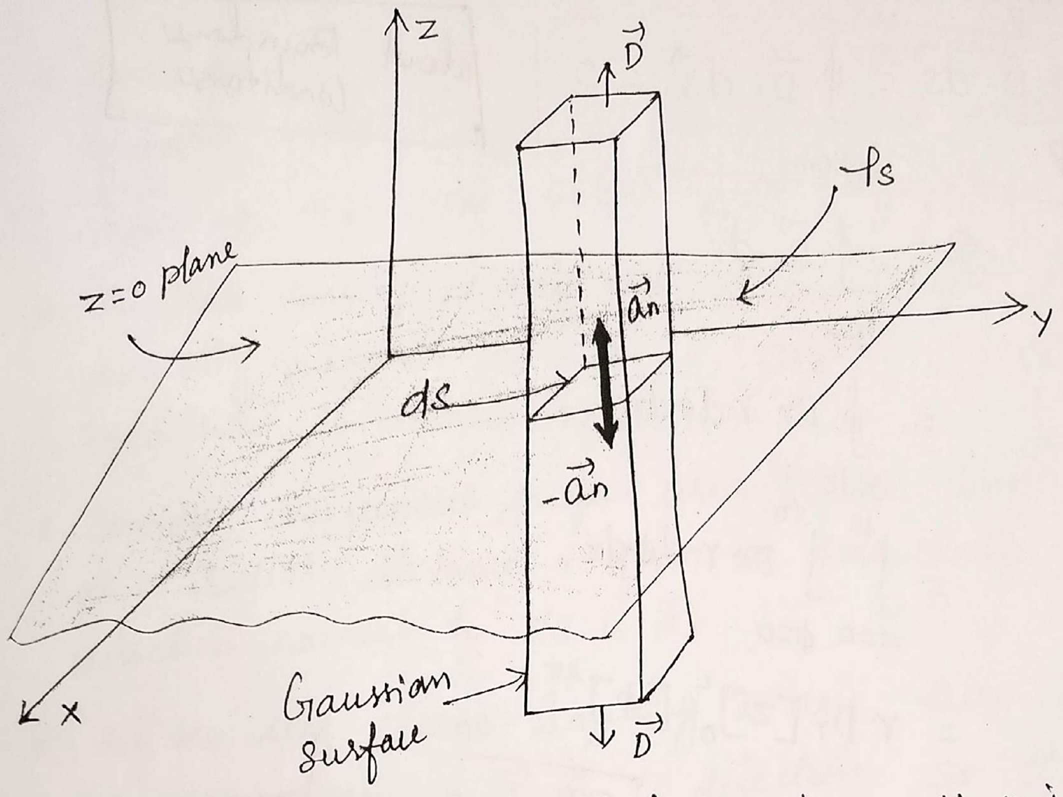
But $\frac{Q}{L} = \rho_L \; C/m$ $\quad \therefore Q = \rho_L . L$

$$\boxed{\vec{D} = \frac{\rho_L}{2\pi r} \vec{a_r} \; C/m^2}$$ ... Due to infinite line charge

$$\boxed{\vec{E} = \frac{\vec{D}}{\varepsilon_0} = \frac{\rho_L}{2\pi \varepsilon_0 r} \vec{a_r} \; V/m}$$

$\therefore$ the results are same as obtained from the coulomb's law.

2. Infinite sheet of Charge



* Consider the infinite sheet of charge of uniform charge density ρ_s C/m², lying in the z=0 plane. i.e xy plane.
* Consider a rectangular box as a Gaussian surface which is cut by the sheet of charge to give $ds = dx\,dy$.

$\vec{D}$ acts normal to the plane i.e $\vec{a_n} = \vec{a}$

$-\vec{a_n} = -\vec{a_z}$ direction.

Hence $\vec{D} = 0$ in x & y directions.

Hence the charge enclosed can be written as,

$$Q = \oint_s \vec{D} \cdot d\vec{s}$$

$$= \oint_{\text{sides}} \vec{D} \cdot d\vec{s} + \oint_{\text{top}} \vec{D} \cdot d\vec{s} + \oint_{\text{bottom}} \vec{D} \cdot d\vec{s}$$

* But $\oint_{sides} \vec{D} \cdot \vec{ds} = 0$ as $\vec{D}$ has no component in x & y directions

Now $\vec{D} = D_z \vec{a_z}$ for top surface.

& $\vec{ds} = dx\, dy\, \vec{a_z}$

$$\vec{D} \cdot \vec{ds} = D_z\, dx\, dy\, (\vec{a_z} \cdot \vec{a_z})$$

$$= D_z\, dx\, dy$$

$$\vec{D} = D_z(-\vec{a_z}) \text{ for bottom surface.}$$

$$\vec{ds} = dx\, dy\, (-\vec{a_z})$$

$$\vec{D} \cdot \vec{ds} = D_z\, dx\, dy\, (\vec{a_z} \cdot \vec{a_z})$$

$$= D_z\, dx\, dy$$

$$Q = \oint_{top} D_z\, dx\, dy + \oint_{bottom} D_z\, dx\, dy$$

Now $\oint_{top} dx dy = \oint_{bottom} dx dy = A$ = Area of surface

$$Q = 2 D_z A$$

But, $Q = \rho_s \times A$ ρ_s = Surface charge density

$$\rho_s = 2 D_z$$

$$D_z = \frac{\rho_s}{2}$$

$$\vec{D} = D_z \vec{a_z}$$

$$\vec{D} = \frac{\rho_s}{2} \vec{a_z} \ C/m^2$$

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0}$$

$$= \frac{\rho_s}{2\varepsilon_0} \vec{a}_z \text{ V/m}$$

The results are same as obtained by the Coulomb's law for the infinite sheet of charge.

Problem : 1

The flux density $\vec{D} = \frac{r}{3}\vec{a}_r$ nC/m² is in free space:

a) Find $\vec{E}$ at $r = 0.2$ m

b) Find the total electric flux leaving the sphere of $r = 0.2$ m

c) Find the total charge within the sphere of $r = 0.3$ m

Sol. a) $\vec{E} = \frac{\vec{D}}{\varepsilon_0} = \frac{r}{3\varepsilon_0}\vec{a}_r$ $r = 0.2$ m

$$\vec{E} = \frac{0.2 \times 10^{-9}}{3 \times 8.854 \times 10^{-12}} = 7.5295\ \vec{a}_r \text{ V/m}$$

b) $Q = \psi = \oint_S \vec{D} \cdot \vec{ds}$

consider a differential area ds normal to $\vec{a}_r$ which is $r^2 \sin\theta\, d\theta\, d\phi$.

$$\vec{ds} = r^2 \sin\theta\, d\theta\, d\phi\, \vec{a}_r$$

&

$$\vec{D} = \frac{r}{3}\vec{a}_r$$

$$\vec{D} \cdot \vec{ds} = \frac{r^3}{3} \sin\theta\, d\theta\, d\phi \qquad (\vec{a}_r \cdot \vec{a}_r = 1)$$

$$Q = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{r^3}{3} \sin\theta\, d\theta\, d\phi = \frac{r^3}{3}\left[-\cos\theta\right]_0^{\pi}\left[\phi\right]_0^{2\pi}$$

$$= \frac{4}{3}\pi r^3 \text{ nC}$$

$\vec{D}$ nC/m³

∴ At $r = 0.2$ m,

$$Q = \frac{4}{3}\pi(0.2)^3 = 0.0335 \text{ nC} = 33.51 \text{ PC}$$

c) At $r = 0.3$ m

$$Q = \frac{4}{3}\pi(0.3)^3 = 0.113 \text{ nC} = 113.097 \text{ PC}$$

ENERGY DENSITY

* Consider a elementary cube of side Δd parallel to the plates of a Capacitor as shown in fig.

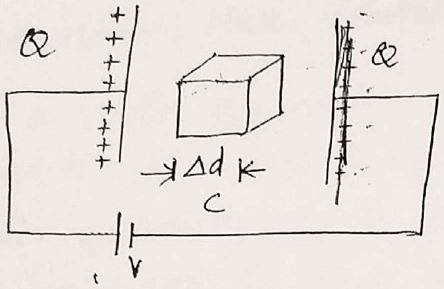


Fig. Energy Storage in Capacitor.

$$C = \frac{\epsilon_1 A}{d}.$$

$$\Delta C = \frac{\epsilon_1 A}{\Delta d}.$$

The Capacitance of elemental Capacitor is

$$\Delta C = \frac{\epsilon_1 A}{\Delta d}$$

$\therefore \boxed{A = \Delta d^2}$

$$\Delta C = \frac{\epsilon_1 (\Delta d)^2}{\Delta d}$$

$$\boxed{\Delta C = \epsilon_1 \Delta d} \quad \text{——— (1)}$$

W.K.T $W_E = \frac{1}{2} CV^2$

Energy stored in the elemental capacitor is

$$\Delta W = \frac{1}{2} \Delta C (\Delta V)^2 \quad \text{——— (2)}$$

But potential difference across the elementary cube is

W.K.T $V = Ed$

$$\boxed{\Delta V = E \cdot \Delta d} \quad \text{——— (3)}$$

where $E \rightarrow$ Electric field exist in the cube.

Sub (1) & (3) in (2)

$$\Delta W = \frac{1}{2} (\epsilon_1 \Delta d)(E \Delta d)^2$$

$$= \frac{1}{2} \epsilon_1 E^2 (\Delta d)^3$$

$$\boxed{\Delta W = \frac{1}{2} \epsilon_1 E^2 \Delta V} \rightarrow (4)$$

$\Delta V = \Delta d^3$ is elementary Volume

From Eqn (4)

The Energy density is given by

W.K.T

$$\frac{\Delta W}{\Delta V} = \frac{1}{2} \epsilon_1 E^2 \qquad (\therefore D = \epsilon_1 E) \qquad \epsilon = \frac{D}{E}$$

$$\boxed{\frac{\Delta W}{\Delta V} = \frac{1}{2} DE \quad \text{Joules}/m^3}$$

$$\frac{1}{2} \boxed{\epsilon_1 \cdot E} E$$

$$\boxed{\frac{\Delta W}{\Delta V} = \frac{1}{2} DE}$$

ELECTRIC POTENTIAL

An electric charge produces an electric field around it, and if a test charge is brought in to this region, it experiences a force. The force is given by

$$\boxed{F = qE} \rightarrow (1)$$

Since there is a movement of charge in the electric field from one point r_1 to another point r_2, there will be work done against the force

$$W = -\int_{+r_2}^{r_2} q . E\, dr$$

$$\boxed{W = -q \int_{+r_2}^{r_2} E\, dr} \rightarrow (2)$$

Potential difference (V) is defined as the work done in moving a unit positive charge from one point to another in an electric field.

Workdone on unit positive charge per charge is

$$\boxed{V = \frac{W}{q}}$$

$$V = -\int_{r_2}^{r_1} E . dr \quad \text{Joules/coulomb}$$

But,

$$\boxed{E = \frac{Q}{4\pi\epsilon_0 r^2}}$$

$$V = -\frac{Q}{4\pi\epsilon_0}\int_{r_2}^{r_1} \frac{1}{r^2}\, dr$$

$$= -\frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_{r_2}^{r_1} = \frac{-Q}{4\pi\epsilon_0}\left[-\frac{1}{r_2} + \frac{1}{r_1}\right]$$

$$V = +\frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \text{volts}$$

This is the potential difference between two points r_1 & r_2

$$V = \frac{Q}{4\pi\epsilon_0 r_1} - \frac{Q}{4\pi\epsilon_0 r_2}$$

$$V = V_1 - V_2$$

(1.48)

If the test charge is moved from infinity to a given point in the electric field.

$V_2 = 0$ $\left[\because \frac{1}{r_2} = 0\right]$

then $V = V_1$

Absolute potential or potential at a point is defined as the work done is moving a unit positive charge from infinity to a given point in an electric field.

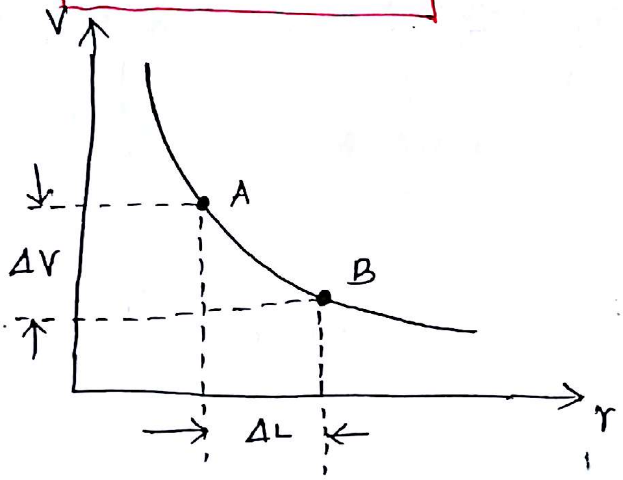
$$V = \frac{Q}{4\pi\epsilon_0 r} \text{ volts.}$$

Potential gradient

* Consider an electric field $\vec{E}$ due to a positive charge placed at the origin of a sphere. Then,

$$V = -\int \vec{E} \cdot d\vec{L}$$

$$\boxed{V = \frac{Q}{4\pi\epsilon_0 r}}$$



* The potential decreases as distance of point from the charge increases. This is shown in the fig.

* It is known that the line integral of $\vec{E}$ between the two points gives a potential difference between the two points.

* For an elementary length ΔL, we can write

$$\therefore V_{AB} = \Delta V$$
$$= -\vec{E} \cdot \Delta\vec{L}$$

* The rate of change of potential with respect to the distance is called the potential gradient.

$$\text{Grad } V = \underset{\Delta l \to 0}{Lt} \left(\frac{\Delta V}{\Delta l}\right) = \frac{dv}{dl}$$

$$dv = -E\,dl$$

$$= -E\cos\theta\,dl$$

$$\boxed{\frac{dv}{dl} = -E\cos\theta}$$

If $\theta = 0$, $\frac{dv}{dl}$ is maximum

$$|\text{Grad } V| = \frac{dv}{dl} = -E$$

$$\nabla V = -E$$

$$\nabla V = \frac{\partial v}{\partial x} i + \frac{\partial v}{\partial y} j + \frac{\partial v}{\partial z} k = -E$$

The gradiant of a scalar function v can be written as

$$\nabla V = i\frac{\partial v}{\partial l} + j\frac{\partial v}{\partial l_2} + k\frac{\partial v}{\partial l_3}.$$

where

$$\partial l_1 = h_1 \partial P_1$$

$$\partial l_2 = h_2 \partial P_2$$

$$\partial l_3 = h_3 \partial P_3$$

$$\nabla V = i\left(\frac{1}{h_1}\frac{\partial v}{\partial P_1}\right) + j\left(\frac{1}{h_2}\frac{\partial v}{\partial P_2}\right) + k\left(\frac{1}{h_3}\frac{\partial v}{\partial P_3}\right)$$

In Cartesian system

$$h_1 = h_2 = h_3 = 1$$

$$P_1 = x, \quad P_2 = y, \quad P_3 = z$$

$$\nabla V = i\frac{\partial v}{\partial x} + j\frac{\partial v}{\partial y} + k\frac{\partial v}{\partial z}$$

In cylindrical Co ordinate System.

$h_1 = 1, \quad h_2 = r, \quad h_3 = 1$

$P_1 = r, \quad P_2 = 0, \quad P_3 = z$

$$\nabla V = i \frac{\partial v}{\partial r} + j \frac{1}{r} \frac{\partial v}{\partial \theta} + k \frac{\partial v}{\partial z}$$

Spherical Co ordinate system

$h_1 = 1, \quad h_2 = P, \quad h_3 = R \sin\theta$

$P_1 = R, \quad P_2 = 0, \quad P_3 = \phi$

$$\nabla V = i \frac{\partial v}{\partial R} + j \frac{1}{R} \frac{\partial v}{\partial \theta} + k \frac{1}{R \sin\theta} \frac{\partial v}{\partial \phi}$$

Problem 1

Find the electric potential at a point (4, 3) m due to a charge of 10^{-9} C located at the origin in free space.

Sol.

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

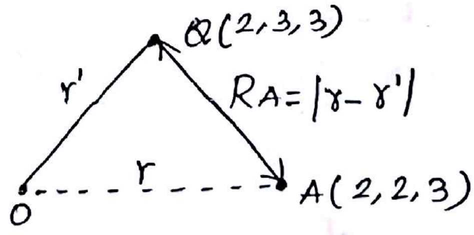
$$r = \sqrt{4^2 + 3^2} = 5m$$

$$V = \frac{10^{-9}}{4\pi \cdot \frac{1}{36\pi \times 10^9} \cdot (5)} = \frac{9}{5} = 1.8V$$

Problem 2

If same charge $Q = 0.4$ nC in above example is located at (2, 3, 3) then obtain the absolute potential of point A(2, 2, 3)

Sol.



Now the Q is located at (2, 3, 3)

The potential at A is given by

$$V_A = \frac{Q}{4\pi\varepsilon_0 R_A}$$

$$R_A = |r - r'| = \sqrt{(2-2)^2 + (2-3)^2 + (3-3)^2} = 1$$

$$V_A = \frac{0.4 \times 10^{-9}}{4\pi \times 8.854 \times 10^{-12} \times 1} = 3.595 V$$

POISSON'S AND LAPLACE'S EQUATIONS

(2.24. P.D)

* According to Gauss's law in point form, the divergence of electric ~~field~~ flux is equal to the volume charge density.

$$\nabla \cdot D = \rho_v$$

W.K.T $\quad D = \varepsilon E$

$D = \varepsilon E$

$B = \mu H$

$$\nabla \cdot (\varepsilon E) = \rho_v$$

$$\varepsilon \nabla \cdot E = \rho_v$$

$$\nabla \cdot E = \frac{\rho_v}{\varepsilon}$$

W.K.T $\quad E = -\nabla V$

$$\nabla \cdot (-\nabla V) = \frac{\rho_v}{\varepsilon}$$

$$\nabla \cdot \nabla V = -\frac{\rho_v}{\varepsilon}$$

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\varepsilon}}$$

→ This is the poisson's equation.

For Cartesian co-ordinates system

$$\nabla \cdot \nabla V = \frac{\partial}{\partial x}\left(\frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial V}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial V}{\partial z}\right)$$

$$= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

poisson's equation for Cartesian co-ordinate system is written as

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho_v}{\varepsilon}$$

for cylindrical co-ordinate system, the poisson's equation is

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho_v}{\varepsilon}$$

For spherical co-ordinate system, the poisson's equation is

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

$$= -\frac{\rho_v}{\varepsilon}$$

If the volume charge density (ρ_v) is zero, then

$\boxed{\nabla^2 V = 0}$ ⟶ This is Laplace equation.

∇^2 ⟶ Laplacian operator.

DIPOLE (or) ELECTRIC DIPOLE (or) POTENTIAL DUE TO DIPOLE :

* An electric dipole or simply dipole is nothing but two equal and opposite charges separated by a very small distance. The product of charge and spacing is called Electric dipole moment.
* Let Q and −Q be the two charges separated by a small distance d. The product of charge Q and spacing d is called dipole moment.

$$m = Qd$$

* Let P be any point at distance of r_1, r_2 & r from $+Q$, $-Q$ and mid point of dipole respectively as shown in fig.

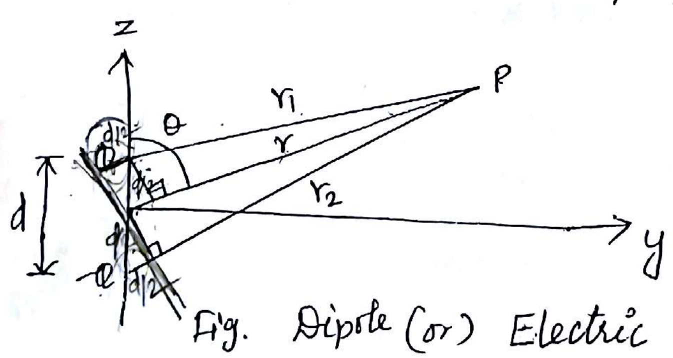


Fig. Dipole (or) Electric Dipole

Potential at P due to +Q is

$$V_1 = \frac{Q}{4\pi\varepsilon r_1}$$

$r = r_1 + d/2 \cos\theta$

$r_1 = r - d/2 \cos\theta$

Potential at P due to $-Q$ is

$$V_2 = \frac{-Q}{4\pi\epsilon_1 r_2}$$

The resultant potential at P

$$V = V_1 + V_2 = \frac{Q}{4\pi\epsilon_1}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

If the point P is too far away from the dipole, the distance r_1 and r_2 are written as

$$r_1 = r - \frac{d}{2}\cos\theta$$

$$r_2 = r + \frac{d}{2}\cos\theta$$

$$V = \frac{Q}{4\pi\epsilon_1}\left[\frac{1}{r - \frac{d}{2}\cos\theta} - \frac{1}{r + \frac{d}{2}\cos\theta}\right]$$

$$= \frac{Q}{4\pi\epsilon_1}\left[\frac{\cancel{r} + \frac{d}{2}\cos\theta - \cancel{r} + \frac{d}{2}\cos\theta}{r^2 - \left(\frac{d}{2}\cos\theta\right)^2}\right]$$

$$\frac{d}{2} + \frac{d}{2} = \cancel{2}\frac{d}{\cancel{2}}$$

$$= \frac{Q}{4\pi\epsilon_1}\left[\frac{d\cos\theta}{r^2 - \left(\frac{d}{2}\cos\theta\right)^2}\right]$$

$$= \frac{Q}{4\pi\epsilon_1}\left[\frac{d\cos\theta}{r^2}\right] \qquad \because \frac{d}{2} << r^2$$

$$V = \frac{m\cos\theta}{4\pi\epsilon_1 r^2} \qquad m = Qd$$

This shows that the potential is directly proportional to the dipole moment & inversely proportional to the square of the distance.

DIPOLE MOMENT

Dipole may be described by its dipole moment m. If Q is the charge and r is the vector (distance) from the negative to the positive charge, the dipole moment is given by

$$m = Qr.$$

If there are n dipole per unit volume Δv, then there are $n\Delta v$ dipoles and the total dipole moment is given by

$$m_{total} = \sum_{i=1}^{n\Delta v} m_i$$

CAPACITOR

* A capacitor is an electric device which consists of two Conductors separated by a dielectric medium.

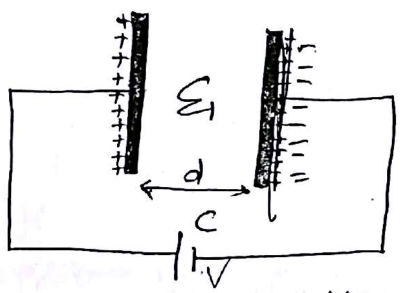


Fig. Capacitor.

* Consider a capacitor composed of two conducting plates of area 'A' separated by a dielectric medium whose permitivity is ε_1.

* The space separation between the plates is d. If potential 'V' is applied across the plates, the positive charge Q is deposited on one plate and the negative charge $-Q$ is deposited on other plate as shown in fig. The net charge is zero.

* (The capacitance of two conducting planes is defined as the ratio of magnitude of charge on either of the conductor to the potential difference between conductors. It is given by

$$C = \frac{Q}{V}.$$

The unit of capacitance is coulombs/volt or Farad.)

* The unit of capacitance

* Assume that there is a uniform charge density D over the plates and dielectric medium

$$D = \frac{Q}{A} \ C/m^2$$

It is also written as in terms of electric field E

$$D = \varepsilon E$$

$$\frac{Q}{A} = \varepsilon E$$

$$Q = A\varepsilon E$$

But electric field is given by

$$E = V/d \quad V/m$$

Sub. the value of E in above equation

$$Q = A\varepsilon \frac{V}{d}$$

$$\frac{Q}{V} = \frac{A\varepsilon}{d}$$

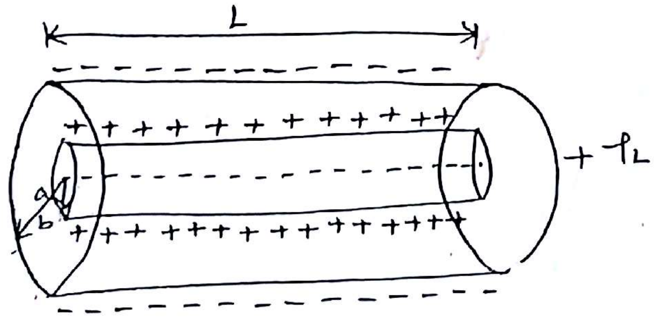
Capacitance is given by

$$C = \frac{Q}{V}$$

$$C = \frac{A\varepsilon}{d}$$

$$\boxed{C = \frac{A\varepsilon_0 \varepsilon_r}{d}} \text{ Farad} \qquad \boxed{\varepsilon = \varepsilon_0 \varepsilon_r}$$

Capacitance of coaxial Cable



* Consider a co-axial cable or co-axial capacitor. Let inner radius is 'a' and outer radius is 'b'.

* Two concentric conductors are separated by dielectric of permitivity of ε_0
* The length of cable l, the inner conductor carries charge density $(+\rho_L)$. Then the equal and opposite charge density $(-ve\ \rho_L)$ exists on the outer conductor.

$$\boxed{\rho_L = \frac{Q}{L}} \rightarrow (1)$$

$$\boxed{Q = \rho_L \cdot L} \rightarrow (2)$$

* Assuming cylindrical co-ordinate system electric field intensity $(\vec{E})$ will be radial from inner to outer conductor and for infinite line of charge the electric field intensity is given by

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\,\vec{a_R}} \rightarrow (3)$$

* $\vec{E}$ is directed from inner conductor to outer conductor

W.K.T

$$C = \frac{Q}{V}$$

$$\boxed{Q = \rho_L \cdot L}$$

$$V = -\int_b^a \vec{E} \cdot dl$$

$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_1 r}\,\vec{a_r} \cdot dl \qquad dl = dr \cdot \vec{a_r}$$

$$\vec{a_r} \cdot \vec{a_r} = 1$$

$$V = -\int_b^a \frac{\rho_L}{2\pi\varepsilon_1 r}\,\vec{a_r} \cdot dr \cdot \vec{a_r}$$

$$= \frac{-\rho_L}{2\pi\varepsilon_1} \int_b^a \frac{1}{r}\,dr$$

$$= -\frac{\rho_L}{2\pi\varepsilon_1}\left[\ln(r)\right]_b^a$$

$$= -\frac{\rho_L}{2\pi\varepsilon_1}\left[\ln(a) - \ln(b)\right]$$

$$V = \frac{\rho_L}{2\pi\varepsilon_1}\left[\ln(b) - \ln(a)\right]$$

$$V = \frac{\rho_L}{2\pi\varepsilon_1}\left[\ln\left(\frac{b}{a}\right)\right]$$

$$\frac{2\pi \varepsilon L}{\ln(b/a)} \text{ farad.}$$

$$C = \frac{\cancel{\rho_L} L}{\frac{\cancel{\rho_L}}{2\pi\varepsilon_1}\left(\ln\left(\frac{b}{a}\right)\right)}$$

Energy density

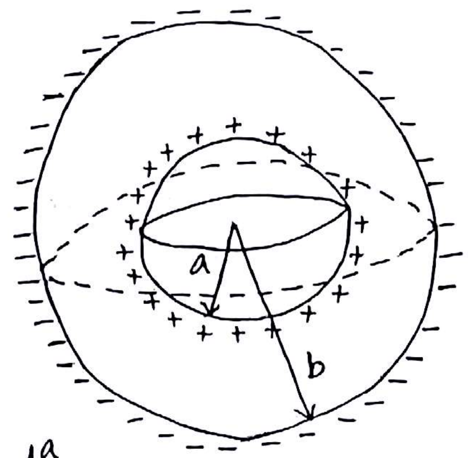
$$\boxed{C = \frac{2\pi\varepsilon_1 L}{\ln(b/a)} \text{ farad}}$$

Capacitance of Spherical Shell (or) Spherical Shape two conductor:

* Consider a spherical capacitor formed by two concentric spherical conducting shells of radius 'a' and 'b'.
* The capacitor radius of outer sphere is 'b' while that of inner sphere is 'a'.
* The permitivity of dielectric between two conductors is ε_1. The inner conductor carries positive charges $(+Q)$ which constituting negative charges on outer conductor.
* Consider Gaussian surface as a sphere of radius 'r' it can be obtained that electric field intensity $(\vec{E})$ is in radial direction.

$$\boxed{C = \frac{Q}{V}} \rightarrow (1)$$

$$\boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_1 r^2}\vec{a_r}} \rightarrow (2) \qquad \rightarrow \vec{E} \text{ due to point charge.}$$



$$V = -\int_b^a E \cdot dl$$

$$dl = dr \cdot \vec{a_r}$$

$$= -\int_b^a \frac{Q}{4\pi\varepsilon_0 r^2} \vec{a_r} \cdot dr \cdot \vec{a_r}$$

$$= -\int_a^b \frac{Q}{4\pi\varepsilon_0 r^2} dr$$

$$\boxed{\int \frac{1}{x^2} dx = \frac{-1}{x}}$$

$$= \frac{-Q}{4\pi\varepsilon_0} \int_a^b \frac{1}{r^2} dr$$

$$= -\frac{Q}{4\pi\varepsilon_0} \left[-\frac{1}{r} \right]_b^a$$

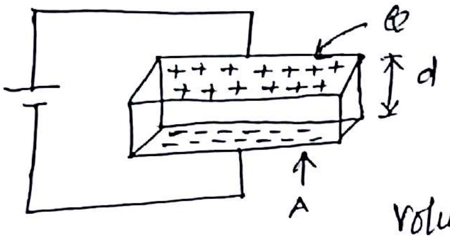
$$= \frac{-Q}{4\pi\varepsilon_0} \left[-\frac{1}{a} + \frac{1}{b} \right]$$

$$V = \frac{Q}{4\pi\varepsilon_0} \left[\frac{1}{a} - \frac{1}{b} \right]$$

$$C = \frac{Q}{\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]}$$

$$\boxed{C = \frac{4\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}} \text{ farad}}$$

Energy stored in a capacitor : [Energy stored in a electrostatic field]



Volume = $A \times d$

Energy stored in a capacitor is given by

$$W_E = \frac{1}{2}\int_V \vec{D}\cdot\vec{E}\cdot dv$$

W.K.T $\boxed{\vec{E} = \frac{V}{d}}$, $\boxed{\vec{D} = \varepsilon_1 \vec{E}}$

$$W_E = \frac{1}{2}\int_V \varepsilon_1 \vec{E}\cdot\vec{E}\, dv$$

$$= \frac{1}{2}\int_V \varepsilon_1 \vec{E}^2 dv$$

$$= \frac{1}{2}\int_V \varepsilon_1 \frac{V^2}{d^2} dv$$

$$= \frac{1}{2} \varepsilon_1 \frac{V^2}{d^2}\int_V dv$$

$$= \frac{1}{2} \frac{\varepsilon_1 V^2}{d^2} (\text{volume})$$

$$= \frac{1}{2} \frac{\varepsilon_1 V^2}{d^{\cancel{2}}} \times A\cancel{d}$$

$$= \frac{1}{2} \left(\varepsilon_1 V^2 \frac{A}{d}\right)$$

W.K.T $\boxed{C = \frac{\varepsilon_1 A}{d}}$

$$\boxed{W_E = \frac{1}{2} C V^2}$$

* If the dielectric is free space then there is increasing ~~the~~ in storing energy.

Problems

1. Determine the capacitance of a parallel plate capacitor with two metal plates of size 30 cm x 30 cm separated by 5 mm in air medium.

Given:

$$A = 0.3 \times 0.3 = 0.09 m^2$$

$$d = 5 \times 10^{-3} m$$

$$\varepsilon_0 = \frac{1}{36\pi \times 10^9}$$

$$C = \frac{A \varepsilon_0}{2}$$

$$= \frac{0.09 \times \frac{1}{36\pi \times 10^9}}{5 \times 10^{-3}}$$

$$= \frac{0.09}{36 \times 5\pi} \times 10^{-6}$$

$$= 1.59 \times 10^{-10}$$

$$= 15.9 nF$$

2. Find the capacitance of a parallel plate capacitor containing two dielectrics $\varepsilon_{r_1} = 2$, & $\varepsilon_{r_2} = 3$, each comprising one-half the volume as shown in fig. Assume $A = 2m^2$ & $d = 1 mm$.

Sol. The capacitance of a capacitor consisting of dielectric 1 (ε_{r1}) alone is C_1.

$$C_1 = \frac{\varepsilon_0 \varepsilon_{r1} A/2}{d}$$

$$= \frac{8.854 \times 10^{-12} \times 2 \times 1}{10^{-3}}$$

$$= 17.708 nF$$

The capacitance of a capacitor consisting of dielectric 2 (ε_{r2}) alone is C_2.

$$C_2 = \frac{\epsilon_0 \epsilon_{r2} A/2}{d}$$

$$= \frac{8.854 \times 10^{-12} \times 3 \times 1}{10^{-3}}$$

$$= 26.562\ nF.$$

These two capacitances C_1 and C_2 are in parallel. The effective capacitance is

$$C = C_1 + C_2$$

$$= 17.708 + 26.562$$

$$C = 44.270\ nF$$

3. Determine the capacitance of a parallel plate capacitor composed of thin foil sheets, 20 cm square for plates separated through a glass-dielectric 0.4cm thick with relative permittivity 6.

Given.

Area of plates, $A = 20cm^2 = 20 \times 10^{-4} m^2$

Distance of separation, $d = 0.4cm = 0.4 \times 10^{-2} m$

Relative permittivity, $\epsilon_r = 6$

For a parallel plate capacitor, Capacitance is given by

$$C = \frac{\epsilon_0 \epsilon_r A}{d} \text{ Farads}$$

$$= \frac{8.854 \times 10^{-12} \times 6 \times 20 \times 10^{-4}}{0.4 \times 10^{-2}}$$

$$= \frac{1.06248 \times 10^{-13}}{0.4 \times 10^{-2}}$$

$$C = 26.562 \times 10^{-12} \text{ Farads}$$

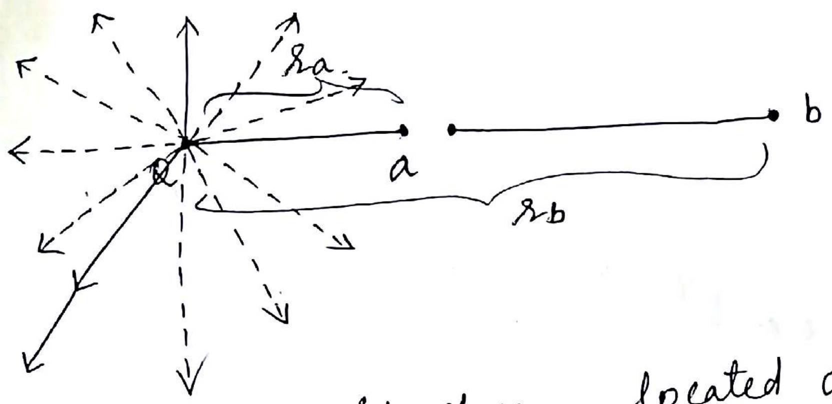
4. Find the energy stored in a parallel plate capacitor of 0.5 m by 1m has a separation of 2cm and a voltage difference of 10V.

Sol. $C = \frac{\epsilon_0 A}{d} = \frac{8.854 \times 10^{-12} \times 0.5 \times 1}{2 \times 10^{-2}} = 2.2135 \times 10^{-10} F$

Energy stored $E = \frac{1}{2} CV^2 = \frac{1}{2} \times 2.2135 \times 10^{-12} \times 10^2$

$= 1.10675 \times 10^{-8}$ Joules.

Potential due to Point Charge



* Consider a point charge, located at the origin of a spherical co-ordinate system, producing $\vec{E}$ radially in all directions as shown in the fig.

* Assuming free space, the field $\vec{E}$ due to a point charge Q at a point having radial distance r from origin is given by,

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2} \vec{a}_r \quad \text{——— (1)}$$

* Consider a unit charge which is placed at a point B which is at a radial distance of r_B from the origin.

* It is moved against the direction of $\vec{E}$ from point B to point A. The point A is at a radial distance of r_A from the origin.

* The potential difference V_{AB} between points A & B is given by

$$V = -\int_a^b \vec{E} \cdot dl \qquad dl = \vec{a_R} \cdot dr$$

$$V = -\int_{ra}^{rb} \frac{Q}{4\pi\varepsilon_0 r^2} \vec{a_R}\, dr \cdot \vec{a_R} \qquad \vec{a_R} \cdot \vec{a_R} = 1$$

$$= -\frac{Q}{4\pi\varepsilon_0} \int_{ra}^{rb} \frac{1}{R^2}\, dr$$

$$V = -\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{R}\right]_{r_a}^{r_b}$$

$$= -\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r_b} + \frac{1}{r_a}\right]$$

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_b} - \frac{1}{r_a}\right]$$

$r_b = \infty$

$$\boxed{V_{ab} = \frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r_a}\right]}$$ → Absolute potential at 'a'.

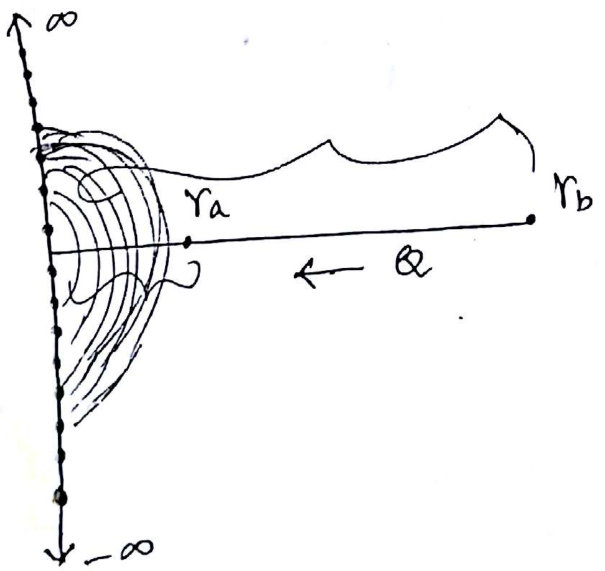
* IIIly Absolute potential at 'b'

$r_a = \infty$

$$\boxed{V_{ab} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_b}\right]}$$

Potential due to infinite line charge

B - 4-23



* The $\vec{E}$ due to infinite line charge along z-axis is

$$\vec{E} = \frac{\rho_L}{2\pi \epsilon_0 r} \vec{a_r} \quad \text{——— (1)}$$

$$V = -\int_a^b \vec{E} \cdot dl$$

$\vec{E}$ due to infinite line charges

$$V = -\int_{r_a}^{r_b} \frac{\rho_l}{2\pi \epsilon_0 r} \vec{a_R}\, dr\, \vec{a_R}$$

$$V = -\frac{\rho_l}{2\pi \epsilon_0} \int_{r_a}^{r_b} \frac{1}{r}\, dr$$

$$= -\frac{\rho_l}{2\pi \epsilon_0} \left[\ln r \right]_{r_a}^{r_b} \qquad \int \frac{1}{r} dr = \ln r$$

$$V = -\frac{\rho_l}{2\pi \epsilon_0} \left[\ln r_b - \ln r_a \right]$$

$$\boxed{V = -\frac{\rho_l}{2\pi \epsilon_0} \left[\ln \left(\frac{r_b}{r_a} \right) \right]}$$

Problem: 1

Find the field intensity and potential at a distance 100 mm from a positive point charge of 10nC.

$$E = \frac{Q}{4\pi \epsilon_0 r^2} = \frac{10 \times 10^{-9}}{4\pi \frac{1}{36\pi \times 10^9} \times (0.1)^2} = 9\, kV/m$$

$$V = \frac{Q}{4\pi \epsilon_0 r} = \frac{10 \times 10^{-9}}{4\pi \frac{1}{36\pi \times 10^9} \times (0.1)} = 900\, V$$

ENERGY DENSITY

* Consider a elementary cube of side Δd parallel to the plates of a Capacitor as shown in fig.

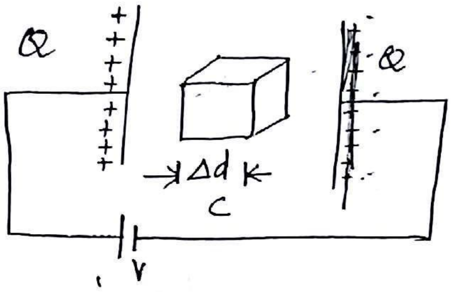


Fig. Energy Storage in Capacitor.

$$C = \frac{\epsilon A}{d}.$$

$$\Delta C = \frac{\epsilon A}{\Delta d}.$$

The capacitance of elemental capacitor is

$$\Delta C = \frac{\epsilon A}{\Delta d}$$

$$\therefore \boxed{A = \Delta d^2}$$

$$\Delta C = \frac{\epsilon (\Delta d)^2}{\Delta d}$$

$$\boxed{\Delta C = \epsilon \Delta d} \quad \text{——— (1)}$$

Energy stored in the elemental capacitor is W.K.T $W_E = \frac{1}{2} CV^2$

$$\Delta W = \frac{1}{2} \Delta C (\Delta V)^2 \quad \text{——— (2)}$$

But potential difference across the elementary cube is W.K.T $V = Ed$

$$\boxed{\Delta V = E \cdot \Delta d} \quad \text{——— (3)}$$

where $E \rightarrow$ Electric field exist in the cube.

Sub (1) & (3) in (2)

$$\Delta W = \frac{1}{2} (\epsilon \Delta d)(E \Delta d)^2$$

$$= \frac{1}{2} \epsilon E^2 (\Delta d)^3$$

From Eqn (4) $\boxed{\Delta W = \frac{1}{2} \epsilon E^2 \Delta V} \Rightarrow (4)$

$\Delta V = \Delta d^3$ is elementary Volume

The Energy density is given by

$$\frac{\Delta W}{\Delta V} = \frac{1}{2} \epsilon E^2$$

W.K.T $(\because D = \epsilon E)$ $\quad \epsilon = \frac{D}{E}$

$$\boxed{\frac{\Delta W}{\Delta V} = \frac{1}{2} DE \text{ Joules}/m^3}$$

$$\frac{1}{2} \boxed{\epsilon \cdot E} E$$

$$\boxed{\frac{\Delta W}{\Delta V} = \frac{1}{2} DE}$$

UNIT – II

MAGNETIC FIELD

Basic concepts

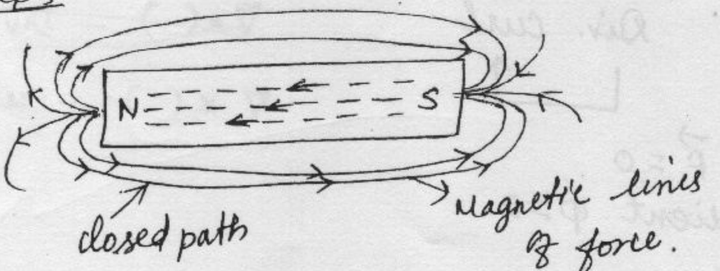


Fig. Permanet magnet

* The region around a permanent magnet or the space around a current carrying conductor which is occupied by magnetic lines of force is called magnetic field.

Magnetic flux lines and their properties

* These are purely imaginary lines used to represent the magnetic field and to facilitate our understanding of magnetic field

1. they are closed lines. they always start from north pole and end at south pole outside the magnet. Inside the magnet they start from south pole and end at north pole. The path followed by the magnetic flux line is called magnetic circuit.
2. They are non- intersecting lines
3. The number of lines passing across unit cross sectional area is proportional to the magnitude of current in the conductor.

Scalar & Vector Magnetic potential

The scalar & vector magnetic potential used to find the current density of a conductor or current density in a particular region.

Two rules of Emf*

Gradients

Div. curl

Div. curl $\vec{F} = 0$

curl. Gradient $\phi = 0$

$\vec{F}$ = vector

ϕ = scalar.

$$\boxed{\vec{E} = -\nabla V}$$
$$\boxed{\vec{H} = -\nabla V_m}$$

Scalar Magnetic Potential (U)

Curl of gradient of a scalar is equal is equal to Zero.

$V_m \rightarrow$ Scalar Magnetic potential

According to the rule $\boxed{\nabla \times \nabla (V_m) = 0} \longrightarrow (1)$

$$\nabla \times \vec{H} = \vec{J} \quad \text{——} (2)$$

$(1) \rightarrow (2)$

$$\vec{H} = \nabla . V_m$$

$$\boxed{\vec{J} = 0}$$

* A scalar magnetic potential can be defined for source free region where current density is equal to zero.

* So scalar magnetic potential is applicable only in permanent magnet since $\vec{J} = 0$ & $I = 0$.

Vector Magnetic potential ($\vec{A}$)

$\boxed{\text{Divergence of curl of a vector} = 0}$

$$\nabla . (\nabla \times \vec{A}) = 0 \quad \text{——} (1)$$

$$\nabla \cdot \vec{B} = 0 \quad \text{—— ②}$$

$$\boxed{\vec{B} = \nabla \times \vec{A}}$$

According to rule,

$$\nabla \times \vec{H} = \vec{J}$$

$$\vec{B} = \mu \vec{H}$$

$$\vec{H} = \frac{\vec{B}}{\mu}$$

$$\nabla \times \frac{\vec{B}}{\mu} = \vec{J}$$

$$\nabla \times \vec{B} = \mu \vec{J} \quad \text{—— ③}$$

$$\nabla \times |\nabla \times \vec{A}| = \mu \vec{J}$$

$$\nabla \times \nabla \times \vec{A} = \mu \vec{J}$$

$$\nabla \times \nabla \times \vec{A} = \begin{vmatrix} \nabla \cdot & \nabla^2 \\ A & \nabla \cdot A \end{vmatrix}$$

$$= \nabla (\nabla \cdot \vec{A}) - \nabla^2 A$$

$$\nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = \mu \vec{J}$$

$$\boxed{\vec{J} = \frac{\nabla (\nabla \cdot \vec{A}) - \nabla^2 A}{\mu}}$$

MAGNETIC FORCE ON A MOVING CHARGE [force on a current element]

* A charged particle in motion in a magnetic field of flux density `B' is experienced a force.

* The force is proportional to the product of the magnitude of the charge Q, its velocity v and flux density B, & to the sine of the angle between v and B.

* The direction of the force is perpendicular to both v and B.

$$F = Q(\vec{V} \times \vec{B})$$

$$F = QvB\sin\theta$$

* The electrical force on a charged particle in electric field of intensity E is

$$F = QE$$

* The force on a moving particle due to combined electric and magnetic field is obtained

$$F = Q\underset{\rightarrow}{E} + Q(\underset{\rightarrow}{v} \times \underset{\rightarrow}{B})$$

$$F = Q\left[\vec{E} + \vec{V} \times \vec{B}\right]$$

This force is called <u>Lorentz force.</u>

* The force on current element is

$$dF = dQ(v \times B)$$

$$= dQ\left(\frac{dl}{dt} \times B\right) \qquad \left[\because v = \frac{dl}{dt} \text{ velocity}\right]$$

$$= \frac{dQ}{dt}(dl \times B) \qquad \left[\because I = \frac{dQ}{dt} \text{ current}\right]$$

$$dF = I\,dl \times B$$

$$F = (I \times B)\,l$$

This is also called Lorentz force.

* The magnitude of the force is

$$F = BIl\sin\theta$$

BIOT- SAVART LAW

* Biot - Savart law states that the magnetic field intensity (H) produced at a point P due to differential current element (Idl) is

1. Proportional to the product of current and differential length

$$dH \propto Idl$$

2. It is proportional to angle between the element and the line joining the point P to the element.

$$dH \propto \sin\theta$$

3. It is inversely proportional to square of distance between the point P and element

$$dH \propto \frac{1}{R^2}$$

$$dH \propto \frac{Idl \sin\theta}{R^2}$$

$$dH = \frac{K\, Idl \sin\theta}{R^2}$$

$$K = \frac{1}{4\pi}$$

$$\vec{dH} = \frac{I\,\vec{dl} \sin\theta\, \vec{a_\phi}}{4\pi R^2}$$

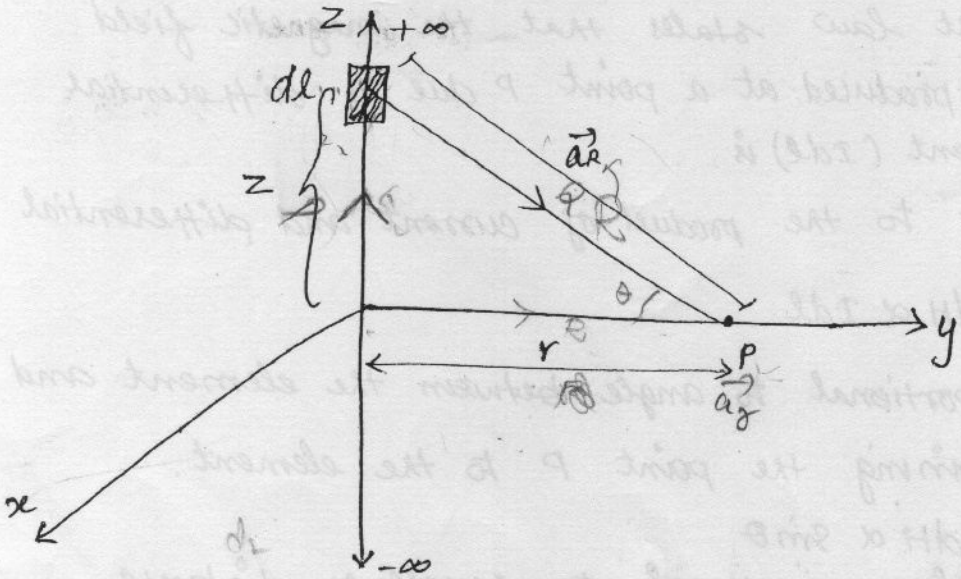
$$d\vec{H} = \frac{I\,d\vec{l}\, \vec{a_\phi} \sin\theta}{4\pi R^2}$$

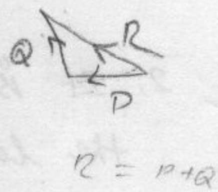
$$d\vec{H} = \frac{I\,d\vec{l} \times \vec{a_\phi}}{4\pi R^2}$$

$$\boxed{\vec{H} = \int \frac{I\,d\vec{l} \times \vec{a_\phi}}{4\pi R^2}}$$

$\vec{H}$ due to infinite long straight conductor

(II.6)





* Consider an infinitely long straight conductor, along Z-axis. The current passing through the conductor is 'I' ampere.
* The field intensity $\vec{H}$ at a point P is to be calculated, which is at a distance 'r' from the Z-axis.
* Consider small differential element (dl), along the Z-axis, at a distance z from origin.

$$\vec{H} = \int \frac{I\,d\vec{l} \times \vec{a_\phi}}{4\pi R^2}$$

$$= \int \frac{I\,d\vec{l} \times \vec{a_R}}{4\pi R^2}$$

$$d\vec{l} = dz \cdot \vec{a_z}$$

$$\vec{a_R} = \frac{r\vec{a_r} - z\vec{a_z}}{\sqrt{r^2+z^2}}$$

Here taking circular Co-ordinates

$$d\vec{l} \times \vec{a_R} = \begin{vmatrix} \vec{a_r} & \vec{a_\phi} & \vec{a_z} \\ 0 & 0 & dz \\ \frac{r}{\sqrt{r^2+z^2}} & 0 & \frac{-z}{\sqrt{r^2+z^2}} \end{vmatrix}$$

~~$dl \times a_R$ ... OR~~

$dz . \vec{a_z}$

$\vec{a_r} = \vec{a_l} + \vec{a_R}$

$\vec{a_R} = \vec{a_r} - \vec{a_l}$

$|\vec{a_R}|\,\hat{a}_R = r\hat{a}_r - z\hat{a}_z$

$$\hat{a}_R = \frac{r\hat{a}_r - z\hat{a}_z}{\sqrt{r^2+z^2}}$$

$$= -\vec{a\phi}\left[0 - dz\left(\frac{r}{\sqrt{r^2+z^2}}\right)\right]$$

$$= \vec{a\phi}\left[dz\left(\frac{r}{\sqrt{r^2+z^2}}\right)\right]$$

$$\vec{dl} \times \vec{a\phi} = \frac{r\,dz}{\sqrt{r^2+z^2}}\ \vec{a\phi}$$

$$R^2 = r^2+z^2$$

Limits are $-\infty$ to ∞

$$\vec{H} = \int_{-\infty}^{\infty} \frac{I.\ r\,dz}{4\pi\left[r^2+z^2\right]\sqrt{r^2+z^2}}\ \vec{a\phi}$$

$$= \int_{-\infty}^{\infty} \frac{I r\,dz}{4\pi\left[r^2+z^2\right]^{3/2}}\ \vec{a\phi}$$

$$z = r\tan\theta$$

$$dz = r\sec^2\theta\,d\theta$$

$$\theta = \tan^{-1}\left(\frac{z}{r}\right)$$

Limit $z = \infty,\ \theta = \pi/2$

$z = -\infty,\ \theta = -\pi/2$

$$\vec{H} = \int_{-\pi/2}^{\pi/2} \frac{I r.r.\sec^2\theta\,d\theta}{4\pi\left[r^2+r^2\tan^2\theta\right]^{3/2}}\ a\phi$$

$$= \int_{-\pi/2}^{\pi/2} \frac{I r^2\sec^2\theta\,d\theta}{4\pi r^3\left[1+\tan^2\theta\right]^{3/2}}\ a\phi$$

$$= \frac{I}{4\pi r}\int_{-\pi/2}^{\pi/2} \frac{\sec^2\theta}{(\sec^2\theta)^{3/2}}\,d\theta\ a\phi$$

$$\vec{H} = \frac{I}{4\pi r}\int_{-\pi/2}^{\pi/2} \frac{1}{\sec\theta}\,d\theta\ a\phi$$

$$= \frac{I}{4\pi r} \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta \, \vec{a\phi}$$

$$= \frac{I}{4\pi r} \left[\sin\theta \right]_{-\pi/2}^{\pi/2} \vec{a\phi}$$

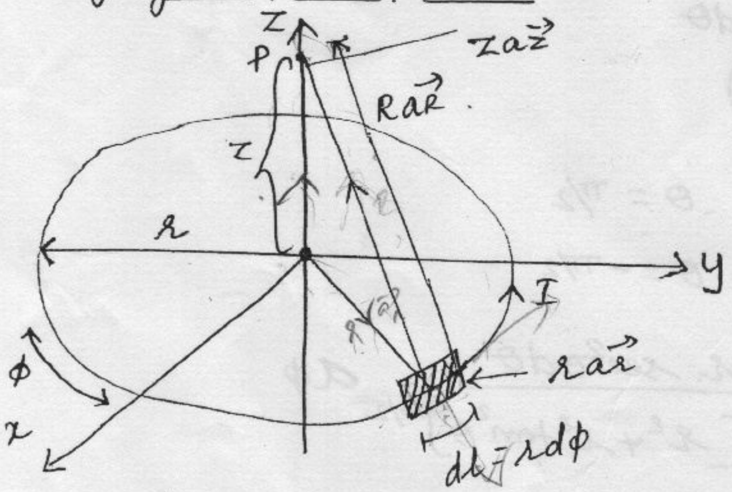
$$= \frac{I}{4\pi r} [1+1] . \vec{a\phi}$$

$$= \frac{2I}{4\pi r} \vec{a\phi}$$

$$\boxed{\vec{H} = \frac{I \vec{a\phi}}{2\pi r}}$$

$$B = \frac{\mu_0 I}{2\pi a} \hat{u}_r$$

$\vec{H}$ due to circular ring conductor (or) $\vec{H}$ due to current carrying circular loop



$\vec{a_r} + \vec{a_R} = \vec{a_z}$

* Consider a circular loop carrying a current I amp, placed in xy plane, with z axis as its axis as shown in fig.
* The magnetic field intensity $\vec{H}$ at point P is to be obtained. The point P is at a distance z from the plane of the circular loop, along its axis.

* The radius of the circular loop is r. Consider the differential length $d\vec{L}$ of the circular loop as shown in the fig.

$$\vec{H} = \int \frac{I\, d\vec{l} \times \vec{a_R}}{4\pi R^2}$$

$$\vec{a_R} = \frac{z\vec{a_z} - r\vec{a_r}}{\sqrt{z^2+r^2}}$$

$$d\vec{l} = r\, d\phi\, \vec{a_\phi}$$

$$d\vec{l} \times \vec{a_R} = \begin{vmatrix} \vec{a_r} & \vec{a_\phi} & \vec{a_z} \\ 0 & r\,d\phi & 0 \\ \frac{-r}{\sqrt{z^2+r^2}} & 0 & \frac{z}{\sqrt{z^2+r^2}} \end{vmatrix}$$

$$= \vec{a_r}\left[(r\,d\phi)\left(\frac{z}{\sqrt{z^2+r^2}}\right)\right] + \vec{a_z}\left[\left(\frac{r}{\sqrt{z^2+r^2}}\right)(r\,d\phi)\right]$$

$$d\vec{l} \times \vec{a_R} = \cancel{\frac{r\,d\phi\, z\, \vec{a_r}}{\sqrt{z^2+r^2}}}_{\,0} + \frac{\vec{a_z}\, r^2\, d\phi}{\sqrt{z^2+r^2}}$$

Here radial component is zero due to magnetic field intensity in radial component is vanish.

$$d\vec{l} \times \vec{a_R} = \frac{\vec{a_z} \cdot r^2\, d\phi}{\sqrt{r^2+z^2}}$$

Limits are 0 to 2π

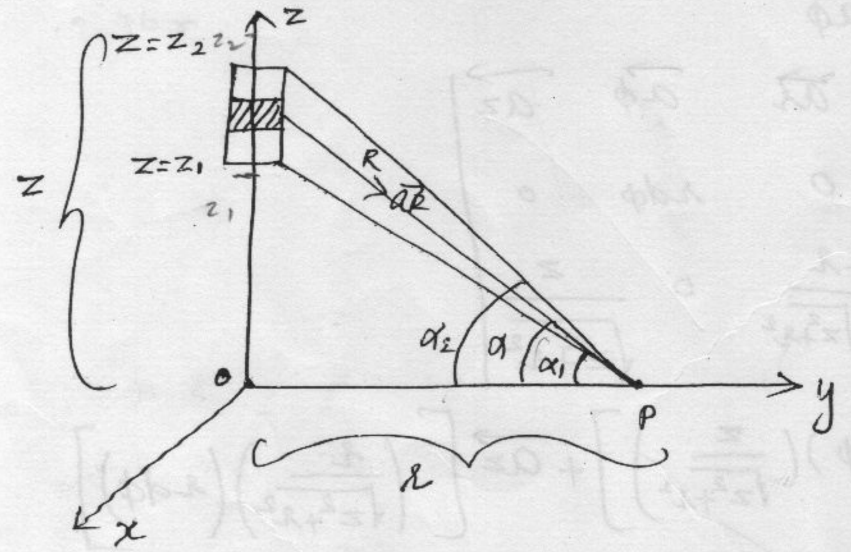
$$\vec{H} = \int_0^{2\pi} \frac{I r^2\, d\phi\, \vec{a_z}}{4\pi[r^2+z^2]\sqrt{r^2+z^2}} = \int_0^{2\pi} \frac{I r^2\, d\phi\, \vec{a_z}}{4\pi[r^2+z^2]^{3/2}}$$

$$= \frac{I r^2 \vec{a_z}}{4\pi[r^2+z^2]^{3/2}} \int_0^{2\pi} d\phi$$

$$= \frac{I r^2 \vec{a_z}}{4\pi [r^2+z^2]^{3/2}} [2\pi]$$

$$\boxed{\vec{H} = \frac{I r^2 \vec{a_z}}{2 [r^2+z^2]^{3/2}}}$$

$\vec{H}$ due to finite length of current carrying conductor :



* consider a conductor of finite length placed in z-axis it carries a direct current I the perpendicular distance of point P from the z-axis is r.
* The conductor is placed such that one end is at $z=z_1$ while the other end is $z=z_2$.
* Consider a differential element dl along z-axis at distance of z from the origin

$$\vec{H} = \int \frac{I(\vec{dl} \times \vec{a_R})}{4\pi R^2} \qquad R = \cancel{r+z}$$

R = Pythagoras theorem.

$$\vec{dl} = dz \vec{a_z}$$

$$\vec{a_R} = \frac{r\vec{a_r} - Z\vec{a_z}}{\sqrt{r^2+z^2}}$$

component of z axis is zero.

$$\vec{a_R} = \frac{r\vec{a_r}}{\sqrt{r^2+z^2}}$$

$$dl \times \vec{a_R} = \begin{vmatrix} \vec{a_r} & \vec{a_\phi} & \vec{a_z} \\ 0 & 0 & dz \\ \frac{r}{\sqrt{r^2+z^2}} & 0 & \frac{-z}{\sqrt{r^2+z^2}} \end{vmatrix}$$

$$= \frac{r\,dz\,a\phi}{\sqrt{r^2+z^2}}$$

Limits z_1 to z_2

$$z \to z_1 \qquad \theta \to \alpha_1$$

$$z \to z_2 \qquad \theta \to \alpha_2$$

$$\vec{H} = \int_{z_1}^{z_2} \frac{I\,r\,dz\,\vec{a\phi}}{4\pi\sqrt{r^2+z^2}\,[r^2+z^2]}$$

$$= \frac{I}{4\pi}\int_{z_1}^{z_2} \frac{r\,dz\,a\phi}{[r^2+z^2]^{3/2}}$$

$$z = r\tan\theta \qquad \theta = \tan^{-1}(z/r)$$

$$dz = r\sec^2\theta\,d\theta$$

$$z = z_1, \quad \theta = \alpha_1$$

$$z = z_2, \quad \theta = \alpha_2$$

$$\vec{H} = \frac{I\,a\phi}{4\pi}\int_{\alpha_1}^{\alpha_2} \frac{r^2\sec^2\theta\,d\theta}{r^3(\sec^3\theta)}$$

$$r^2 + r^2\tan^2\theta = r^2(1+\tan^2\theta)^{3/2} = r^2(\sec^2)^{3/2} = r^3\sec^3\theta.$$

$$= \frac{I\,a\phi}{4\pi r}\int_{\alpha_1}^{\alpha_2} \frac{\sec^2\theta}{\sec^3\theta}\,d\theta$$

$$\vec{H} = \frac{I a_\phi}{4\pi r} \int_{\alpha_1}^{\alpha_2} \cos\theta \, d\theta$$

$$= \frac{I a_\phi}{4\pi r} \left[\sin\theta \right]_{\alpha_1}^{\alpha_2}$$

$$\boxed{\vec{H} = \frac{I a_\phi}{4\pi r} \left[\sin\alpha_2 - \sin\alpha_1 \right]}$$

---

* Magnetic flux density due to circular ring (or) circular loop current carrying conductors is

$$\vec{B} = \mu \vec{H}$$

$$\boxed{\vec{B} = \frac{\mu I r^2 \vec{a_z}}{2\left[r^2+z^2\right]^{3/2}}}$$

* Magnetic flux density due to infinite straight conductor is

$$\boxed{B = \frac{\mu I a_\phi}{2\pi r}}$$

Problem 1

A circular coil of radius 2m carries a current of 4A what is the value of magnetic field intensity at the centre?

Given

$I = 4A$, $a = 2m$

$$H = \frac{I}{2a}$$

$$H = \frac{4}{2 \times 2} = 1 \text{ A/m}$$

MAGNETIC FLUX DENSITY (B)

* Magnetic flux density is defined as the magnetic flux passing per unit area. Its unit is weber/metre^2 (or) Tesla

$$B = \frac{\phi}{A} \text{ weber/metre}^2$$

Also defined as

$$B = \mu H$$

$\mu \rightarrow$ permeability of medium

$\mu = \mu_0 \mu_r$ (H/m)

$\mu_0 = 4\pi \times 10^{-7}$ (H/m) (permeability of free space)

μ_r = relative permeability

$H \rightarrow$ Magnetic field intensity (A/m)

Magnetic flux (ϕ)

Magnetic flux is defined as the flux ϕ passing through any area.

$$\phi = \int_S \vec{B}\, ds$$

In a particular surface the incoming flux is equal to the outgoing magnetic flux

$$\phi = 0$$

$$\boxed{\int_S \vec{B}\, ds = 0} \rightarrow \text{integral form of Gauss Law}$$

By applying divergence theorem

$$\iint_S B \cdot ds = \nabla \cdot B$$

then, $\nabla \cdot B = 0$

The total magnetic flux passing through any closed surface is equal to zero.

Magnetic field intensity ($\vec{H}$)

* The quantitative measure of strongness or weakness of the magnetic field is given by magnetic field intensity or Magnetic field strength.
* Magnetic field intensity is defined as force experienced by unit north pole.
* The magnetic field intensity H can be made independent of the medium & is defined by equn.

$$H = \frac{B}{\mu}$$

$$B = \mu H$$

Problem 1

A round copper conductor is carrying a current of 250A. Determine the magnetising force and flux density at a distance of 10 cm from the conductor.

sol.

$I = 250A$, $r = 10\,cm = 10 \times 10^{-2}\,m = 0.1\,m$

$$B = \frac{\mu_0 N I}{2\pi r} \quad (N=1)$$

$$B = \frac{4\pi \times 10^{-7} \times 1 \times 250}{2 \times \pi \times 0.1}$$

$$B = 0.5 \times 10^{-3}\ Wb/m^2$$

$$B = 0.5\ m\,Wb/m^2$$

Magnetic force $H = \frac{B}{\mu_0}$

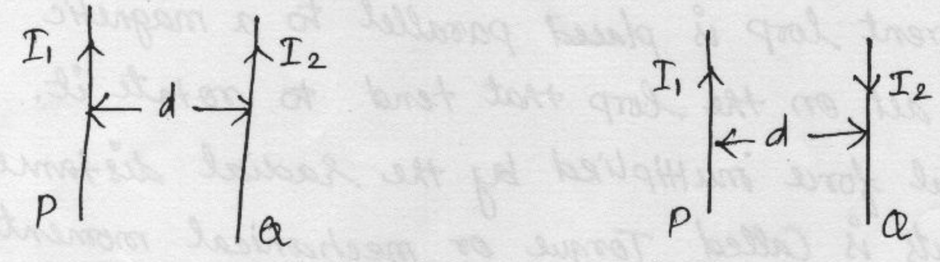
$$H = \frac{0.5 \times 10^{-3}}{4 \times \pi \times 10^{-7}}$$

$$= 397.88\ AT/m$$

Force between current carrying conductors

* Consider two straight, long parallel conductors P and Q separated by a distance 'd'.

* Let I_1 and I_2 be the currents flowing in conductors P and Q respectively



a) Force of attraction b) Force of repulsion

Fig. Parallel Conductors

* Consider a conductor P produces a magnetic field whose flux density is B at conductor Q

$$B = \frac{\mu_0 I_1}{2\pi d}$$

The force on conductor Q due to P

$$F = B I_2 l$$

where l is the length of the conductor

$$F = \frac{\mu_0 I_1}{2\pi d} I_2 l$$

* If currents of conductors P and Q are flowing in same direction, there is a force of attraction.

* If currents are flowing in opposite direction, there is a force of repulsion. However the value of force is same.

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi d} \text{ N}$$

* If the conductors are infinitely long the force per unit length is

$$F = \frac{\mu_0 I_1 I_2}{2\pi d} \ N/m$$

Torque on closed conductors: (Magnetic torque)

* When a current loop is placed parallel to a magnetic field, forces act on the loop that tend to rotate it. The tangential force multiplied by the radial distance at which it acts is called Torque or mechanical moment on the loop.

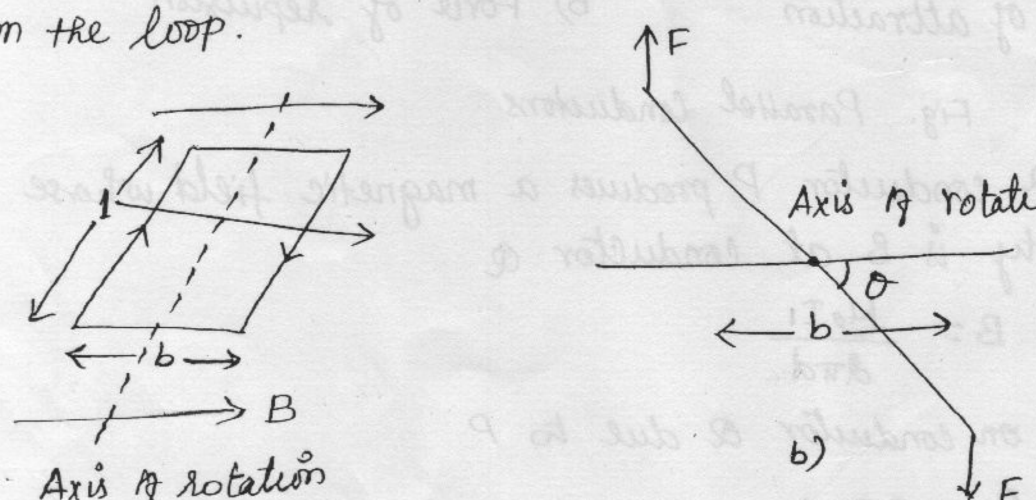


Fig. Current loop.

* Consider the rectangular loop of length 'l' and breadth 'b' carrying a current I in a uniform magnetic field of flux density B as shown in fig.)
* The loop makes an angle θ with respect to magnetic flux density B.

The force acting on the loop

$$F = BIl \sin\theta$$

* If the loop plane is parallel to the magnetic field, the total torque on the loop

$$T = 2 \times \text{torque on each side}$$

$T = 2 \times$ force $\times$ distance

$= 2 BIl \sin\theta \frac{b}{2}$

$= BIlb \sin\theta$

$= BIA \sin\theta$ (Area $A = lb$)

The magnetic moment of loop is IA

* The magnetic moment is a vector with the direction given by the unit normal $\hat{n}$ to the plane of the loop

$$m = IA\hat{n}$$

$$T = mB\sin\theta\hat{n} \qquad dT = BI\,dA\sin\theta\hat{n}$$

* In vector form, Torque can be represented as

$$\vec{T} = \vec{m} \times \vec{B}$$

$$\boxed{m = \frac{T}{B}}$$

* The magnetic moment is defined as the maximum torque on loop per unit magnetic induction.

Problem 1

1. Find the maximum torque on an 100 turn rectangular coil, 0.2 by 0.3m, carrying a current of 2A in the field of flux density 5 web/m^2.

Given

$N = 100$

$A = 0.2 \times 0.3 = 0.06\,m^2$

$I = 2A$

$B = 5$ web/m^2

$T_{MAX} = NIAB$

$= 100 \times 2 \times 0.06 \times 5$

$= 60$ Newton-meter

---

2. Two wires carrying current in the same direction of 3A and 6A are placed with their axes 5 cm apart, free space permeability = $4\pi \times 10^{-7}$ H/m. Calculate the force between them in kg/m length. (II.18)

Sol. P-98 B

Force between two parallel conductors is given by

$$F = \frac{\mu I_1 I_2 l}{2\pi d}$$

d = distance of separation = 5 cm

$= 5 \times 10^{-2}$ m

$I_1 = 3A$,

$I_2 = 6A$

l = Length of conductors

Hence force per unit meter length is given by,

$$\frac{F}{l} = \frac{\mu I_1 I_2}{2\pi d} = \frac{\mu_0 \mu_r I_1 I_2}{2\pi d}$$

For free space $\mu_r = 1$

$$\frac{F}{l} = \frac{4 \times \pi \times 10^{-7} \times 3 \times 6}{2 \times \pi \times 5 \times 10^{-2}}$$

$= 7.2 \times 10^{-5}$ N/m

$\frac{F}{l} = 72\ \mu$ N/m

The force expressed in kg/m is given by

$$\frac{F}{l} = \frac{72 \times 10^{-6} \text{ N/m}}{9.8}$$

$= 7.3469 \times 10^{-6}$ kg/m

Ampere's Law

Ampere's law states that line integral of magnetic field intensity around a closed path is exactly equal to the current enclosed by the path.

$$\boxed{\oint \vec{H}\, dl = I}$$

Another form of Ampere's Law

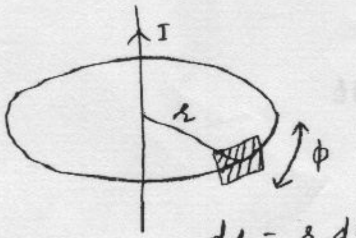
divided by Δs

$$\oint \frac{\vec{H}\, dl}{\Delta s} = \frac{I}{\Delta s}$$

$$\boxed{\text{curl } \vec{H} = J}$$

APPLICATIONS OF AMPERES LAW

* Ampere's law used to finding the magnetic field intensity due to current carrying conductor



$$dl = r\, d\phi .\, a\phi$$
$$\vec{H} = \vec{H} .\, a\phi$$

* Consider a direct current going through a conductor is I, which is inducing magnetic field intensity around the conductor.

$$\oint \vec{H} . dl = I$$

$$\int_0^{2\pi} \vec{H} . \vec{a\phi} . r . d\phi . \vec{a\phi} = I$$

$$\int_0^{2\pi} \vec{H} . r . d\phi = I$$

$$\vec{H} \int_0^{2\pi} d\phi = \frac{I}{r}$$

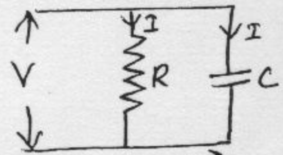
$$\vec{H}\, 2\pi = I / r$$

$$\boxed{\vec{H} = \frac{I}{2\pi r}}$$

MODIFIED AMPERE'S LAW

* Amperes law deals with conductor
* The modified amperes law deals with both conductor & Dielectric.

Note



$$\nabla \times \vec{H} = \vec{J}$$

$$\vec{J} = \vec{J}_{con} + \vec{J}_{dis}$$

$$\vec{J}_{con} = \sigma \vec{E}, \quad \vec{J}_{dis} = \varepsilon \frac{d\vec{E}}{dt}$$

$$\boxed{\vec{J} = \sigma \vec{E} + \varepsilon \frac{d\vec{E}}{dt}}$$

$$\nabla \times \vec{H} = \sigma \vec{E} + \varepsilon \frac{d\vec{E}}{dt}$$

$$\boxed{\vec{E} = E_0 e^{j\omega t}}$$

$$\nabla \times \vec{H} = \sigma \vec{E} + \frac{\varepsilon \, dE_0 e^{j\omega t}}{dt}$$

time varying $\vec{E}$

$$\vec{E} = \vec{E}_0 e^{j\omega t}$$

$$\nabla \times \vec{H} = \sigma \vec{E} + \varepsilon j\omega E_0 e^{j\omega t}$$

$$\boxed{\nabla \times \vec{H} = \sigma \vec{E} + j\omega \varepsilon \vec{E}}$$

$$\nabla \times \vec{H} = \vec{J}_{con} + \vec{J}_{dis}$$

$$\nabla \times \vec{H} = \sigma \vec{E} + j\omega \varepsilon \vec{E}$$

$$J_c = \sigma \vec{E}$$

$$J_d = \omega \varepsilon \vec{E}$$

$$\frac{J_c}{J_d} = \frac{\sigma \vec{E}}{\omega \varepsilon \vec{E}}$$

$J_c >> J_d \Rightarrow \frac{J_c}{J_d} >> 1 \rightarrow$ conductor

$J_c <<< J_d \Rightarrow \frac{J_c}{J_d} << 1 \rightarrow$ Dielectric

Helmholtz's theorem

$F = ma$, $1 N = (1)\, 9.8\ kg$

A vector A is uniquely prescribed within a region by its divergence and its curl.

let
$$\nabla \cdot A = \rho_v \qquad \text{(A)}$$
$$\nabla \times A = \rho_s \qquad \text{(B)}$$

where, $\rho_v \Rightarrow$ source density of A

$\rho_s \Rightarrow$ circulation density of A

* Any vector A satisfying equation (A) & (B) with both ρ_v and ρ_s vanishing at infinity can be written as the sum of two vectors: one irrotational (zero curl), the other solenoidal (zero divergence). This is called Helmholtz's theorem.

$$A = -\nabla V + \nabla \times B$$

Let $A_i = -\nabla V$

$A_s = \nabla \times B$

where $A_i \rightarrow$ irrotational field

$A_s \rightarrow$ solenoidal field

From the result of $\boxed{\nabla^2 A = \nabla(\nabla \cdot A) - \nabla \times \nabla \times A}$

$\Downarrow$

Gradient of Divergence of A minus curl of curl of A

$$\boxed{\nabla^2 A = \nabla \rho_v - \nabla \times \rho_s}$$

Two Marks (unit-2)

1. Define magnetic vector potential.

It is defined as that quantity whose curl gives the magnetic flux density.

$$B = \nabla \times A$$

where A is the magnetic vector potential

$$A = \frac{\mu}{4\pi} \iiint_v \frac{J}{r} \, dr \text{ web/m}$$

2. Define Biot-Savart Law?

the magnetic field intensity (H) produced at a point P due to differential current element (Idl) is

1. Proportional to the product of current and differential length

$$dH \propto Idl$$

2. It is proportional to angle between the element and the line joining the point P to the element

$$dH \propto \sin\theta$$

3. It is inversely proportional to square of the distance between the point P and element

$$dH \propto \frac{1}{R^2}$$

5. Give the Magnetic field intensity due to a finite and infinite wire carrying a current.

For infinite wire.

$$\vec{H} = \frac{I\vec{a_\phi}}{2\pi r}$$

For finite wire.

$$\vec{H} = \frac{I a_\phi}{4\pi r}\left[\sin\alpha_2 - \sin\alpha_1\right]$$

4) Define Magnetic flux density.

Magnetic flux density is defined as the magnetic flux passing per unit area. Its unit is weber/meter^2 (or) Tesla. Magnetic flux through any ~~closed~~ surface is surface integral of normal component B, $\phi = \int_S B_n \, da = \int_S B \cdot da$.

$$B = \frac{\phi}{A} \text{ weber/meter}^2$$

5) what is the difference between scalar and vector magnetic potential.

→ Magnetic scalar potential is a quantity whose negative gives the magnetic intensity

→ Magnetic vector potential is a quantity whose curl gives the magnetic flux density.

6) Give Lorentz force equation

$$F = Q[\vec{E} + \vec{V} * \vec{B}]$$

where

F = Lorentz force

Q = Charge of the moving charge

v = velocity of charge

B = Magnetic field

E = Electric field intensity

7) State Gauss law for Magnetic field

→ The total magnetic flux passing through any closed surface is equal to zero.

8. State Ampere's law

Ampere's law states that line integral of magnetic field intensity around a closed path is exactly equal to the current enclosed by the path.

$$\boxed{\oint \vec{H} \, dl = I}$$

9. Find the maximum torque on a 100 turn rectangular coil 0.2 m by 0.3 m, carrying a current of 2 A in the field of flux density 5 Wb/m^2?

Sol.

Given

$N = 100$

$A = 0.2 \times 0.3 = 0.06\,m^2$

$B = 5\ Wb/m^2$

$T_{max} = NIAB$

$= 100 \times 2 \times 0.06 \times 5$

$$\boxed{T_{max} = 60\ N\text{-}m}$$

10. Give the force on a current element

The force on a current element Idl is given by

$dF = I \times B\,dl$

$= BI\,dl\sin\theta$ Newton.

11. Define Magnetic dipole

A small bar magnetic with pole strength Q_m and length l may be treated as magnetic dipole whose magnetic moment is $Q_m l$.

12. State Ampere's circuital law.

The ampere's circuital law states that the line integral of magnetic field intensity $\vec{H}$ around a closed path is exactly equal to the direct current enclosed by that path.

The mathematical representation of Ampere's circuital law is,

$$\boxed{\oint \vec{H} \cdot d\vec{L} = I}$$

14/08/17

UNIT-4

ELECTROMAGNETIC INDUCTION

Faraday's law of electromagnetic induction :

The total electromagnetic force (emf) induced in a circuit is equal to rate of decrease of total magnetic flux in a circuit.

$$v = -\frac{\partial \phi}{\partial t} \rightarrow ①$$

considering a coil with n number of turns then it is given by

$$v = -N\frac{\partial \phi}{\partial t} - ②$$

For an electric field intensity the emf in a circuit is the line integral of electric field around a closed path

$$v = \oint E \cdot dl - ③$$

According to Gauss law the total flux passing through the surface is equal to the surface integral of magnetic flux density enclosed in it

$$\phi = \iint B \cdot ds - ④$$

sub ④ in ①

$$v = -\frac{\partial}{\partial t} \iint B \cdot ds$$

$$v = -\iint \frac{\partial B}{\partial t}\, ds \quad -⑤$$

Equating equ ③ and ⑤

$$\oint E \cdot dl = -\iint \frac{\partial B}{\partial t}\, ds \quad -⑥$$

WKT stoke theorem states, the line integral of normal component of any vector field is equal to surface integral of curl of that particular vector field ∴

$$\oint E \cdot dl = \iint \nabla \times E\, ds \quad -⑦$$

compare equ ⑥ and ⑦

$$-\cancel{\iint} \frac{\partial B}{\partial t} \cancel{ds} = \cancel{\iint} \nabla \times E \cancel{ds}$$

$$\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$$

This equation is called maxwells equation

Self inductance:

It is defined as the property of the circuit by which changing current induces emf in a circuit to oppose changing current passing through it.

By Faraday's law the changing current will produce an ~~emf~~ induced emf in a circuit to oppose change in flux. This phenomenon is called as self inductance.

Considering a coil with n number of turns and current I passing through it.

The induced emf is proportional to the rate of change of current

$$V \propto \frac{di}{dt}$$

$$V = L\frac{di}{dt} \quad \text{—} ①$$

where L - self inductance of the coil

From Faraday's law

$$V = N\frac{d\phi}{dt} \quad \text{—} ②$$

Equating eqn ① and ②

$$L\frac{di}{\cancel{dt}} = N\frac{d\phi}{\cancel{dt}}$$

$$L = N\frac{d\phi}{di}$$

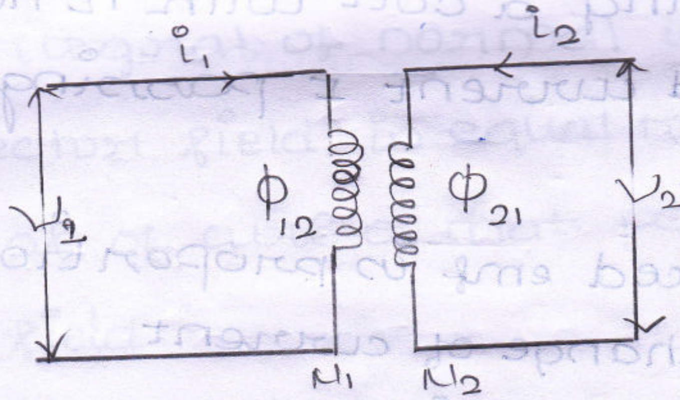
If it is constant

$$L = \frac{N\phi}{I}.$$

we conclude that the inductance is defined as the rate of magnetic flux linkage in a coil to the current passing through the coil.

18/08/17

Mutual inductance:



Mutual inductance is defined as rate of rate of magnetic flux linkage in one coil to the current passing through other coil

consider two coil where N_1 and N_2 are the number of turns in the coil. current i_1 passing through coil one which induce flux ϕ_{21} in other coil. Similarly current I_2 passing through coil two which induce flux ϕ_{12} in coil one.

WKT from inductance

$$V_2 \propto \frac{di_1}{dt} \quad -①$$

$$V_2 = M \frac{di_1}{dt} \quad \text{—②}$$

From Faraday's law

$$V_2 = \frac{N_2 d\phi_{12}}{dt} \quad \text{—③}$$

compare equation ② and ③

$$M \cancel{\frac{di_1}{dt}} = N_2 \frac{d\phi_{12}}{\cancel{dt}}$$

$$M = N_2 \frac{d\phi_{12}}{di_1}$$

(or)

$$M = N_2 \frac{\phi_{12}}{I_1}$$

Inductance of solenoid:

consider a solenoid of N number of turns carrying a current I.

B is the flux density and A is the cross section of solenoid.

Then flux linkage to the solenoid is $N\phi = NBA$

$$L = \frac{N\phi}{I} \quad \text{(From self inductance)}$$

$$L = \frac{NBA}{I} \quad \text{—①} \qquad \because \phi = BA$$

But for long solenoid

$$B = \frac{\mu_0 N I}{l} \quad -②$$

Sub the value of B

Sub ② in ①

$$L = \frac{NA}{\cancel{I}}\left(\frac{\mu_0 N \cancel{I}}{l}\right)$$

$$\boxed{L = \frac{\mu_0 N^2 A}{l}}$$

Inductance of Toroid:

consider a Toroid of N Number of turns carrying current I with mean radius

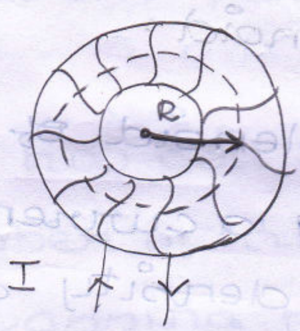
Hint:

① To find L

② $\phi = BA$

③ $B = \frac{\mu_0 N I}{l}$

④ Sub $l = 2\pi r$ and $A = \pi r^2$



If B is the flux density of the toroid then $\Rightarrow \phi = BA$

$$B = \frac{\mu_0 N I}{l}$$

where l is the mean length of coil

$$l = 2\pi R$$

$$B = \frac{\mu_0 N I}{2\pi R}$$

The flux linkage in the toroid is $N\phi$

$$N\phi = NBA$$

$$= \frac{NA\mu_0 N I}{2\pi R}$$

$$B = \frac{\mu_0 N^2 A I}{2\pi R}$$

where A - area of cross section.

if r is the Radius of coil

$$\boxed{A = \pi r^2}$$

$$N\phi = \frac{\mu_0 N^2 I \cancel{\pi} r^2}{2\cancel{\pi} R}$$

$$= \frac{\mu_0 N^2 r^2 I}{2R}$$

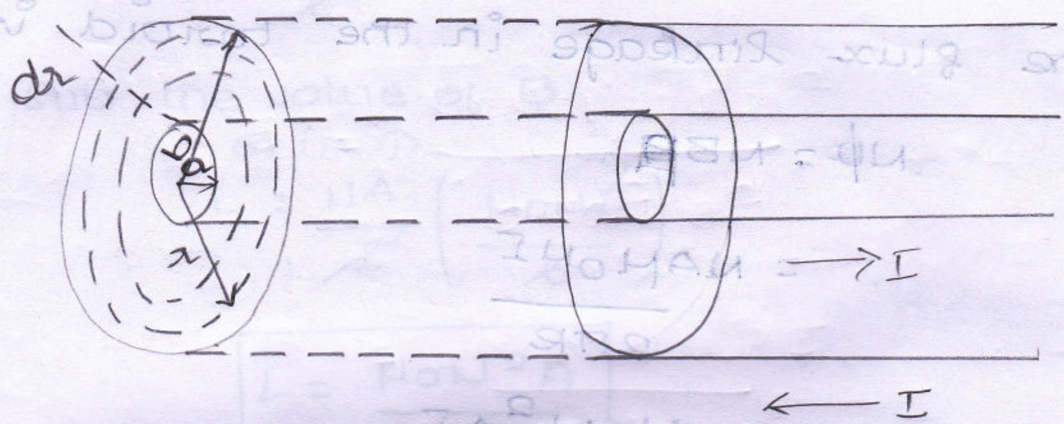
Inductance of Toroid is

$$L = \frac{N\phi}{I}$$

$$= \frac{\mu_0 N^2 r^2 I}{2RI}$$

$$\boxed{L = \frac{\mu_0 N^2 r^2}{2R}}$$

Inductance of co-axial cable:



consider a coaxial cable of inner radius 'a' and outer radius 'b' as shown in figure.

Let I be the current in inner cylinder and -I be the current in the outer cylinder.

consider a angular ring of thickness dr at a distance 'r' from the centre of the cable, the flux density B is given by

$$B = \frac{\mu_0 I}{2\pi r}$$

The total flux linkage per unit length between A and B

$$\phi = \int_a^b \frac{\mu_0 I}{2\pi r} \, dr$$

$$= \frac{\mu_0 I}{2\pi} [\ln r]_a^b$$

$$\phi = \frac{\mu_0 I}{2\pi} \ln\left(\frac{b}{a}\right)$$

Inductance of coaxial cable per unit length is given by wkt $L = \phi/I$

$$\boxed{L = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right)}$$

Inductance of a co-axial cable with of solid in a conductor:

Consider a co-axial cable with solid inner conductor of radius 'a' and outer radius 'b'.

Let I be the current in solid conductor and -I be the current in outer conductor. The flux density with the solid conductor at a distance r from the axis of cable is

$$B = \frac{\mu_0 \mu_r I' r}{2\pi a^2}$$

$0 < r < a$ current flowing through solid in a conductor between a to r is $I' = \frac{I}{\pi a^2} \times \pi r^2$

$$I' = \frac{I r^2}{a^2}$$

Now

$$B = \frac{\mu_0 \mu_r r I r^2}{2\pi a^2 \cdot a^2}$$

$$B = \frac{\mu_0 \mu_r r^3 I}{2\pi a^4}$$

The total flux linkage area between 0 and a

$$= \int_0^a \frac{\mu_0 \mu_r I r^3}{2\pi a^4} dr$$

$$= \frac{\mu_0 \mu_r I}{2\pi a^4} \int_0^a r^3 dr$$

$$= \frac{\mu_0 \mu_r I}{2\pi a^4} \left[\frac{r^4}{4}\right]_0^a$$

$$= \frac{\mu_0 \mu_r I}{2\pi a^4} \frac{a^4}{4}$$

$$= \frac{\mu_0 \mu_r I \cancel{a^4}}{8\pi \cancel{a^4}}.$$

$$\phi = \frac{\mu_0 \mu_r I}{8\pi}.$$

Inductance of solid conductor between 0 and a

$$L = \frac{\mu_0 \mu_r}{8\pi}$$

Inductance of a coaxial cable per unit length between a and b is

$$L_2 = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right)$$

Inductance of a coaxial cable per unit length is given by

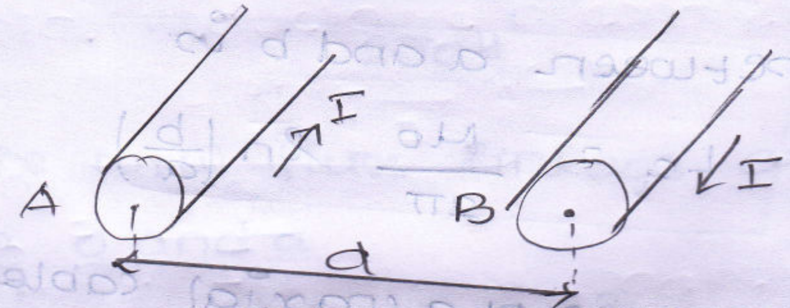
$$L = L_1 + L_2$$

$$L = \frac{\mu_0 \mu_r}{8\pi} + \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$\boxed{L = \frac{\mu_0}{4\pi}\left[\frac{1}{2}\mu_r + 2\ln\left(\frac{b}{a}\right)\right] H/m}$$

Inductance of two transmission line :

Consider two conductors A and B with radius a and b respectively seperated with a distance d. The conductor A carries a current I and conductor B carries a current -I



The internal flux linkage of conductor A is given by $\phi_1 = \frac{\mu_0 \mu_r I}{8\pi}$

The external flux linkage of conductor A is given by

$$\phi_2 = \frac{\mu_0 I}{2\pi} \ln\left(\frac{d}{a}\right)$$

The total flux linkage A is given by $\phi = \phi_1 + \phi_2$

$$\phi = \frac{\mu_0 \mu_r I}{8\pi} + \frac{\mu_0 I}{2\pi} \ln\left(\frac{d}{a}\right)$$

$$= \frac{\mu_0 I}{2\pi}\left(\frac{\mu_r}{4} + \ln\left[\frac{d}{a}\right)\right]$$ (no need)

The total inductance of conductor A is given by

$$L_a = \frac{\phi}{I}$$

sub ϕ in above equation

$$L_a = \frac{\frac{\mu_0 I \mu_r}{8\pi} + \frac{\mu_0 I}{2\pi} \ln(d/a)}{I}$$

$$L_a = \frac{\mu_0 \cancel{I}}{4\pi \cancel{I}} \left[\frac{\mu_r}{2} + 2 \ln(d/a) \right]$$

$$L_a = \frac{\mu_0}{4\pi} \left[\frac{\mu_r}{2} + 2 \ln(d/a) \right]$$

Similarly for conductor B

The total flux linkage $\phi = \phi_1 + \phi_2$

The internal flux linkage of conductor B is given by $\phi_1 = \frac{\mu_0 \mu_r I}{8\pi}$

The external flux linkage of conductor B is given by

$$\phi_2 = \frac{\mu_0 I}{2\pi} \ln\left(\frac{d}{b}\right)$$

$$\phi = \phi_1 + \phi_2$$

$$= \frac{\mu_0 \mu_r I}{8\pi} + \frac{\mu_0 I}{2\pi} \ln\left(\frac{d}{b}\right)$$

$$\phi = \frac{\mu_0 I \mu_r}{8\pi} + \frac{\mu_0 I}{2\pi} \ln\left(\frac{d}{b}\right)$$

The total inductance of conductor B is given by

$$L_b = \frac{\phi}{I}$$

$$= \frac{\frac{\mu_0 I \mu_r}{8\pi} + \frac{\mu_0 I}{2\pi} \ln\left(\frac{d}{b}\right)}{I}$$

$$= \frac{\mu_0 \cancel{I}}{4\pi \cancel{I}} \left[\frac{\mu_r}{2} + 2 \ln(d/b)\right]$$

$$L_b = \frac{\mu_0}{4\pi} \left[\frac{\mu_r}{2} + 2 \ln(d/b)\right]$$

The total inductance of the transmission line by unit length is given by

$$L = L_a + L_b$$

$$= \frac{\mu_0}{4\pi} \left[\frac{\mu_r}{2} + 2 \ln(d/a)\right] +$$

$$\frac{\mu_0}{4\pi} \left[\frac{\mu_r}{2} + 2 \ln(d/b)\right]$$

$$= \frac{\mu_0}{4\pi}\left[\frac{\mu_r}{2} + 2\ln(d/a) + \frac{\mu_r}{2} + 2\ln(d/b)\right]$$

$$= \frac{\mu_0}{4\pi}\left[\frac{\cancel{2}\mu_r}{\cancel{2}} + 2\ln(d/a) + 2\ln(d/b)\right]$$

$$= \frac{\mu_0}{4\pi}\left[\mu_r + 2\ln(d/a) + 2\ln(d/b)\right]$$

$$\boxed{L = \frac{\mu_0}{4\pi}\left[\mu_r + 2\ln\left(\frac{d^2}{ab}\right)\right]} \text{ Henry/m}$$

Energy stored in the magnetic field:

When a current passing through the inductor is increases gradually from 0 to i and the corresponding potential difference is assumed to be V. The energy stored with respect to time is given by dW

$$dW = Vi\,dt \quad -①$$

Energy stored in the magnetic field is given by

$$W = \int_0^i Vi\,dt.$$

$$= V\int_0^i i\,dt$$

$$= V\int_0^i \frac{di}{dt}\Big]_0^i \quad -②$$

workdone in the inductance is given by

$$W_L = \int_0^i L i \, di$$

$$W_L = L \int_0^i i \, di$$

$$W_L = L \left[\frac{i^2}{2}\right]_0^i$$

$$\boxed{W_L = L\left[\frac{i^2}{2}\right]} \quad (3)$$

Energy density in the magnetic field

Energy density in the magnetic field is given by

$$W_L = L\left[\frac{i^2}{2}\right] \quad (1)$$

The inductance of solenoid is given by

$$L = \frac{\mu_0 N^2 A}{l} \quad (2)$$

Sub (2) in (1)

$$W_L = \frac{\mu_0 N^2 A}{l}\left(\frac{i^2}{2}\right)$$

For the magnetic field with N number of turns H can be written as

$$H = \frac{NI}{l}$$

Replace NI as Hl

$$W = \frac{\mu_0 H^2 \cancel{l^2} A}{2\cancel{l}}$$

$$W = \frac{\mu_0 H^2 l A}{2} \text{ Joules}$$

To find energy density

Replace

$$B = \mu_0 H$$

$$W = \frac{BHlA}{2}$$ — 1st half: 1

EM WAVES AND WAVE EQUATIONS

Maxwell's equation in point and integral form

* Micheal Faraday showed that the electric field produced by a changing magnetic field. James clerk Maxwell introduced a concept that the magnetic field produced by a changing electric field.

* Maxwell derived four equations to describe the electromagnetic field. These four electromagnetic equation are also known as Maxwell's equation. These equations based on the fundamental laws of Gauss, Faraday and Ampere.

Maxwells 1st Law (or) 1st equation (Ampere's law)

Ampere law states that line integral of magnetic field intensity around a closed path is exactly equal to the current enclosed by the path

$$\boxed{\oint \vec{H} \cdot dl = I}$$

÷ by ΔV

$$\frac{\oint \vec{H} \cdot dl}{\Delta V} = \frac{I}{\Delta V}$$

$$\nabla \times \vec{H} = \vec{J}$$

$$\boxed{\nabla \times \vec{H} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t}}$$

→ Differential form (or) vector form (or) point form of Maxwell's 1st equation. (or) Ampere's Law.

$$\oint \vec{H} \cdot dl = I$$

$$I = \int J \cdot ds$$

$$\oint \vec{H} \cdot dl = \int_S \vec{J} \cdot ds$$

$$\boxed{\oint \vec{H} \cdot dl = \int_S \left(\sigma \vec{E} + \varepsilon \frac{d\vec{E}}{dt}\right) ds}$$

Integral form of Maxwell's 1st equation.

Maxwells 2nd equation (or) Maxwells 2nd Law (Faraday's Law)

Time varying magnetic field produce an electromotive force or emf which may be established a current in a closed surface

$$\boxed{\oint \vec{E}.dl = -\int_S \frac{\partial \vec{B}}{\partial t}\, ds} \rightarrow$$ Integral form of maxwells 2nd equation.

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}} \longrightarrow$$ Differential form of maxwells 2nd equ.

Maxwells 3rd law (or) III rd equation (Gauss law for electric field)

Electric flux passed through any closed surface is equal to the total charge enclosed by the surface.

$$\psi = Q$$

$$\psi = \int_S \vec{D}.ds$$

$$\int_S \vec{D}\, ds = Q$$

$$Q = \int_V \rho_v\, dv$$

$$\boxed{\int_S \vec{D}\, ds = \int_V \rho_v\, dv} \rightarrow$$ Integral form of maxwells 3rd equation

$$\boxed{\nabla.\vec{D} = \rho_v} \rightarrow$$ Differential equation

Maxwells 4th equation (Gauss law for magnetic field)

The magnetic flux passing through unit area in a plane at right angle to the direction of flux.

* For magnetic fields, the surface integral of $\vec{B}$ over a closed surface S is always zero. (i.e $\phi = 0$, incoming = outgoing)

$$\boxed{\int_S \vec{B}\cdot ds = 0}$$

$\rightarrow$ Integral form of Maxwell's 4th equation

Differential form

$$\int_S \vec{B}\, ds = 0$$

÷ by area on both sides

$$\int_S \frac{\vec{B}\cdot ds}{\Delta V} = 0$$

$$\boxed{\nabla\cdot\vec{B} = 0}$$

Maxwell equation	Significance	Integral form	Differential form
1st equation	Amperes law	$\oint \vec{H} dl = \int \left(\sigma E + \varepsilon_1 \frac{d\vec{E}}{dt}\right) ds$	$\nabla \times \vec{H} = \sigma\vec{E} + \varepsilon_1 \frac{\partial \vec{E}}{dt}$
2nd equation	Faradays Law	$\oint E\, dl = -\int \frac{\partial B}{\partial t} ds$	$\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$
3rd equation	Gauss law for Electric field	$\int_S \vec{D}\cdot ds = \int_V \rho_v \cdot dv$	$\nabla\cdot\vec{D} = \rho_v$
4th equation	Gauss law for Magnetic field	$\int_S \vec{B}\cdot ds = 0$	$\nabla\cdot\vec{B} = 0$

Table. Maxwell's equation

Maxwells equation for free space

$\rho = 0$, $\sigma\vec{E} = 0$, $[\sigma = 0]$

Maxwells equation 1	Significance	Integral form	Differential form
1st equation	Amperes law	$\oint \vec{H} dl = \int_S \left(\varepsilon_1 \frac{d\vec{E}}{dt}\right) ds$	$\nabla \times \vec{H} = \varepsilon_1 \frac{\partial \vec{E}}{\partial t}$
2nd equation	Faradays Law	$\oint \vec{E} dl = -\int_S \frac{\partial B}{\partial t} ds$	$\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$

3rd equation	Gauss law for electric field	$\int_S \vec{D} \cdot ds = 0$	$\nabla \cdot \vec{D} = 0$
4th equation	Gauss law for Magnetic field	$\int_S \vec{B} \cdot ds = 0$	$\nabla \cdot \vec{B} = 0$

Maxwells equal for conductor

condition for conductor $\rho = 0$, $J_D = \frac{\varepsilon_1 \partial \vec{E}}{\partial t} = 0$

1st equation	Ampere Law	$\oint \vec{H} \cdot dl = \sigma \vec{E}$	$\nabla \times \vec{H} = \sigma \vec{E}$
2nd equation	Faradays law	$\oint \vec{E} \cdot dl = -\int_S \frac{\partial B}{\partial t} ds$	$\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$
3rd equation	Gauss law for electric field	$\int_S D \cdot ds = 0$	$\nabla \cdot \vec{D} = 0$
4th equation	Gauss law for Magnetic field	$\int_S \vec{B} \cdot ds = 0$	$\nabla \cdot \vec{B} = 0$

Maxwells equation for time varying fields

$$\frac{\partial \vec{B}}{\partial t} = j\omega \vec{B}$$

1st equation	Ampere Law	$\oint \vec{H} \cdot dl = \left(\sigma \vec{E} + \frac{j\omega \vec{B}}{dt}\right) ds$	$\Delta \times \vec{H} = \sigma \vec{E} + \vec{E}(j\omega)$
2nd equation	Faradays Law	$\oint \vec{E} dl = -\int_S j\omega \vec{B} \, ds$	$\Delta \times B = -j\omega \vec{B}$
3rd equation	Gauss law for electric field	$\int_S \vec{D} ds = \int_V \rho_v dv$	$\nabla \cdot \vec{D} = \rho_v$
4th equation	Gauss law for magnetic field	$\int_S \vec{B} \cdot ds = 0$	$\nabla \cdot \vec{B} = 0$.

Poynting's theorem:

The vector product of electric field intensity and Magnetic field intensity at any point is a measure of the rate of energy flow per unit area at that point

$$\vec{P} = \vec{E} \times \vec{H}$$

The direction of flow (P) is perpendicular to E & H

Proof:

The energy flow equation can be obtained from maxwell's equation.

$$\nabla \times \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \quad \text{--- (1)} \rightarrow \text{from Amperes law}$$

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \text{--- (2) from faradays law}$$

Dot product of $\vec{E}$ on both side of (1)

$$\vec{E} \cdot (\nabla \times \vec{H}) = \vec{E} \cdot \left(\sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t}\right) \quad \text{--- (3)}$$

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{C}(\vec{A} \times \vec{B}) - \vec{B}(\vec{A} \times \vec{C})$$

$$\vec{E}(\nabla \times \vec{H}) = \vec{H}(\nabla \times \vec{E}) - \nabla(\vec{E} \times \vec{H}) \quad \text{--- (4)}$$

$$\vec{H} \cdot (\nabla \times \vec{E}) - \nabla(\vec{E} \times \vec{H}) = \vec{E} \cdot \left(\sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t}\right)$$

$$\vec{H}\left(-\mu \frac{\partial \vec{H}}{\partial t}\right) - \nabla \cdot \vec{P} = \sigma E^2 + \vec{E} \cdot \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$-\nabla \cdot \vec{P} = \sigma E^2 + \vec{E} \cdot \epsilon \frac{\partial \vec{E}}{\partial t} + \vec{H} \mu \frac{\partial \vec{H}}{\partial t}$$

$$-\nabla \cdot \vec{P} = \sigma E^2 + \epsilon \frac{\partial \vec{E}}{\partial t} \vec{E} + \mu \frac{\partial \vec{H}}{\partial t} \vec{H} \quad \text{--- (5)}$$

$$\frac{\partial \vec{E} \cdot \vec{E}}{\partial t} = \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} + \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}$$

$$\frac{\partial E^2}{\partial t} = 2 \vec{E} \frac{\partial \vec{E}}{\partial t} \quad \text{--- (6)}$$

⑥ $\Rightarrow \dfrac{\partial E^2}{\partial t} = 2\vec{E} \cdot \dfrac{\partial \vec{E}}{\partial t}$

$\vec{E} \cdot \dfrac{\partial \vec{E}}{\partial t} = \dfrac{1}{2} \dfrac{\partial E^2}{\partial t}$ —— ⑦

III^ly $\vec{H} \dfrac{\partial \vec{H}}{\partial t} = \dfrac{1}{2} \dfrac{\partial H^2}{\partial t}$ —— ⑧

Sub ⑦ & ⑧ in ⑤

⑤ $\Rightarrow -\nabla \vec{P} = \sigma E^2 + \frac{1}{2} \varepsilon \frac{\partial E^2}{\partial t} + \frac{1}{2} \mu \frac{\partial H^2}{\partial t}$

$$\boxed{-\nabla \vec{P} = \sigma E^2 + \frac{1}{2} \frac{\partial}{\partial t}\left[\varepsilon E^2 + \mu H^2 \right]}$$

→ Point form of Poynting theorem.

* Taking volume integral on both side of the above equation.

$$\int_V -\nabla \vec{P} dv = \int_V \left[\sigma E^2 + \frac{1}{2} \frac{\partial}{\partial t}\left(\varepsilon E^2 + \mu H^2 \right) \right] dv$$

$$\boxed{-\int_S \vec{P} . ds = \int_V \left[\sigma E^2 + \frac{1}{2} \frac{\partial}{\partial t}\left[\varepsilon E^2 + \mu H^2 \right] \right] dv}$$

→ Integral form of Poynting theorem.

ELECTROMAGNETIC WAVE EQUATION

Definition of a wave

If a physical phenonmenon that occurs at one place at a given point of time is reproduced at other place at later instants, the time delay being proportional to the space separation from the first location.

1. WAVE EQUATION FOR CONDUCTING MEDIUM:

* the application of Maxwell's equations is the prediction of existence of electromagnetic wave.

wave equation for electric field

* The Maxwell's equation from faraday's Law in point form is given by

$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad \boxed{\rho = 0} \text{ volume charge density.}$$

$$= -\mu \frac{\partial H}{\partial t}$$

Takes curl on both sides

$$\nabla \times \nabla \times E = -\mu \nabla \times \frac{\partial H}{\partial t} \quad \text{——— (1)}$$

But Maxwell's equation from Ampere's law in point form is

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$= \sigma E + \epsilon \frac{\partial E}{\partial t}$$

Diff.

$$\nabla \times \frac{\partial H}{\partial t} = \frac{\partial \nabla \times H}{\partial t}$$

$$= \frac{\partial}{\partial t}\left(\sigma E + \epsilon \frac{\partial E}{\partial t}\right)$$

$$\nabla \times \frac{\partial H}{\partial t} = \sigma \frac{\partial E}{\partial t} + \epsilon \frac{\partial^2 E}{\partial t^2} \quad \text{——— (2)}$$

Sub equ (2) in (1)

$$\nabla \times \nabla \times E = -\mu\left[\sigma \frac{\partial E}{\partial t} + \epsilon \frac{\partial^2 E}{\partial t^2}\right]$$

$$= -\mu \sigma \frac{\partial E}{\partial t} - \mu \epsilon \frac{\partial^2 E}{\partial t^2} \quad \text{——— (3)}$$

But according to the identity

$$\nabla \times \nabla \times E = \nabla(\nabla \cdot E) - \nabla^2 E \quad \text{—— (4)}$$

But

$$\nabla \cdot E = \frac{1}{\varepsilon_1} \nabla \cdot D$$

Since there is no net charge within the conductor, the charge density $\rho = 0$.

$$\nabla \cdot D = 0$$

$$\nabla \cdot E = 0$$

Then eqn (4) becomes

$$\nabla \times \nabla \times E = -\nabla^2 E \quad \text{—— (5)}$$

Compare (3) & (5)

$$\nabla^2 E = \mu\sigma \frac{\partial E}{\partial t} + \mu\varepsilon_1 \frac{\partial^2 E}{\partial t^2}$$

$$\boxed{\nabla^2 E - \mu\sigma \frac{\partial E}{\partial t} - \mu\varepsilon_1 \frac{\partial^2 E}{\partial t^2} = 0} \quad \text{—— (6)}$$

↳ This is the wave equation for electric field E.

<u>The wave equation for magnetic field H:</u>

The maxwell's equation from Ampere's law in point form is given by

$$\nabla \times H = \sigma E + \varepsilon_1 \frac{\partial E}{\partial t}$$

Taking curl on both sides

$$\nabla \times \nabla \times H = \sigma \nabla \times E + \varepsilon_1 \nabla \times \frac{\partial E}{\partial t} \quad \text{—— (7)}$$

But Maxwell's equation from faraday's law

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

Diff w.r.t 't'

$$\nabla \times \frac{\partial E}{\partial t} = -\mu \frac{\partial^2 H}{\partial t}$$

Sub. the values of $\nabla \times E$ & $\nabla \times \frac{\partial E}{\partial t}$ in eqn (7)

$$\nabla \times \nabla \times H = -\mu\sigma \frac{\partial H}{\partial t} - \mu \varepsilon_1 \frac{\partial^2 H}{\partial t^2} \quad \text{--- (8)}$$

The identity is

$$\nabla \times \nabla \times H = \nabla(\nabla \cdot H) - \nabla^2 H$$

But $\quad \nabla \cdot B = \mu \nabla \cdot H = 0$

Then, $\quad \nabla \times \nabla \times H = -\nabla^2 H \quad \text{--- (9)}$

Compare (8) & (9)

$$\nabla^2 H = \mu\sigma \frac{\partial H}{\partial t} + \mu\varepsilon_1 \frac{\partial^2 H}{\partial t^2}$$

$$\boxed{\nabla^2 H - \mu\sigma \frac{\partial H}{\partial t} - \mu\varepsilon_1 \frac{\partial^2 H}{\partial t^2} = 0} \longrightarrow (10)$$

This is the wave equation for magnetic field H

Wave equation for free space

* For free space (dielectric medium) the conductivity of the medium is Zero. (i.e $\sigma = 0$) & there is no charge containing in it (i.e $\rho = 0$).

The electromagnetic wave equations for free space can be obtained from maxwell's equations.

The wave equation for free space in terms of electric field:

* The maxwell's equation from faraday's law for free space in point form is

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$= -\mu \frac{\partial H}{\partial t}$$

Taking curl on both sides

$$\nabla \times \nabla \times E = -\mu \nabla \times \frac{\partial H}{\partial t} \quad \text{--- (1)}$$

* The Maxwell's equation from Ampere's law for free space in point form is

$$\nabla \times H = \frac{\partial D}{\partial t}$$

$$= \varepsilon_1 \frac{\partial E}{\partial t}$$

then

$$\nabla \times \frac{\partial H}{\partial t} = \frac{\partial \nabla \times H}{\partial t}$$

$$= \frac{\partial}{\partial t}\left(\varepsilon_1 \frac{\partial E}{\partial t}\right)$$

$$\nabla \times \frac{\partial H}{\partial t} = \varepsilon_1 \frac{\partial^2 E}{\partial t^2} \quad \text{———} \quad (2)$$

Sub (2) in (1)

$$\nabla \times \nabla \times E = -\mu \varepsilon_1 \frac{\partial^2 E}{\partial t^2} \quad \text{———} \quad (3)$$

But the identity is given by

$$A \times B \times C = A(B \cdot C) - (A \cdot B)C$$

$$\nabla \times \nabla \times E = \nabla(\nabla \cdot E) - \nabla^2 E$$

But $\nabla \cdot E = \frac{1}{\varepsilon_1} \nabla \cdot D = \frac{\rho}{\varepsilon_1} = 0$

then, $\nabla \times \nabla \times E = -\nabla^2 E$ ——— (4)

Compare (3) & (4)

$$\nabla^2 E = \mu \varepsilon_1 \frac{\partial^2 E}{\partial t^2}$$

$$\boxed{\nabla^2 E - \mu \varepsilon_1 \frac{\partial^2 E}{\partial t^2} = 0} \quad \text{———} \quad (5)$$

↳ This is the wave equation for free space in terms of electric field.

The wave equation for free space in terms of magnetic field H.

* The Maxwell's equation from Ampere's law for free space in point form is given by

$$\nabla \times H = \varepsilon_1 \frac{\partial E}{\partial t}$$

Take curl on both sides

$$\nabla \times \nabla \times H = \varepsilon_1 \nabla \times \frac{\partial E}{\partial t} \quad \text{——— (6)}$$

But Maxwell's equation from faraday's law

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

Differentiating

$$\nabla \times \frac{\partial E}{\partial t} = -\mu \frac{\partial^2 H}{\partial t^2} \quad \text{——— (7)}$$

Sub equ (7) in (6)

$$\nabla \times \nabla \times H = -\mu \varepsilon_1 \frac{\partial^2 H}{\partial t^2} \quad \text{——— (8)}$$

The identity is given by

$$\nabla \times \nabla \times H = \nabla(\nabla \cdot H) - \nabla^2 H \quad \text{——— (9)}$$

But $\nabla \cdot H = \frac{1}{\mu}$

then, $\nabla \times \nabla \times H = -\nabla^2 H$ ——— (10)

compare the equation (8) & (10)

$$\nabla^2 H = \mu \varepsilon_1 \frac{\partial^2 H}{\partial t^2}$$

$$\boxed{\nabla^2 H - \mu \varepsilon_1 \frac{\partial^2 H}{\partial t^2} = 0} \quad \text{——— 11}$$

This is the wave equation for free space in terms of H.

for free space $\mu_r = 1$, $\varepsilon_r = 1$ (air)

the wave equation becomes.

$$\nabla^2 H - \mu_0 \varepsilon_0 \frac{\partial^2 H}{\partial t^2} = 0$$

$$\mu_0 \varepsilon_0 = 4\pi \times 10^{-7} \times \frac{1}{36\pi \times 10^9}$$

$$= \frac{1}{9 \times 10^{16}}$$

$$\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \text{ m/sec}$$

$$= V \text{ (Velocity of light)}$$

then wave equation

$$\boxed{\nabla^2 H - \frac{1}{V^2} \frac{\partial^2 H}{\partial t^2} = 0} \quad \text{or} \quad \boxed{\nabla^2 E - \frac{1}{V^2} \frac{\partial^2 E}{\partial t^2} = 0}$$

2 Mark (UNIT - IV)

1. write the Maxwell's equations from Ampere's law both in integral and point forms.

Maxwell's equation from Ampere's law

$$\oint \vec{H} dl = \int \left(\sigma E + \varepsilon \frac{d\vec{E}}{dt}\right) ds \quad \text{(Integral form)}$$

$$\nabla \times \vec{H} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} \quad \text{(point forms)}$$

2. write the Maxwell's equation from Faraday's law both in integral and point forms.

Maxwell's equation from faraday's law

$$\oint E\, dl = -\int \frac{\partial B}{\partial t} ds \quad \text{(Integral form)}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{(Point form)}$$

3. write down the Maxwell's equation from electric Gauss's law in integral and point form.

Maxwell's equation from electric Gauss's law

$$\int_S \vec{D} \cdot ds = \int_V \rho_v dv \quad \text{(Integral form)}$$

$$\nabla \cdot \vec{D} = \rho_v \quad \text{(Point form)}$$

4. write down the Maxwell's equations from magnetic Gauss's law in integral and point form.

Maxwell's equation from magnetic Gauss's law

$$\int_S \vec{B} \cdot ds = 0 \quad \text{(Integral form)}$$

$$\nabla \cdot \vec{B} = 0 \quad \text{(point form)}$$

5. write down the wave equations for E and H in a non- dissipative (free space) medium.

$$\nabla^2 E - \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} = 0$$

$$\nabla^2 H - \mu_0 \varepsilon_0 \frac{\partial^2 H}{\partial t^2} = 0$$

6. write down the wave equations for E and H in a conducting medium.

$$\nabla^2 E - \mu \varepsilon_1 \frac{\partial^2 E}{\partial t^2} - \mu \sigma \frac{\partial E}{\partial t} = 0$$

$$\nabla^2 H - \mu \varepsilon_1 \frac{\partial^2 H}{\partial t} - \mu \sigma \frac{\partial H}{\partial t} = 0$$

7. State poynting theorem.

The vector product of electric field intensity and magnetic field intensity at any point is a measure of the rate of energy flow per unit area at that point

$$\vec{P} = \vec{E} \times \vec{H}$$

UNIT - V

ELECTROMAGNETIC WAVES

UNIFORM PLANE WAVE

* If the phase of a wave is the same for all points on a plane surface it is called plane wave. If the amplitude is also constant in a plane wave, it is called uniform plane wave.

Properties of uniform plane waves

1. At every point is space electric field (E) and Magnetic field (H) are perpendicular to each other & to the direction of travel.
2. The fields vary harmonically with time and at the same frequency, everywhere in space.
3. Each field has the same direction, magnitudes and phase at every point in any plane perpendicular to the direction of wave travel.

If the electric field is in x direction and the magnetic field in y direction, then the wave is travelling in z direction.

The wave equation for free space is given by

$$\nabla^2 E = \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

Consider electric field E varions in x direction & E is independent of directions y and z.

then the wave equation becomes

$$\frac{\partial^2 E}{\partial x^2} = \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \qquad \left[\because \frac{\partial^2 E}{\partial y^2} = \frac{\partial^2 E}{\partial z^2} = 0\right]$$

It can be written as in terms of the components of E

$$\frac{\partial^2 E_x}{\partial x^2} = \mu \varepsilon_1 \frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{\partial^2 E_y}{\partial x^2} = \mu \varepsilon_1 \frac{\partial^2 E_y}{\partial t^2}$$

$$\frac{\partial^2 E_z}{\partial x^2} = \mu \varepsilon_1 \frac{\partial^2 E_z}{\partial t^2}$$

For free space there is no charge density

$$\nabla \cdot D = \varepsilon_1 \nabla \cdot E = 0$$

$$\nabla \cdot E = 0$$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

For uniform plane wave, E is independent of y and z

then

$$\frac{\partial E_x}{\partial x} = 0$$

This equation shows that there is no variation of E_x in the x direction.

Diff. this equ. w.r.t 't'

$$\frac{\partial^2 E_x}{\partial x^2} = 0$$

* It requires that either E_x be zero or constant.

* If E_x is constant, it would not part of wave motion & so E_x must be zero.

* Therefore a uniform plane wave propagating in the x direction has no x component of E. A similar Analysis would show that there is no x component of H.

$$\nabla \cdot B = \mu \nabla \cdot H = 0$$

$$\nabla \cdot H = 0$$

$$\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 0$$

Since H is propagating in x direction, it is independent of y and z. (V.3)

Then
$$\frac{\partial H_x}{\partial x} = 0 \quad \& \quad \frac{\partial^2 H_x}{\partial x^2} = 0$$

Since H_x is not a constant, H_x must be Zero.

$\therefore$ $H_x = 0$ for uniform plane wave.

Characteristic impedance or Intrinsic impedance (η_0)

* Consider the plane wave propagating in x direction. The wave equation for free space is

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

The general solution of this differential equation is in the form.

$$E = f_1(x - V_0 t) + f_2(x + V_0 t)$$

where
$$V_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

f_1 & f_2 are any functions of $(x - V_0 t)$ & $(x + V_0 t)$ respectively.

* The solution of wave equation consists of two waves, one travelling in positive direction and other travelling in negative direction.

* Consider the wave travel in positive direction alone.

$$f_2(x + V_0 t) = 0 \qquad \text{(negative direction)}$$

The general solution of wave equation becomes

$$E = f(x - V_0 t)$$

$$\nabla \times E = \begin{vmatrix} \vec{x} & \vec{y} & \vec{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$$= \vec{x}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + \vec{y}\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) + \vec{z}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) \quad V.4$$

Since wave travelling in x direction, E & H are both independent of y and z. i.e $E_x = H_x = 0$ & $\frac{\partial E}{\partial y} = \frac{\partial E}{\partial z} = 0$.

$$\nabla \times E = -\frac{\partial E_z}{\partial x}\vec{y} + \frac{\partial E_y}{\partial x}\vec{z}$$

Similarly

$$\nabla \times H = -\frac{\partial H_z}{\partial x}\vec{y} + \frac{\partial H_y}{\partial x}\vec{z}$$

But

$$\nabla \times H = \varepsilon_1 \frac{\partial E}{\partial t}$$

Comparing these two equations

$$-\frac{\partial H_z}{\partial x}\vec{y} + \frac{\partial H_y}{\partial x}\vec{z} = \varepsilon_1\left[\frac{\partial E_y}{\partial t}\vec{y} + \frac{\partial E_z}{\partial t}\vec{z}\right] \quad [\because E_x = 0]$$

Equating $\vec{y}$ & $\vec{z}$ terms

$$-\frac{\partial H_z}{\partial x} = \varepsilon_1 \frac{\partial E_y}{\partial t}$$

$$\frac{\partial H_y}{\partial x} = \varepsilon_1 \frac{\partial E_z}{\partial t}$$

From second Maxwell's equation for free space

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

But

$$\nabla \times E = -\frac{\partial E_z}{\partial x}\vec{y} + \frac{\partial E_y}{\partial x}\vec{z}$$

Equating these two equations

$$-\frac{\partial E_z}{\partial x}\vec{y} + \frac{\partial E_y}{\partial x}\vec{z} = -\mu\left[\frac{\partial H_y}{\partial t}\vec{y} + \frac{\partial H_z}{\partial t}\vec{z}\right] \quad [\because H_x = 0]$$

Equating $\vec{y}$ & $\vec{z}$ terms

$$\frac{\partial E_z}{\partial x} = \mu \frac{\partial H_y}{\partial t}$$

$$\frac{\partial E_y}{\partial x} = -\mu \frac{\partial H_z}{\partial t}$$

Let the solution of this equation is given by

$$E_y = f(x - V_0 t)$$

Diff. $$\frac{\partial E_y}{\partial t} = \frac{\partial f}{\partial (x - V_0 t)} \cdot \frac{\partial (x - V_0 t)}{\partial t}$$

$$\frac{\partial E_y}{\partial t} = f'(x - V_0 t)(-V_0)$$

Simplify $f'(x - V_0 t)$ can be written as f'

$$\frac{\partial E_y}{\partial t} = -V_0 f'$$

But

$$-\frac{\partial H_z}{\partial x} = \varepsilon_1 \frac{\partial E_y}{\partial t}$$

$$\frac{\partial H_z}{\partial x} = -\varepsilon_1 (-V_0 f')$$

$$= V_0 \varepsilon_1 f'$$

$$= \frac{1}{\sqrt{\mu \varepsilon_1}} \varepsilon_1 f'$$

$$\frac{\partial H_z}{\partial x} = \sqrt{\varepsilon_1/\mu}\, f'$$

integrating

$$H_z = \sqrt{\varepsilon_1/\mu} \int f'\, dx$$

$$H_z = \sqrt{\varepsilon_1/\mu}\, f$$

$$= \sqrt{\varepsilon_1/\mu}\, E_y$$

$$\frac{E_y}{H_z} = \sqrt{\frac{\mu}{\varepsilon_1}}$$

Similarly,

$$\frac{E_z}{H_y} = -\sqrt{\frac{\mu}{\varepsilon_1}}$$

If E is the total electric field

$$E = \sqrt{E_y^2 + E_z^2}$$

If H is the total magnetic field,

$$H = \sqrt{H_y^2 + H_z^2}$$

then $\boxed{\frac{E}{H} = \sqrt{\frac{\mu}{\epsilon}}}$

* It is referred to as characteristics impedance or intrinsic impedance of the medium.
* It is the ratio of square root of permeability to the dielectric constant of the medium and it is denoted by η.

$$\eta = \frac{E}{H} = \sqrt{\frac{\mu}{\epsilon}}$$

For free space $\epsilon_r = \mu_r = 1$ (air), the characteristics impedance or intrinsic impedance for free space is given by

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$= \sqrt{\frac{4\pi \times 10^{-7}}{\frac{1}{36\pi \times 10^9}}}$$

$$= \sqrt{4\pi \times 36\pi \times 10^2}$$

$$\eta_0 = 120\pi$$

$$\boxed{\eta_0 \approx 377\Omega}$$

Problem No: 1

Find the characteristics impedance of the medium whose relative permittivity is 3 and relative permeability is 1.

$\epsilon_r = 3$

$\mu_r = 1$

characteristics impedance $\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}}$

$$= \eta_0 \sqrt{\frac{\mu_r}{\epsilon_r}}$$

where $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi$

$$\eta = 120\pi\sqrt{\frac{1}{3}}$$

$$\eta = 217.66 \text{ ohms}$$

2. Uniform plane wave in free space is described by $E = 100e^{-(\pi z/3)}\vec{a_x}$ Determine the frequency and wave-length.

$$E = 100e^{-(\pi z/3)}\vec{a_x}$$

$$\beta = \frac{2\pi}{\lambda} = \frac{\pi}{3}$$

$$\lambda = 6m$$

$$f = \frac{c}{\lambda} = \frac{3\times10^8}{6} = 50MHZ$$

3. The velocity of uniform plane wave in a loss-less dielectric is 1×10^8 m/sec. Find the dielectric constant.

for loss-less dielectric $\mu_r = 1$

$$V = \frac{1}{\sqrt{\mu_r\mu_0\varepsilon_0\varepsilon_r}}$$

$$= \frac{1}{\sqrt{\mu_0\varepsilon_0}\sqrt{\mu_r\varepsilon_r}}$$

$$1\times10^8 = \frac{3\times10^8}{\sqrt{\varepsilon_r}}$$

$$\sqrt{\varepsilon_r} = 3$$

$$\boxed{\varepsilon_r = 9}$$

WAVE PROPAGATION IN A LOSSLESS MEDIUM

The wave equation for free space (lossless medium) is

$$\nabla^2 E = \mu \varepsilon_1 \frac{\partial^2 E}{\partial t^2}$$

The phasor value of E is

$$E(x,t) = Re\left[E(x) e^{j\omega t}\right]$$

Applying to the wave equation

$$\nabla^2 Re\left[E e^{j\omega t}\right] = \mu \varepsilon_1 \frac{\partial^2}{\partial t^2} Re\left[E e^{j\omega t}\right]$$

$$\nabla^2 Re\left[E e^{j\omega t}\right] = \mu \varepsilon_1 Re\left[-\omega^2 E e^{j\omega t}\right]$$

$$Re\left[(\nabla^2 E + \mu \varepsilon_1 \omega^2 E) e^{j\omega t}\right] = 0$$

$$\nabla^2 E + \mu \varepsilon_1 \omega^2 E = 0$$

This is the wave equation for lossless medium in phasor form and it is called Vector Helmhdtz equation

$$\nabla^2 E + \beta^2 E = 0$$

where,

$$\beta^2 = \mu \varepsilon_1 \omega^2$$

$$\beta = \sqrt{\mu \varepsilon_1}\, \omega$$

$\beta \rightarrow$ phase shift constant

The velocity of propagation is

$$V = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu \varepsilon_1}}$$

The wave propagates in x direction i.e no variation in y and z.

$$\frac{\partial^2 E}{\partial x^2} + \beta^2 E = 0$$

The solution of the equation is

$$E = C_1 e^{-j\beta x} + C_2 e^{j\beta x}$$

WAVE PROPAGATION IN A CONDUCTING MEDIUM:

The wave equation for conducting medium is

$$\nabla^2 E - \mu \varepsilon_1 \frac{\partial^2 E}{\partial t^2} - \mu \sigma \frac{\partial E}{\partial t} = 0.$$

The phasor form of wave equation is

$$\nabla^2 E - j^2 \mu \varepsilon_1 \omega^2 E - j\omega\mu\sigma E = 0$$

$$\nabla^2 E - j(\omega\mu\sigma + j\mu\varepsilon_1\omega^2) E = 0$$

$$\nabla^2 E - j\omega\mu(\sigma + j\omega\varepsilon_1) E = 0$$

$$\nabla^2 E - \gamma^2 E = 0$$

where, $\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon_1)$

γ is called Propagation constant, which has both real & imaginary parts.

$$\gamma = \alpha + j\beta$$

where, α is attenuation constant

β is Phase shift

$$\gamma = \alpha + j\beta$$

$$= \sqrt{j\omega\mu(\sigma + j\omega\varepsilon_1)}$$

squaring on both sides

$$\alpha^2 - \beta^2 + 2j\alpha\beta = j\omega\mu\sigma - \omega^2\mu\varepsilon_1$$

Equating real & imaginary parts

$$\alpha^2 - \beta^2 = -\omega^2\mu\varepsilon_1$$

& $$2\alpha\beta = \omega\mu\sigma$$

To solve these two equations

W.K.T

$$\alpha^2 + \beta^2 = \sqrt{(\alpha^2 - \beta^2)^2 + 4\alpha^2\beta^2}$$

But $(\alpha^2 - \beta^2)^2 = (-\omega^2\mu\varepsilon_1)^2$

$$(2\alpha\beta)^2 = (\omega\mu\sigma)^2$$

$$\alpha^2 + \beta^2 = \sqrt{\omega^4\mu^2\varepsilon_1^2 + \omega^2\mu^2\sigma^2}$$

$$\alpha^2 - \beta^2 = -\omega^2\mu\varepsilon_1$$

Adding these two equations

$$2\alpha^2 = -\omega^2\mu\varepsilon_1 + \sqrt{\omega^4\mu^2\varepsilon_1^2 + \omega^2\mu^2\sigma^2}$$

$$\alpha^2 = \frac{-\omega^2\mu\varepsilon_1}{2} + \frac{\omega^2\mu\varepsilon_1}{2}\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon_1^2}}$$

$$\alpha = \sqrt{\frac{\omega^2\mu\varepsilon_1}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon_1^2}}-1\right]}$$

Attenuation factor is given by

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon_1}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon_1^2}}-1\right]}$$

By subtracting $\alpha^2 - \beta^2$ from $\alpha^2 + \beta^2$, the value of β becomes

$$\beta = \omega\sqrt{\frac{\mu\varepsilon_1}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon_1^2}}+1\right]}$$

WAVE PROPAGATION IN GOOD DIELECTRICS

* The ratio of conduction current density to displacement current density in the medium is $\frac{\sigma}{\omega\varepsilon_1}$. Hence $\frac{\sigma}{\omega\varepsilon_1} = 1$ can be considered to mark the dividing line between conductor and dielectrics.
* For good conductors $\frac{\sigma}{\omega\varepsilon_1}$ is much greater than unity. For good dielectrics $\frac{\sigma}{\omega\varepsilon_1}$ is very much less than unity.

For dielectrics

$$\frac{\sigma}{\omega\varepsilon_1} << 1$$

$$\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon_1^2}} = \left(1+\frac{\sigma^2}{\omega^2\varepsilon_1^2}\right)^{1/2}$$

$$\simeq \left(1+\frac{\sigma^2}{2\omega^2\varepsilon_1^2}\right)$$

the attenuation factor is

$$\alpha \simeq \omega \sqrt{\frac{\mu\epsilon_1}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon_1^2}}-1\right]}$$

$$\simeq \omega \sqrt{\frac{\mu\sigma^2}{4\omega^2\epsilon_1}}$$

$$\alpha \simeq \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon_1}}$$

The phase shift is

$$\beta \simeq \omega \sqrt{\frac{\mu\epsilon_1}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon_1^2}}+1\right]}$$

$$\simeq \omega \sqrt{\frac{\mu\epsilon_1}{2}\left[2+\frac{\sigma^2}{2\omega^2\epsilon_1^2}\right]}$$

$$\simeq \omega \sqrt{\mu\epsilon_1\left[1+\frac{\sigma^2}{4\omega^2\epsilon_1^2}\right]}$$

$$\simeq \omega\sqrt{\mu\epsilon_1}\left[1+\frac{\sigma^2}{4\omega^2\epsilon_1^2}\right]^{1/2}$$

$$\beta \simeq \omega\sqrt{\mu\epsilon_1}\left[1+\frac{\sigma^2}{8\omega^2\epsilon_1^2}\right]$$

The velocity of the wave in the dielectric is

$$v = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\epsilon_1}\left(1+\frac{\sigma^2}{8\omega^2\epsilon_1^2}\right)}$$

$$\simeq \frac{1}{\sqrt{\mu\epsilon_1}}\left(1-\frac{\sigma}{8\omega^2\epsilon_1^2}\right) \qquad \left[\because V_0 = \frac{1}{\sqrt{\mu\epsilon_1}}\right]$$

$$v \simeq v_0\left(1-\frac{\sigma}{8\omega^2\epsilon_1^2}\right)$$

The intrinsic or characteristic impedance of medium is given by

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma+j\omega\epsilon_1}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon_1\left(1+\frac{\sigma}{j\omega\varepsilon_1}\right)}} = \sqrt{\frac{\mu}{\varepsilon_1}\left(1+\frac{\sigma}{j\omega\varepsilon_1}\right)^{-1}}$$

$$= \sqrt{\frac{\mu}{\varepsilon_1}\left[1-\frac{\sigma}{j\omega\varepsilon_1}\right]}$$

$$= \sqrt{\mu/\varepsilon_1}\left[1 \mp \frac{j\sigma}{\omega\varepsilon_1}\right]^{1/2}$$

$$\eta = \sqrt{\mu/\varepsilon_1}\left[1+\frac{j\sigma}{2\omega\varepsilon_1}\right]$$

WAVE PROPAGATION IN GOOD CONDUCTOR:

For good conductor $\frac{\sigma}{\omega\varepsilon_1} \gg 1$

$$\gamma = \sqrt{j\omega\mu\,(\sigma+j\omega\varepsilon_1)}$$

$$= \sqrt{j\omega\mu\sigma\left[1+\frac{j\omega\varepsilon_1}{\sigma}\right]}$$

$$\approx \sqrt{j\omega\mu\sigma}$$

$$\gamma \approx \sqrt{\omega\mu\sigma}\ \angle 45^\circ$$

$$\therefore\ \alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

The velocity of the wave in conductor

$$V = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\frac{\omega\mu\sigma}{2}}}$$

$$V = \sqrt{\frac{2\omega}{\mu\sigma}}$$

The intrinsic impedance of the conductor

$$\eta \approx \sqrt{\frac{j\omega\mu}{j\omega\varepsilon_1\left(1+\frac{\sigma}{j\omega\varepsilon_1}\right)}}$$

$$\approx \sqrt{\frac{j\omega\mu}{j\omega\varepsilon_1 \cdot \frac{\sigma}{j\omega\varepsilon_1}}}$$

$$\approx \sqrt{\frac{j\omega\mu}{\sigma}}$$

$$\eta \approx \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$$

* It is found that in good conductors, α & β are large since σ is large., (i.e) The wave is attenuated greatly as it progresses through the conductor.
* But the velocity & characteristics impedance are reduced considerably.

Problem 1

A uniform plane wave propagating in semi-infinite region $z > 0$ is expressed (at zero) as $100 \cos 10^9 \pi t \, \vec{a_x}$ V/m. The region has $\sigma = 0.25$ $\varepsilon_r = 9$ & $\mu_r = 1000$. Determine the propagation and attenuation constants.

Sol.

$E = 100 \cos 10^9 \pi t \, \vec{a_x}$ V/m

$\omega = \pi \times 10^9$

$\sigma = 0.25$

$\varepsilon_r = 9$

$\mu_r = 1000$

Propagation constant $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

$$\gamma = \sqrt{j\omega\mu\sigma\left(1 + \frac{j\omega\varepsilon}{\sigma}\right)}$$

But

$$\frac{\omega\varepsilon}{\sigma} = \frac{\pi \times 10^9 \times 9}{0.25 \times 36\pi \times 10^9}$$

$$= 1$$

$$\gamma = \sqrt{j\omega\mu\sigma(1+j)}$$

$$= \sqrt{j\pi \times 10^9 \times 4\pi \times 10^{-7} \times 1000 \times 0.25 \times (1+j)}$$

$$= \sqrt{j\, 986960\, (1+j)}$$

$= 1181.428 \angle 67.5^\circ$

V.13

Attenuation constant $\alpha = w\sqrt{\frac{\mu\varepsilon}{2}\left[\left(1+\frac{\sigma^2}{2w^2\varepsilon^2}\right)-1\right]}$

$$\frac{\sigma}{w\varepsilon} = \frac{0.25}{\pi \times 10^9 \times \frac{9}{36\pi \times 10^9}} = 1$$

$$\alpha = \pi \times 10^9 \sqrt{\frac{4\pi \times 10^{-7} \times 1000 \times \frac{9}{36\pi \times 10^9}}{2}\left[1+\frac{1}{2}-1\right]}$$

$$\boxed{\alpha = 496.73}$$

---

2. A lossy dielectric is characterized by $\varepsilon_r = 2.5$, $\mu_r = 4$ & $\sigma = 10^{-3}$ mho/m at 10MHz. Let $E = 10e^{-Vz}\vec{a_x}$ V/m. find i) α, ii) β, iii) λ, iv) V_p v) η?

Given

$\sigma = 10^{-3}$ mho/m

$\varepsilon_r = 2.5$

$\mu_r = 4$

$E = 10e^{-Vz}\vec{a_x}$

$f = 10$ MHz

$$\frac{\sigma}{w\varepsilon} = \frac{10^{-3}}{2\pi \times 10 \times 10^6 \times \frac{2.5}{36\pi \times 10^9}}$$

$= 0.72$

$\frac{\sigma}{w\varepsilon} \leq 1$

i) $\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$

$= \frac{10^{-3}}{2}\sqrt{\frac{4}{2.5}} \times 120\pi$

$= 238.43 \times 10^{-3}$

ii) $\beta = \omega\sqrt{\mu\varepsilon}\left[1+\frac{1}{2}\left(\frac{\sigma}{2\omega\varepsilon}\right)^2\right]$ (V.14)

$= 2\pi \times 10 \times 10^6 \sqrt{4\pi\times10^{-7} \times 4 \times \frac{2.5}{36\pi\times10^9}}\left[\left(1+\frac{1}{2}\left(\frac{0.72}{2}\right)^2\right]\right.$

$= 0.6623 \times 1.0648$

$= 0.7052$

iii) $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.7052} = 8.91m$

iv) $V_p = f\lambda$

$= 10\times10^6 \times 8.91$

$= 89.1\times10^6$ m/sec

v) $\eta = \sqrt{\frac{\mu}{\varepsilon}}$

$= \sqrt{\frac{\mu_r}{\varepsilon_r}} \times 120\pi$

$= \sqrt{\frac{4}{2.5}} \times 120\pi$

$= 476.86$ ohms

---

3. Determine the propagation constant for material with $\mu_r = 1$ and $\varepsilon_r = 8$ and conductivity $\sigma = 0.25$ PS/m and wave at frequency 1.6 MHz.

sol. $\gamma = \sqrt{j\omega\mu(\sigma + j\omega)\varepsilon}$

$= \sqrt{j\omega\mu(0.25 + j(1.6\times10^{-6})\varepsilon_0\varepsilon_r}$

$= \sqrt{j(1.6\times10^{-6})(4\pi\times10^{-7})(0.25+j(1.6\times10^{-6})(8)(8.854\times10^{-12})}$

$= \sqrt{j(2\times10^{-12})[0.25+j(1.133)\times10^{-16}}$

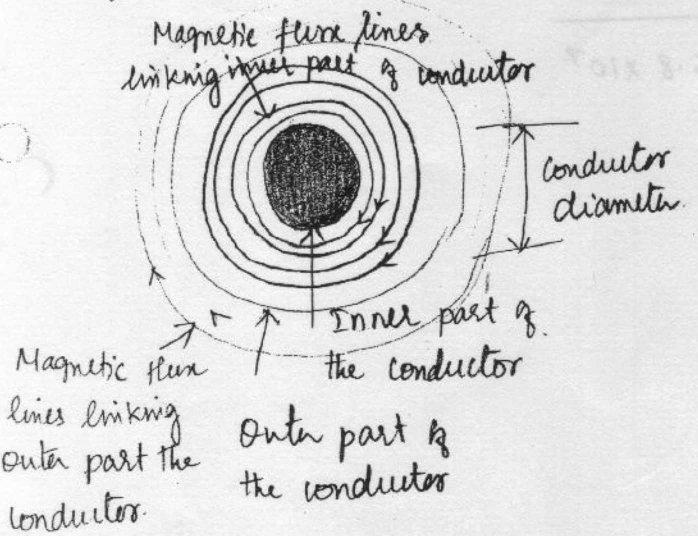
$= \sqrt{j5\times10^{-13} - (2.66\times10^{-28})}$

$\boxed{\gamma = 7.07\times10^{-7} m^{-1}}$

DEPTH OF PENETRATION

* The depth of penetration (δ) is defined as that depth in which the wave has been attenuated to 1/e or approximately 37 percent of its original value. It is also known as skin effect.

* The alternating current flows more easily through outer portion of a conductor than through inner portion. This effect effect is called skin effect.



* The amplitude of the wave decreases by the factor $e^{-\alpha x}$ as it propagates through a distance x.

By definition

$$e^{-\alpha x} = \frac{1}{e}$$

$$e^{-\alpha \delta} = e^{-1}$$

$$\alpha \delta = 1$$

$$\boxed{\delta = \frac{1}{\alpha}}$$ → This is the depth of peneration (or) skin effect

$$\delta = \frac{1}{\alpha}$$

$$= \frac{1}{\omega \sqrt{\frac{\mu \epsilon_1}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2 \epsilon_1^2}} - 1\right)}}$$

For a good conductor the depth of peneration is

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega \mu \sigma}}$$

Problem 1

Find the depth of penetration of a plane wave in copper at a power frequency of 60 Hz and at microwave frequency 10^{10} Hz. Given $\sigma = 5.8 \times 10^7$ mho/m.

sol.

Depth of penetration $\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega \mu \sigma}}$

i) $f = 60\,Hz$, $\sigma = 5.8 \times 10^7$ mho/m, $\mu_r = 1$

$$\delta = \sqrt{\frac{2}{2\pi \times 60 \times 4\pi \times 10^{-7} \times 5.8 \times 10^7}}$$

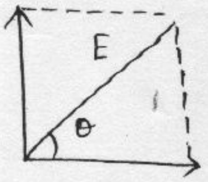
$$\delta = 8.53 \times 10^{-3}\, m$$

POLARIZATION

* The polarization of a uniform plane wave refers to the time-varying behaviour of the electric field strength vector at some fixed point in space.

* Consider a uniform plane wave travelling in the Z-direction, with E & H lying in x-y plane. If $E_y = 0$ & E_x only is present, the wave is said to be polarized in the x-direction. Similarly $E_x = 0$ & only E_y is present, the wave is said to be polarized in the y direction.

Linear polarization



$$\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right)$$

$$E = \sqrt{E_x^2 + E_y^2}$$

Fig. Linear polarization

* If both E_x & E_y are present and are in phase, the resultant electric field has a direction at an angle of $\tan^{-1}(E_y / E_x)$.

* If the direction of the resultant vector is constant with time, the wave is said to be linearly polarized.

Circular Polarization:

* If E_x and E_y have equal magnitudes and a $\pi/2$ phase difference, the locus of the resultant E is a circle and the wave is said to be circularly polarized.

* E_x & E_y have the same magnitude E_a & differ $90°$ in phase.

The resultant electrifield in vector form is

$$E = \vec{a_x} Ea + j \vec{a_y} Ea$$

The corresponding time varying field is

$$E = \vec{a_x} Ea \cos\omega t \mp \vec{a_y} Ea \sin\omega t$$

The components are

$$Ex = Ea \cos\omega t$$

$$Ey = -Ea \sin\omega t$$

then

$$Ex^2 + Ey^2 = Ea^2.$$

This equation shows that the locus of the resultant E is a circle whose radius is Ea.

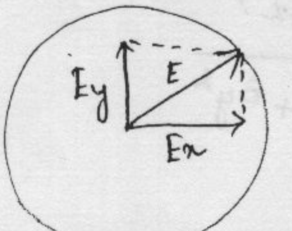


Fig. Circular polarization

Elliptical polarization:

Ex and Ey have different amplitudes and $\pi/2$ phase difference, the locus of the resultant E is an ellipse and the wave is said to be elliptically polarized.

Let Ex has the magnitude A and Ey has the magnitude B and differ 90^0 in phase.

The resultant electric field in vector form is

$$E = \vec{a_x} A + j \vec{a_y} B.$$

The corresponding time varying field is

$$E = \vec{a_x} A \cos\omega t - \vec{a_y} B \sin\omega t$$

The components are

$$Ex = A \cos\omega t$$

$$E_y = -B \sin\omega t$$

Then,

$$\frac{E_x}{A} = \cos\omega t$$

$$\frac{E_y}{B} = -\sin\omega t$$

$$\frac{E_x^2}{A^2} + \frac{E_y^2}{B^2} = 1$$

This equation shows that the locus of the resultant E is an ellipse. it is shown in fig.

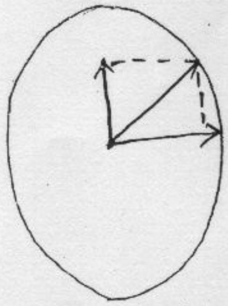


fig. Elliptical polarization.

Reflection and Refraction of plane waves

Reflection by a perfect conductor

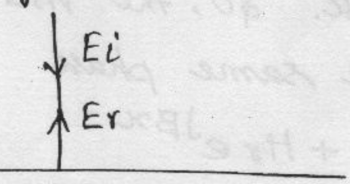
* when the electromagnetic wave travelling in one medium strikes upon a second medium, the wave will be partially transmitted and partially reflected.

* It depends upon types of incidence. The two types of incidence are normal and oblique.

i) wave incident normally on a perfect conductor:

* when the plane wave incident normally upon the surface of a perfect conductor, the wave is entirely reflected

* Since there can be no loss within a perfect conductor, none of the energy is absorbed.



Normal Incidence

* As a result the amplitudes of E and H in the incident wave are the same as in the reflected wave and differ by π (out of phase). i.e $E_i = -E_r$

* Let the Electric field of the incident wave is $E_i e^{\gamma x}$. since attenuation is zero ($\alpha = 0$)

   propagation constant, $\gamma = j\beta$. then incident wave is $E_i e^{-j\beta x}$ and reflected wave is $E_r e^{+j\beta x}$ (opposite direction)

* The resultant electric field is the sum of the Electric field of incident and reflected waves.

$$E_T(x) = E_i e^{-j\beta x} + E_r e^{j\beta x}$$

But, $E_i = -E_r$

$$E_T(x) = E_i\left[e^{-j\beta x} - e^{j\beta x}\right]$$

$$\boxed{\sin\beta x = \frac{e^{-j\beta x} - e^{j\beta x}}{2j}}$$

$$= -j2E_i \sin\beta x$$

Expressing in time variation

$$E_T(x,t) = -2jE_i \sin\beta x\, e^{j\omega t}$$

If E_i is chosen to be real

$$\boxed{E_T(x,t) = 2E_i \sin\beta x \sin\omega t}$$

This equation shows that the incident and reflected waves combine to produce a standing wave, which does not progress.

* In order to maintain the reversal of direction of energy propagation magnetic field must be reflected without reversal of phase. So, the incident H_i and reflected H_r are in the same phase

$$H_T(x) = H_i e^{-j\beta x} + H_r e^{j\beta x}$$

$$= H_i\left(e^{-j\beta x} + e^{j\beta x}\right) \qquad \because H_i = H_r$$

$$= 2H_i(\cos\beta x)$$

H_i is real

$$H_T(x,t) = R_e\left[2H_i \cos\beta x\, e^{j\omega t}\right]$$

$$\boxed{H_T(x,t) = 2H_i \cos\beta x \cos\omega t}$$

This equation shows that the magnetic field H has a standing wave distribution. These two equations indicate that the E and H are differ $\pi/2$ in phase.

ii) <u>wave incident obliquely on a perfect conductor:</u>

* when a wave is incident obliquely on a perfect

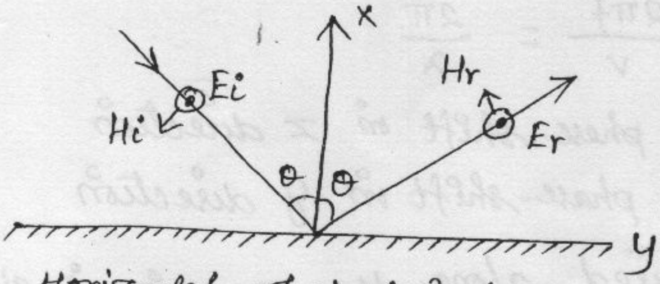
conductor, it is necessary to consider two special cases. In first case the electric vector is parallel to the boundary surface or perpendicular to the plane of incidence. This case is termed horizontal polarization. (1.22)

* In the second case the electric vector is parallel to the plane of incidence. This case is termed vertical polarization.

Horizontal Polarization

* Electric field E is perpendicular to the plane of incidence in this case.

* Let the incident and reflected waves makes angles $\theta i = \theta r = \theta$ with the z-axis as shown in fig.



Horizontal polarization (Perpendicular polarization)

The incident wave is expressed as

$$E_{in} = E_i e^{-j\beta \bar{n}.r}$$

For the normal of the incident wave

$$\bar{n}.r = x \cos\frac{\pi}{2} + y\cos\left(\frac{\pi}{2}-\theta\right) + z\cos(\pi-\theta)$$

$$= y \sin\theta - z\cos\theta$$

$$E_{in} = E_i e^{-j\beta(y\sin\theta - z\cos\theta)}$$

The reflected wave is expressed as

$$E_{ref} = E_r e^{-j\beta \bar{n}.r}$$

For the normal of the incident wave

$$\bar{n}.r = x\cos\frac{\pi}{2} + y\cos\left(\frac{\pi}{2}-\theta\right) + z\cos\theta$$

$$= y\sin\theta + z\cos\theta$$

$$E_{ref} = E_r e^{-j\beta(y\sin\theta + z\cos\theta)}$$ (V.23)

But $E_{ref} = -E_i$

The total Electric field

$$E_T = E_{in} + E_{ref}$$

$$= E_i\left[e^{-j\beta(y\sin\theta - z\cos\theta)} - e^{-j\beta(y\sin\theta + z\cos\theta)}\right]$$

$$= E_i\left[e^{j\beta z\cos\theta} - e^{-j\beta z\cos\theta}\right]e^{-j\beta y\sin\theta}$$

$$= 2jE_i\sin(\beta z\cos\theta)e^{-j\beta y\sin\theta}$$

$$E_T = 2jE_i\sin\beta_z z\, e^{-j\beta_y y}$$

$$E_T = 2jE_i\sin\beta_z z\, e^{j\beta_y y}$$

where, $\beta = \frac{\omega}{v} = \frac{2\pi f}{v} = \frac{2\pi}{\lambda}$

$\beta_z = \beta\cos\theta$, phase-shift in z direction

$\beta_y = \beta\sin\theta$, phase-shift in y direction

The wavelength, measured along the z-axis is given by

$$\lambda_z = \frac{2\pi}{\beta_z}$$

$$= \frac{2\pi}{\beta\cos\theta}$$

$$= \frac{\lambda}{\cos\theta}$$

$$\lambda_z = \frac{\lambda}{\cos\theta}$$

The velocity in y direction

$$V_y = \frac{\omega}{\beta_y}$$

$$= \frac{\omega}{\beta\sin\theta}$$

$$V_y = \frac{v}{\sin\theta}$$

The corresponding wave length

$$\boxed{\lambda_y = \frac{\lambda}{\sin\theta}}$$

Vertical Polarization

The electric field E is parallel to the plane of incidence. Let the incident and reflected waves (H_i and H_r) make angles $\theta_i = \theta_r = \theta$ as shown in fig.

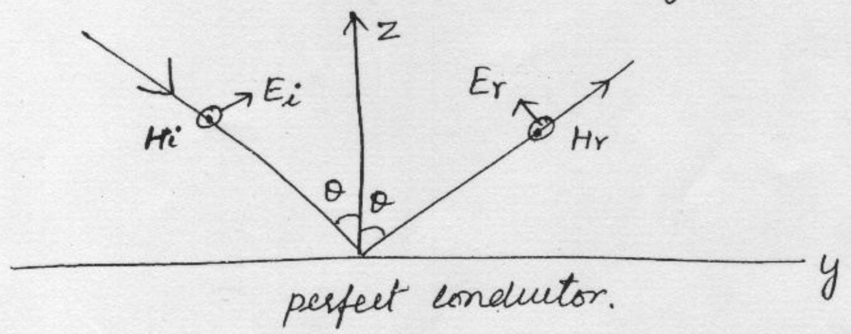


Fig. Vertical polarization (parallel polarization)

The incident wave is expressed as

$$H_{in} = H_i e^{-j\beta(y\sin\theta - z\cos\theta)}$$

& reflected waves are

$$H_{ref} = H_r e^{-j\beta(y\sin\theta + z\cos\theta)}$$

since $H_{in} = H_{ref}$ (no phase reversal), the total magnetic field is

$$H_T = H_{in} + H_{ref}$$

$$H_T = H_i\left[e^{-j\beta(y\sin\theta - z\cos\theta)} + e^{-j\beta(y\sin\theta + z\cos\theta)}\right]$$

$$= H_i e^{j\beta y\sin\theta}\left[e^{-j\beta z\cos\theta} + e^{-j\beta z\cos\theta}\right]$$

$$= 2H_i \cos\beta z\cos\theta . e^{-j\beta y\sin\theta}$$

$$\boxed{H_T = 2H_i \cos\beta_z z\, e^{-j\beta_y y}}$$

where,

$$\beta_z = \beta\cos\theta$$

$$\beta_y = \beta\sin\theta$$

Reflection by a perfect dielectric

(V. 25)

* When a plane electromagnetic wave is incident on the surface of a perfect dielectric, part of the energy is transmitted and part of it is reflected.

* A perfect dielectric is one with zero conductivity, so that there is no loss or absorption of power in propagation through the dielectric.

* Consider two case
  i) wave incident normally
  ii) wave incident obliquely.

i) Wave Incident normally on a perfect dielectric

* Consider two perfect dielectric media separated by a boundary as shown in fig.

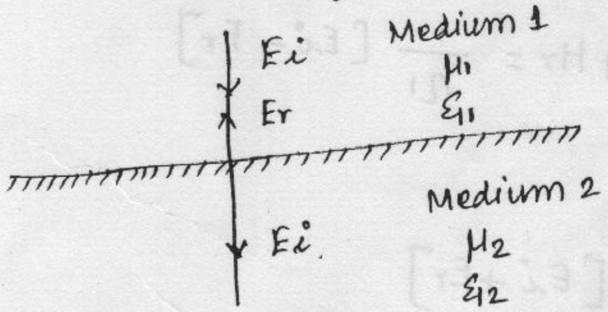


fig. Normal Incidence on a perfect dielectric

* Let ε_1 and μ_1 are the permittivity and permeability of medium 1 respectively.
* Let ε_2 and μ_2 are the permittivity and permeability of medium 2 respectively.
* Let E_i be the electric field of incident wave, E_r be the electric field of reflected wave and at E_t be the electric field of transmitted (refracted) wave.
* Let H_i, H_r and H_t be the magnetic field of incident, reflected and transmitted wave respectively.

* The intrinsic impedance of medium 1 is $\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}$ and that of medium 2 is $\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$

$$E_i = \eta_1 H_i$$

$$E_r = -\eta_1 H_r$$

$$E_t = \eta_2 H_t$$

* According to boundary condition, the tangential component of E or H is continuous across the boundary

$$H_i + H_r = H_t$$

$$\& \quad E_i + E_r = E_t$$

from above equations

$$H_i = \frac{E_i}{\eta_1}, \quad H_r = -\frac{E_r}{\eta_1} \quad \& \quad H_t = \frac{E_t}{\eta_2},$$

$$H_t = H_i + H_r = \frac{1}{\eta_1}[E_i - E_r]$$

$$H_t = \frac{E_t}{\eta_2}$$

$$H_t = \frac{1}{\eta_2}[E_i + E_r]$$

Equating H_t equations

$$\frac{1}{\eta_1}[E_i - E_r] = \frac{1}{\eta_2}[E_i + E_r]$$

$$\eta_2[E_i - E_r] = \eta_1[E_i + E_r]$$

$$E_i[\eta_2 - \eta_1] = E_r[\eta_1 + \eta_2]$$

Reflection Coefficient

$$\boxed{\frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}}$$

Also $\quad \frac{E_t}{E_i} = \frac{E_i + E_r}{E_i} = 1 + \frac{E_r}{E_i}$

$$= 1 + \frac{\eta_2 - \eta_1}{\eta_2 - \eta_1}$$

$$= \frac{\eta_1 + \eta_2 + \eta_2 - \eta_1}{\eta_1 + \eta_2}$$

Transmission coefficient $\Bigg\}$ $\boxed{\frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1+\eta_2}}$

Similarly for magnetic field

$$\frac{H_r}{H_i} = -\frac{E_r}{E_i}$$

$$= -\left[\frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}\right]$$

$$\boxed{\frac{H_r}{H_i} = \frac{\eta_1 - \eta_2}{\eta_2 + \eta_1}}$$

$$\frac{H_t}{H_i} = \frac{\eta_1}{\eta_2}\frac{E_t}{E_i}$$

$$= \frac{\eta_1}{\eta_2}\left[\frac{2\eta_2}{\eta_1 + \eta_2}\right]$$

$$\frac{H_t}{H_i} = \left[\frac{2\eta_1}{\eta_1 + \eta_2}\right]$$

* Since the permeabilities of perfect dielectrics do not differ appreciably from that of free space

$\mu_1 = \mu_2 = \mu_0$

Then $\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_1}}$ and

$$\eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_2}}$$

$$\frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$= \frac{\sqrt{\frac{\mu_0}{\varepsilon_2}} - \sqrt{\frac{\mu_0}{\varepsilon_1}}}{\sqrt{\frac{\mu_0}{\varepsilon_1}} + \sqrt{\frac{\mu_0}{\varepsilon_2}}}$$

$$= \frac{\sqrt{\frac{1}{\varepsilon_2}} - \sqrt{\frac{1}{\varepsilon_1}}}{\sqrt{\frac{1}{\varepsilon_1}} + \sqrt{\frac{1}{\varepsilon_2}}}$$

$$\boxed{\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}}$$

Similarly $\frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1 + \eta_2}$

$$= \frac{2\sqrt{\frac{\mu_0}{\varepsilon_2}}}{\sqrt{\frac{\mu_0}{\varepsilon_1}} + \sqrt{\frac{\mu_0}{\varepsilon_2}}}$$

$$\boxed{\frac{E_t}{E_i} = \frac{2\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}}$$

$$\frac{H_r}{H_i} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}$$

$$= \frac{\sqrt{\frac{\mu_0}{\varepsilon_1}} - \sqrt{\frac{\mu_0}{\varepsilon_2}}}{\sqrt{\frac{\mu_0}{\varepsilon_1}} + \sqrt{\frac{\mu_0}{\varepsilon_2}}}$$

$$\boxed{\frac{H_r}{H_i} = \frac{\sqrt{\varepsilon_2} - \sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}}$$

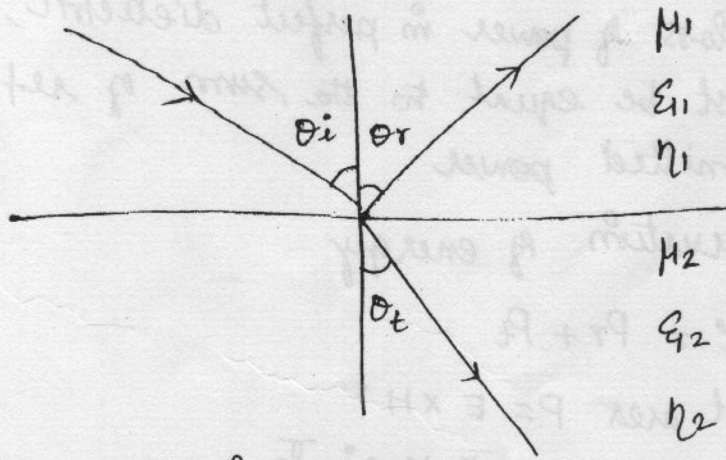
$$\frac{H_t}{H_i} = \frac{2\eta_1}{\eta_1 + \eta_2}$$

$$= \frac{2\sqrt{\frac{\mu_0}{\varepsilon_1}}}{\sqrt{\frac{\mu_0}{\varepsilon_1}} + \sqrt{\frac{\mu_0}{\varepsilon_2}}}$$

$$\frac{H_t}{H_i} = \frac{2\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$$

ii) wave Incident obliquely on a perfect Dielectric:

* when a plane electromagnetic wave is incident obliquely on the boundary, a part of the wave transmitted and part of it reflected, but in this case transmitted wave will be refracted. i.e the direction of propagation will be changed.



* when the wave is incident obliquely at an angle of θ_i with normal part of the wave reflected at an angle of θ_r in the same medium and part of it transmitted (refracted) at angle of θ_t in second medium as shown in fig.

By Snell's law

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{v_1}{v_2}$$

where, v_1 is the velocity of wave in medium 1

v_2 is the velocity of wave in medium 2

$$v_1 = \frac{1}{\sqrt{\mu_1 \varepsilon_{1}}} \quad \& \quad v_2 = \frac{1}{\sqrt{\mu_2 \varepsilon_{2}}}$$

then

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{\sqrt{\mu_2 \varepsilon_{2}}}{\sqrt{\mu_1 \varepsilon_{1}}}$$

Since the permeability of the dielectrics do not vary much from that of free space.

$$\mu_1 = \mu_2 = \mu_0$$

substitude in above equation

$$\frac{\sin\theta_i}{\sin\theta_r} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

* Since there is no loss of power in perfect dielectric, the incident power must be equal to the sum of reflected power and transmitted power

By the conservation of energy

$$P_i = P_r + P_t$$

The power / unit area $P = E \times H$

$$= E.H \sin\frac{\pi}{2}$$

$$= E.H$$

$$= \frac{E^2}{\eta}$$

$$P_i = E_i H_i \cos\theta_i$$

$$P_i = \frac{E_i^2}{\eta_1}\cos\theta_i$$

$$P_r = \frac{E_r^2}{\eta_1} \cos\theta_r$$

$$P_t = \frac{E_t^2}{\eta_2} \cos\theta_t$$

$$\boxed{\frac{E_i^2}{\eta_1}\cos\theta_i = \frac{E_r^2}{\eta_1}\cos\theta_r + \frac{E_t^2}{\eta_2}\cos\theta_t}$$

* By law of reflection, the angle of incidence is equal to the angle of reflection

$$\theta_i = \theta_r$$

$$\frac{E_i^2}{\eta_1}\cos\theta_i = \frac{E_r^2}{\eta_1}\cos\theta_r + \frac{E_t^2}{\eta_2}\cos\theta_t$$

$$\frac{\cos\theta_i}{\eta_1}\left[E_i^2 - E_r^2\right] = \frac{E_t^2}{\eta_2}\cos\theta_t$$

Divide by E_i^2 on both sides

$$\frac{\cos\theta_i}{\eta_1}\left[1 - \frac{E_r^2}{E_i^2}\right] = \frac{1}{\eta_2}\frac{E_t^2}{E_i^2}\cos\theta_t$$

$$1 - \frac{E_r^2}{E_i^2} = \frac{\eta_1}{\eta_2}\frac{E_t^2}{E_i^2}\frac{\cos\theta_t}{\cos\theta_i}$$

$$\frac{E_r^2}{E_i^2} = 1 - \frac{\eta_1 E_t^2 \cos\theta_t}{\eta_2 E_i^2 \cos\theta_i}$$

$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} \text{ and } \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$$

$$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_1}} \text{ and } \eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_r}} \quad [\because \mu_1 = \mu_2 = \mu_0]$$

$$\boxed{\frac{Er^2}{Ei^2} = 1 - \frac{\sqrt{\varepsilon_2}\, Et^2 \cos\theta t}{\sqrt{\varepsilon_1}\, Ei^2 \cos\theta i}}$$

Horizontal polarization (perpendicular polarization)

In this case, electric field E is perpendicular to the plane of incidence and parallel to the reflecting surface.

By applying the boundary condition that the tangential component of E is continuous across the boundary i.e E in one medium is same as E in other medium.

$$Ei + Er = Et$$

$$1 + \frac{Er}{Ei} = \frac{Et}{Ei}$$

But

$$\frac{Er^2}{Ei^2} = 1 - \frac{\sqrt{\varepsilon_2}\, Et^2 \cos\theta t}{\sqrt{\varepsilon_1}\, Ei^2 \cos\theta i}$$

$$= 1 - \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}} \left(1 + \frac{Er}{Ei}\right)^2 \frac{\cos\theta t}{\cos\theta i}$$

$$1 - \left(\frac{Er}{Ei}\right)^2 = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \left(1 + \frac{Er}{Ei}\right)^2 \frac{\cos\theta t}{\cos\theta i}$$

$$\left(1 - \frac{Er}{Ei}\right)\left(1 + \frac{Er}{Ei}\right) = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \left(1 + \frac{Er}{Ei}\right)^2 \frac{\cos\theta t}{\cos\theta i}$$

$$1 - \frac{Er}{Ei} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \left(1 + \frac{Er}{Ei}\right) \frac{\cos\theta t}{\cos\theta i}$$

$$\frac{Er}{Ei}\left[1 + \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \frac{\cos\theta t}{\cos\theta i}\right] = 1 - \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \frac{\cos\theta t}{\cos\theta i}$$

$$\frac{Er}{Ei} = \frac{1 - \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \frac{\cos\theta t}{\cos\theta i}}{1 + \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \frac{\cos\theta t}{\cos\theta i}}$$

$$= \frac{\sqrt{\varepsilon_1}\cos\theta_i - \sqrt{\varepsilon_2}\cos\theta_t}{\sqrt{\varepsilon_1}\cos\theta_i + \sqrt{\varepsilon_2}\cos\theta_t}$$

$\sqrt{\varepsilon_2}\cos\theta_t$ can be written as

$$\sqrt{\varepsilon_2}\cos\theta_t = \sqrt{\varepsilon_2}\sqrt{1-\sin^2\theta_t}$$

But $\dfrac{\sin\theta_i}{\sin\theta_t} = \dfrac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}$

$$\sin^2\theta_t = \frac{\varepsilon_1 \sin^2\theta_i}{\varepsilon_2}$$

Substituting this value in above equation

$$\sqrt{\varepsilon_2}\cos\theta_t = \sqrt{\varepsilon_2}\sqrt{1-\frac{\varepsilon_1 \sin^2\theta_t}{\varepsilon_2}}$$

$$= \sqrt{\varepsilon_2 - \varepsilon_1 \sin^2\theta_i}$$

Substituting this value in $\frac{E_r}{E_i}$ equation

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_1}\cos\theta_i - \sqrt{\varepsilon_2-\varepsilon_1\sin^2\theta_i}}{\sqrt{\varepsilon_1}\cos\theta_i + \sqrt{\varepsilon_2-\varepsilon_1\sin^2\theta_i}}$$

Reflection co-efficient is given by

$$\frac{E_r}{E_i} = \frac{\cos\theta_i - \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}}$$

* This gives the ratio of reflected to incident electric field for horizontally polarized wave. It is the reflection co-efficient for horizontal polarization.

Vertical polarization (Parallel Polarization)

* In this case electric field E is parallel to the plane of incidence. By applying the boundary conditions that the tangential component of E is continuous across the boundary.

from the fig.

$$(E_i - E_r)\cos\theta_i = E_t \cos\theta_t$$

$\div$ Ei on both sides

$$1 - \frac{E_r}{E_i} = \frac{E_t}{E_i}\frac{\cos\theta_t}{\cos\theta_i}$$

$$\frac{E_t}{E_i} = \left(1 - \frac{E_r}{E_i}\right)\frac{\cos\theta_i}{\cos\theta_t}$$

But

$$\frac{E_r^2}{E_i^2} = 1 - \frac{\sqrt{\varepsilon_2}\,E_t^2\cos\theta_t}{\sqrt{\varepsilon_1}\,E_i^2\cos\theta_i}$$

sub. the value of $\frac{E_t}{E_i}$ in above equation

$$= 1 - \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\left(1 - \frac{E_r}{E_i}\right)^2\frac{\cos\theta_i}{\cos\theta_t}$$

$$1 - \left(\frac{E_r}{E_i}\right)^2 = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\frac{\cos\theta_i}{\cos\theta_t}\left(1 - \frac{E_r}{E_i}\right)^2$$

$$\left(1 - \frac{E_r}{E_i}\right)\left(1 + \frac{E_r}{E_i}\right) = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\frac{\cos\theta_i}{\cos\theta_t}\left(1 - \frac{E_r}{E_i}\right)$$

$$\frac{E_r}{E_i}\left[1 + \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\frac{\cos\theta_i}{\cos\theta_t}\right] = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\frac{\cos\theta_i}{\cos\theta_t} - 1$$

$$\frac{E_r}{E_i}\left[\frac{\sqrt{\varepsilon_1}\cos\theta_t + \sqrt{\varepsilon_2}\cos\theta_i}{\sqrt{\varepsilon_1}\cos\theta_i}\right] = \frac{\sqrt{\varepsilon_2}\cos\theta_i - \sqrt{\varepsilon_1}\cos\theta_t}{\sqrt{\varepsilon_1}\cos\theta_i}$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_2}\cos\theta_i - \sqrt{\varepsilon_1}\cos\theta_t}{\sqrt{\varepsilon_2}\cos\theta_i + \sqrt{\varepsilon_1}\cos\theta_t}$$

But $\cos\theta_t = \sqrt{1-\sin^2\theta_t}$

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_2}\cos\theta_i - \sqrt{\varepsilon_1(1-\sin^2\theta_t)}}{\sqrt{\varepsilon_2}\cos\theta_i + \sqrt{\varepsilon_1(1-\sin^2\theta_t)}}$$

But

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}$$

$$\sin\theta_t = \frac{\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_2}}\sin\theta_i$$

$$\sin^2\theta_t = \frac{\varepsilon_1}{\varepsilon_2}\sin^2\theta_i$$

sub,

$$\frac{E_r}{E_r} = \frac{\sqrt{\varepsilon_2}\cos\theta_i - \sqrt{\varepsilon_1 - \frac{\varepsilon_1^2}{\varepsilon_2}\sin^2\theta_i}}{\sqrt{\varepsilon_2}\cos\theta_i + \sqrt{\varepsilon_1 - \frac{\varepsilon_1^2}{\varepsilon_2}\sin^2\theta_i}}$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_2}\cos\theta_i - \frac{1}{\sqrt{\varepsilon_2}}\sqrt{\varepsilon_1\varepsilon_2 - \varepsilon_1^2\sin^2\theta_i}}{\sqrt{\varepsilon_2}\cos\theta_i + \frac{1}{\sqrt{\varepsilon_2}}\sqrt{\varepsilon_1\varepsilon_2 - \varepsilon_1^2\sin^2\theta_i}}$$

$$= \frac{\sqrt{\varepsilon_2}\cos\theta_i - \frac{\varepsilon_1}{\varepsilon_2}\sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}}{\sqrt{\varepsilon_2}\cos\theta_i + \frac{\varepsilon_1}{\varepsilon_2}\sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}}$$

Dividing numerator & Denominator by $\frac{\sqrt{\varepsilon_2}}{\varepsilon_1}$, the reflection coefficient is given by

Reflection co-efficient $$\frac{E_r}{E_i} = \frac{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_i - \sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_i}}{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\cos\theta_i - \sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \sin^2\theta_i}}$$

* This equation gives that ratio of reflected to incident electric field for vertical polarized wave. It is nothing but a reflection coefficient for parallel or vertical polarization.

Brewster Angle:

* Brewster angle is a particular angle at which no reflection takes place. This occurs when the numerator of the above equation zero.

$$\frac{\varepsilon_2}{\varepsilon_1}\cos\theta_i - \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i} = 0$$

$$\frac{\varepsilon_2}{\varepsilon_1}\cos\theta_i - \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}$$

$$\frac{\varepsilon_2}{\varepsilon_1}\sqrt{(1-\sin^2\theta_i)} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}$$

square on both sides

$$\frac{\varepsilon_2^2}{\varepsilon_1^2}(1-\sin^2\theta_i) = \frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i$$

$$\frac{\varepsilon_2^2}{\varepsilon_1^2} - \frac{\varepsilon_2^2}{\varepsilon_1^2}\sin^2\theta_i = \frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i$$

$$\sin^2\theta_i\left(1 - \frac{\varepsilon_2^2}{\varepsilon_1^2}\right) = \frac{\varepsilon_2}{\varepsilon_1} - \frac{\varepsilon_2^2}{\varepsilon_1^2}$$

$$\sin^2\theta_i(\varepsilon_1^2 - \varepsilon_2^2) = \varepsilon_1\varepsilon_2 - \varepsilon_2^2$$

$$= \varepsilon_2(\varepsilon_1 - \varepsilon_2)$$

$$\sin^2\theta_i = \frac{\varepsilon_2(\varepsilon_1 - \varepsilon_2)}{\varepsilon_1^2 - \varepsilon_2^2}$$

$$\sin^2\theta_i = \frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}$$

$$\cos^2\theta_i = \frac{\varepsilon_1}{\varepsilon_1 + \varepsilon_2}$$

$$\tan^2\theta_i = \frac{\varepsilon_2}{\varepsilon_1}$$

$$\boxed{\tan\theta_i = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}}$$

$$\theta_i = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

* This is called Brewster angle at which there is no reflected wave when the incident wave is parallel polarized.

Total Internal Reflection:

* when a wave is incident from the denser medium into rarer medium at an angle equal to or greater than the critical angle, the wave will be totally internally reflected back. This phenomena is called Total internal reflection.

* If ε_1 is greater than ε_2 both the reflection coefficients for vertical and horizontal polarizations become complex when $\sin\theta_i > \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$

At critical angle the reflection coefficients have unity value.

$$\frac{E_r}{E_i} = \frac{\cos\theta_i - \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i}} = 1$$

But, $\theta_i = \theta_c$ (critical angle)

$$\cos\theta_c - \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_c} = \cos\theta_c + \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_c}$$

$$= \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_c} = 0$$

$$\sin^2\theta_c = \frac{\varepsilon_2}{\varepsilon_1}$$

$$\boxed{\theta_c = \sin^{-1}\sqrt{\varepsilon_2/\varepsilon_1}}$$ (v.38')

* This is the critical angle at which there is no refracted (transmitted wave), i.e the wave traversed along the boundary surface.

UNIT - V (2 Marks)

1. Define a wave.

If a physical phenomenon that occurs at one place at a given time is reproduced at other places at later times, the time delay being produced to the space separation from the first location, then the group of phenomena constitutes a wave.

2. Mention the properties of uniform plane wave.

The properties of uniform plane wave are as follows.

1. At every point in space, the electric field E and Magnetic field H are perpendicular to each other and to the direction of the travel.
2. The fields vary harmonically with time and at the same frequency, every where in space.
3. Each field has the same direction, magnitude and phase at every point in any plane perpendicular to the direction of wave travel.

3. Define intrinsic impedance or characteristic impedance.

It is the ratio of electric field to magnetic field. or It is the ratio of square root of permeability to permittivity of the medium.

$$\eta = \frac{E}{H} = \sqrt{\frac{\mu}{\epsilon}} \text{ ohms.}$$

4. Define Propagation constant.

The propagation constant (γ) is a complex number, & it is given by

$$\gamma = \alpha + j\beta$$

where α is attenuation constant

β is phase constant

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

5.) Define skin depth or depth of penetration.

Skin depth or depth of peneration (δ) is defined as that of depth in which the wave has been attenuated to $1/e$ or approximately 37% of its original value

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{j\omega\sigma}} \text{ for good conductor.}$$

6) Define polarization.

Polarization of a uniform plane wave refers to the time varying nature of the electric field vector at some fixed point in space.

7) what is Brewster angle?

Brewster angle is an incident angle at which there is no reflected wave for parallely polarized wave.

$$\theta = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

8. State slepian vector.

Slepian vector is a vector which defined at every point such that its flux coming out of any volume is zero. ($\nabla \cdot S = 0$). Slepian vector is given by

$$S = \nabla \times (VH)$$

where, V is electric potential

H is magnetic field intensity